We wish to thank Mr. M. McKeown for his valuable assistance. This work was carried out under the auspices of the Atomic Energy Commission.

* Now at University of Illinois.
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Analytic Behavior of Heisenberg's S-Matrix

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IF scattering of a photon on a hydrogen atom is studied by usual quantum perturbation theory, the wave matrix takes the form[1]

$$(p\alpha'|\psi_1|p^0\alpha^0)=(p\alpha'|p^0\alpha^0)+\delta_+(\nu-\nu')(p\alpha'|r_1|p^0\alpha^0), \quad (1)$$

where p is the momentum, ν is the energy of outgoing photon, and $\nu'=E^0-H_S(\alpha')$ is determined by energy conservation. The "partial" S-matrix is defined by

$$(p\alpha'|S_1|p^0\alpha^0)=(p\alpha'|p^0\alpha^0)+\delta(\nu-\nu')(p\alpha'|r_1|p^0\alpha^0). \quad (2)$$

This should be distinguished from the "complete" S-matrix, which is defined only for energies large enough to ionize the atom and contains elements for electron photon scattering on a proton center. The form of r_1 will depend on the approximation used. For total energy E^0 near to $H_S(\alpha'')$, the energy of an excited state, it can be shown that the matrix element $(p\alpha'|r_1|p^0\alpha^0)$ taken on the energy shell is an analytic function of E^0 if $H_S(\alpha')<H_S(\alpha'')$, and has a singularity at $E^0=H_S(\alpha'')+a-ib$, where b is positive. As remarked by Wentzel,[2] b is related to the total emission probability from state (α'').

If $\alpha^0\alpha'\alpha''$ denote ground level, first excited level, etc., the matrix element $(p\alpha^0|r_1|p^0\alpha^0)$ for elastic scattering is an analytic function of E^0 even at the critical values $H_S(\alpha')$, but the matrix element $(p\alpha'|r_1|p^0\alpha^0)$ for inelastic scattering is zero for $E^0<H_S(\alpha')$ but non-zero and analytic for $E^0>H_S(\alpha')$. This has two consequences: the partial S-matrix goes into the complete S-matrix for high enough energies, and the partial S-matrix is not an analytic function of energy. If the complete S-matrix is analytically continued into the low energy region, it will satisfy the unitary condition $SS\dagger=I$; hence the partial S-matrix is not unitary.

It is desirable that a consistent physical picture be given by the complete S-matrix, and this requires an interpretation of the analytic continuation of matrix elements $(p\alpha'|r|p^0\alpha^0)$ in region $E^0<H_S(\alpha')$ where inelastic scattering is not possible, i.e., where $(p\alpha'|r_1|p^0\alpha^0)=0$. This is obtained by defining probability as $(p\alpha'|r|p^0\alpha^0)(p^0\alpha^0|r\dagger|p\alpha')$, where $r\dagger$ is the analytic continuation in the E^0 plane of the Hermitian conjugate to r taken for E^0 greater than ionization energy. Thus the probability is $|(p\alpha'|r|p^0\alpha^0)|^2$ for $E^0>H_S(\alpha')$, but because of a factor $(\nu')^{-\frac{1}{2}}$ in $(p\alpha'|r|p^0\alpha^0)$ is $-|(p\alpha'|r|p^0\alpha^0)|$ for $E^0<H_S(\alpha')$. In S-matrix theory we require the interpretation that a negative probability corresponds to a physically impossible process.

Since this work was completed a note by T. S. Chang[3] has appeared, mentioning negative probabilities in connection with the unitary condition on S, but without further details it is not possible to tell if his conclusions are the same as ours. A fuller account of the calculations reported here will be published later. I wish to thank Dr. N. Kemmer for advice and encouragement.

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Neutron Deficient Isotopes of Cerium and Lanthanum

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FAST particle fission of bismuth in this laboratory has produced neutron deficient isotopes in the region of cerium.[1] The studies described herein were undertaken partly to identify some of these.

On the neutron deficient side of stability, two radioactive isotopes of lanthanum assigned to La^{137} [2,3] and La^{138} [4] have been reported, but recent measurements[5] show La^{138} to be stable or extremely long-lived and present in naturally occurring lanthanum to the extent of 0.089 percent. Evidence is presented below for assigning the reported radioactive properties of La^{137} to La^{135} and those of La^{138} to La^{136}.

To prepare the lanthanum activities cesium in the form of $CsNO_3$ was irradiated with 30-Mev helium ions in the 60-inch cyclotron. The lanthanum fraction was separated and the decay curve resolved into two components of 2.1-hour and 19.5-hour half-lives. Both of these activities, therefore, must belong to isotopes of lanthanum of mass number 136 or lower.

A low resolution beta-ray spectrometer showed a positron of 0.82 Mev, decaying with the 2-hour half-life, the energy estimated from the visual end point of the spectrometer curve. No negative particles were detected. A Feather analysis of a beryllium absorption curve, taken soon after

TABLE I.

Isotope	Type of radiation	Half-life	Energy of radiation in Mev: Particle	Energy of radiation in Mev: Electromagnetic	Produced by	Cross section $\times 10^{24}$ cm^2
La^{137}		>400 yr.			decay of Ce^{137}	
La^{136}	β^+	2.1 hr.	0.84	None	$Cs^{133}(\alpha, n)$	0.0003*
La^{135}	K, γ	19.5 hr.	None	0.76	$Cs^{133}(\alpha, 2n)$	0.03*
Ce^{139}	K, γ, e^-	140 days	0.15	0.18 1.8	$La^{139}(d, 2n)$	0.09**
Ce^{137}	K, γ, e^-	36 hr.	0.18	0.28 0.75	$La^{139}(d, 4n)$	0.06† 0.002** 0.04††
Ce^{135}	β^+	16 hr.			$La^{139}(d, 6n)$	

* 30-Mev helium ions.
** 20-Mev deuterons.
† 40-Mev deuterons.
†† 60-Mev deuterons.

the bombardment, gave a range in beryllium of 310 mg/cm², corresponding to an energy of 0.84 Mev.

A beryllium absorption curve was taken about twenty-four hours after the bombardment, when the 2.1-hour activity had decayed out. This curve showed no particle radiation, but a 5.2-kev L x-ray component could be resolved from the harder electromagnetic radiation. The positron activity determined earlier therefore belongs entirely to the 2.1-hour activity. Earlier absorption curves taken with aluminum showed the presence of a 34-kev x-ray while a 0.76-Mev gamma-ray in low intensity was determined by lead absorption. All of the electromagnetic radiation noted soon after bombardment could be attributed to the 19.5-hour activity that was present; thus the 2.1-hour activity probably has no electromagnetic radiation. In the 19.5-hour activity x-rays were more abundant than gamma-rays by a factor of about 50, indicating that most of the electron capture decay goes directly to the ground state.

As indicated in Table I, the 2.1-hour activity was formed in about 1/100 the yield of the 19.5-hour activity. With the energy of helium ions employed (30 Mev), the $(\alpha, 2n)$ reaction is more prolific than the (α, n) reaction.[6] The order of the decay energies is also compatible with the assignment of the 2.1-hour period to La^{136} and the 19.5-hour period to La^{135}.

The radioactive isotope of cerium decaying by K-electron capture and assigned to mass number 137 is discussed below. The lanthanum daughter of this isotope was not definitely observed, and a minimum half-life could be set at 400 years, based on x-ray counting rates.

Cerium activities were produced by irradiating La_2O_3 with 20-Mev deuterons in the 60-inch cyclotron. A sample of the chemically separated cerium resolved into two components: 140-day Ce^{139} and a 36-hour activity assigned below to Ce^{137}. Pool and Krisberg[7] have recently reported work with Ce^{139} in which they found conversion electrons and gamma-radiation of 0.184 and 0.8 Mev, respectively. In the present study, the conversion electrons were confirmed, the low energy gamma-ray was determined as 0.18 Mev, but a value of 1.8 Mev was found for the hard component. The conversion electrons were determined as 0.15 Mev, both with the beta-ray spectrometer and by absorption in beryllium.

In another bombardment of La_2O_3, 40-Mev deuterons from the 184-inch cyclotron were employed. This bombardment produced the 36-hour activity in far greater yield compared to the 140-day activity than did the 20-Mev deuteron bombardment. These two cerium activities are therefore probably not isomers, and the most likely assignment for the 36-hour activity is Ce^{137}.

A sample containing the 36-hour activity showed a well defined conversion electron peak, with an energy of about 0.23 Mev. There was no positron activity with this isotope. A beryllium absorption curve indicated a range of about 45 mg/cm², corresponding to an energy of about 0.22 Mev. An aluminum absorption curve showed the presence of 34-kev x-rays while gamma-rays of 0.28 and 0.75 Mev were measured with lead absorbers.

A third bombardment of La_2O_3 was made, this time with 60-Mev deuterons, in an attempt to prepare the parent of La^{135} by the $d, 6n$ reaction. The 36-hour Ce^{137} activity was formed in such high yield that the Ce^{135} activity was completely masked in the gross decay curve, but a sample of the cerium activity placed on the beta-ray spectrometer showed positron activity. Because of the great background from scattered radiation, the positron energy could only be estimated as 0.4 Mev. Integration of the area under the positron peaks of successive beta-ray spectrometer curves taken for two days indicated a half-life of about 16 hours. It was impossible to determine any other characteristics of this isotope. The activity is assigned to Ce^{135} because it had not appeared in bombardments at lower energies, and 19.5-hour La^{135} could be shown to grow into the cerium fraction at a rate corresponding to a half-life for its parent of about 16 hours.

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Anomalous Values of the Thermionic Emission Constant

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THE theoretical value for the thermionic emission constant, A, in the Richardson-Dushman equation, $I=AT^2\exp(-e\phi/kT)$, is 120 amp./cm² deg.² In general, measured values are smaller than this figure. Recently two papers[1,2] have appeared in which an attempt is made to explain theoretically the anomalously large values of A reported for some metals, and in particular for nickel. Both of these authors have accepted the experimental value of Fox and Bowie[3] for nickel, $A=1380$ amp./cm² deg.² $\times(\phi=5.03$ ev). As is well known and indeed is stressed by Wohlfarth,[2] the emission constants are very sensitive to surface impurities. Later, measurements by Wahlin[4] under very exacting experimental conditions have resulted in values of $A=30$ amp./cm² deg.² and $\phi=4.61\pm0.05$ ev for nickel. If these values are assumed correct, then the emission constant for nickel is not greater than the theoretical value.

A similar situation exists in the case of platinum. The often quoted DuBridge[5] measurements gave $A=17{,}000$ amp./cm² deg.² and $\phi=6.27$ ev. Later measurements by