the λ_0-quanta. What results is that light goes by retarded waves, λ_0-quanta by advanced waves,[7] and the radiation resistance of both contribute positively. Thus an accelerating charge will emit light, but it is predestined that negative energy λ_0-quanta were coming toward it to be absorbed, still further increasing the radiation resistance. This avoids the divergent solutions only to predict observable advanced effects.

For these reasons it is better to restrict oneself to the case of a decaying f-function (distribution of λ) for which a consistent theory can be made. Then the modifications of classical electrodynamics will only appear at very small distances from a charge. On the other hand, these distances are well within the Compton wave-length so that modifications caused by quantum mechanics would in any case appear before the ones here discussed. There is, therefore, little reason to believe that the ideas used here to solve the divergences of classical electrodynamics will prove fruitful for quantum electrodynamics. Nevertheless, the corresponding modifications were attempted with quantum electrodynamics and appear to solve some of the divergence difficulties of that theory. This will be discussed in a future paper.

[7] This may be understood in that, as indicated above, the energy-absorbing walls of the box absorb retarded light waves, but cannot be presumed to absorb retarded λ_0-quanta. Instead, in fact, they spontaneously emit such waves (warming up in the process) and non-divergent solutions result only if they emit just exactly the λ_0-quanta which can later be absorbed by the accelerating charge at the center.

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Concentration of He^3 by Thermal Diffusion

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A thermal diffusion plant for the enrichment of He^3 has been constructed consisting of cylindrical columns followed by a hot wire column. In the case of the cylindrical columns excellent agreement with the theory of Jones and Furry has been obtained. In the case of the hot wire column, discrepancies exist between the observed performance and that predicted by theory. Under continuous operating conditions it was possible to produce with the expenditure of 16.6 kw, 14 std. cc of helium per day having a He^3/He^4 ratio of 0.0021 when well helium was used as a source of gas.

IN view of the great interest in He^3, both as a tool in superflow studies of liquid helium and as one of the simplest nuclei whose properties need to be determined, an investigation has been started to determine the effectiveness of thermal diffusion as a means of concentrating this isotope. A thermal diffusion plant was constructed consisting of two cylindrical columns followed by a hot wire column. While the general design was similar to that proposed by Jones and Furry,[1] the dimensions were somewhat different.

Figure 1 shows a schematic drawing of the arrangement of the columns. The concentric cylindrical tube sections, 1 and 2, consisted of electrically heated steel tubes surrounded by brass water jackets. The hot wire section (3) consisted of a fine platinum wire surrounded by a brass water jacket. Table I gives dimensions and miscellaneous operating details for the plant.

Well helium, $He^3/He^4 = 1.5 \times 10^{-7}$, is caused to circulate past the bottom of section 1. Since sections 1 and 2 are joined by a tube approximately 11 cm long and 6 cm in diameter, no special circulating system is needed at this point. The circulating system between sections 2 and 3 consists of 900 cm of 0.4-cm i. d. copper tubing in series with a small centrifugal blower. A continuously heated palladium thimble 10 cm long and

[1] R. C. Jones and W. H. Furry, Rev. Mod. Phys. 18, 151 (1946).

0.3 cm in diam. connected to the circulating line removes most of the hydrogen which is concentrated at the top of section 2. A second palladium thimble at the top of section 3 removes any hydrogen missed by the first thimble. The effectiveness of the columns in separating hydrogen from helium is such that without the palladium thimbles even a trace of hydrogen impurity concentrates so strongly at the top of the plant as to make the helium isotope separation process ineffective.

PERFORMANCE OF CYLINDRICAL AND HOT WIRE SECTIONS INDIVIDUALLY

Steady State Operation

If helium is removed at a constant rate σ from the top of a uniform column of length L while the isotopic composition is held constant at the bottom, the ratio of He^3 concentration at the top, c_f, to that at the bottom, c_i, is given by formula (231) of reference 1 as

$$c_f/c_i=(1+n)/(e^{-2AL(1+n)}+n), \qquad (A)$$

where $n=\sigma/H$ and A and H may be computed in terms of constants of the gas and dimensions of the column. For the columns and operations discussed here $n\ll 1$ and $2AL\gg 0$. Thus (A) reduces to (B)

$$c_f/c_i=1/n\fallingdotseq H/\sigma, \qquad (B)$$

from which H may be computed for comparison with theory inasmuch as c_f/c_i and σ may be determined experimentally. Table II shows pertinent data as well as a comparison between the experimentally and theoretically determined values for H for the two columns. The gas constants used were those cited by Jones and Furry,[1] except that α, the thermal diffusion factor, was taken as 0.059[2] rather than 0.0758. H (theoretical) was computed by the methods employed by Jones and Furry. Although the cylindrical column employed here consisted of two sections of different cross sections, it is assumed that the entire column has the cross section of the larger section inasmuch as the change in cross section has negligible effect on Eq. (A).

It is to be noted that there is good agreement between the experimentally and theoretically determined values of H for the cylindrical column. On the other hand, although the ratio of the experimentally determined values of H for the hot wire column at two pressures varies as the square of the pressure as predicted by theory, the absolute values are in serious disagreement. This disagreement will be discussed later.

TABLE I. Dimensions and operating details for thermal diffusion plant.

Type	Section 1 concentric cylinder	Section 2 concentric cylinder	Section 3 hot wire
Length (cm)	350	350	254
Hot wall			
Material	steel	steel	platinum
Radius (cm)	3.02	1.75	0.018
Temperature °K	600	600	1100
Power (kw)	10	5	1.6
Voltage	220 a.c.	220 a.c.	130 d.c.
Cold wall			
Material	brass	brass	brass
Radius (cm)	3.65	2.38	0.466*
Temperature °K	300	300	300

* This dimension was incorrectly reported in reference 2.

TABLE II. Experimental determination of H.

	Pressure atmospheres	σ Std. liters/day	c_f/c_i	H (std. liters/day) Experimental $H=\sigma c_f/c_i$	Theoretical
Hot wire column	7.8	0.050	330	16.5	9.5
	9.7	0.080	310	24.8	14.8
Cylindrical column	7.8	0.667	386	257	266*

* This value has been corrected for cylindricity by Jones and Furry's Eq. (100) assuming a Maxwellian gas. Without the correction the value would have been 247.

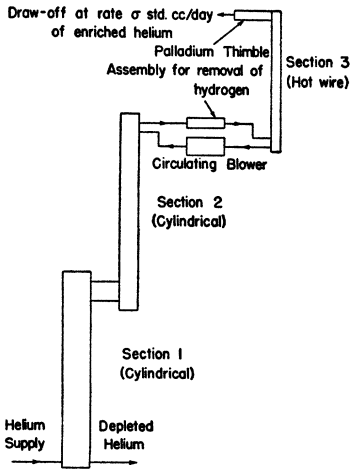

FIG. 1. Schematic diagram showing flow system for three-section thermal diffusion plant.

[2] B. B. McInteer, L. T. Aldrich, and A. O. Nier, Phys. Rev. 72, 510 (1947).

TABLE III. Experimental determination of $2A$.

	Cylindrical column Sections 1 and 2		Hot wire column Section 3	
Pressure (atmos.)	7.8	9.7	7.8	9.7
$(c_f/c_i)/c_i t$ (day^{-1})	60	69	80	112
μ (std. liters/cm)	0.039*	0.048	0.0033	0.0041
M (std. liters)	2.0	2.5	0.060	0.000
H (std. liters/day)	257	400**	16.5	24.8
$2A$ experimental (cm^{-1})	0.017	0.012	0.023	0.019
$2A$ theoretical (cm^{-1})	0.020***	0.014***	0.052	0.062

* These values for μ and the results for ($2A$) refer to section 2 while (H) is that for section 1.
** Calculated from the experimental result for H obtained for 7.8 atmos.
*** These values have been corrected for cylindricity by Jones' and Furry's Eqs. (100)-(108).

Time Rate of Increase of c_f with No Draw-Off ($\sigma=0$)

Another test of the performance of the columns which can be made conveniently is to observe the manner in which the concentration increases with time at the top ends with no draw-off, while the concentration at the lower ends is maintained constant. Such a test was made and for both the cylindrical columns (sections 1 and 2) and the hot wire column (section 3) a linear increase in concentration is observed as may be seen in Fig. 2.

Jones' and Furry's Eq. (384) may be modified slightly to give an approximate value of the slope which may then be compared with that observed here. Jones and Furry showed that for a column with no reservoir at the top, operated as were the columns in the present test, the initial concentration rise with time would be given by

$$c_f - c_i = Hc_i t/(\mu/2A), \qquad (C)$$

where μ = amount of gas contained in a unit length of column. This equation assumes that the entire transport of He^3 at the lower end, Hc_i, accumulates in a short length at the upper end which effectively acts as a reservoir containing an amount of gas $\mu/2A$ having a He^3 concentration c_f. Because of the fact that the actual columns employed had small reservoirs at the top, the equation was modified to include the gas stored in them. Thus one obtains

$$c_f - c_i = Hc_i t/((\mu/2A)+M), \qquad (D)$$

where M is the mass of gas stored in the small reservoir at the top of the column. This equation may be used to calculate $2A$ inasmuch as all of the other quantities in the equation may be determined experimentally. The value obtained may then be compared with that calculated directly from the constants of the gas and the dimensions of the column. Table III shows the results obtained. As was true in our determination of H, the agreement between experimental and theoretical values is good for the cylindrical column but poor for the hot wire column.

The reason for the disagreement in the latter case is by no means obvious. The low theoretical value for H could be due to the fact that α, at the higher temperature of the hot wire column, is more than was assumed in the computations. This would not, however, explain the low value of A actually observed unless imperfections in the column or other spurious effects reduced A more than enough to make up for the fact that the value of α is undoubtedly more than was assumed. Such a hypothesis is not completely unreasonable inasmuch as spurious effects are generally believed to reduce A more than H. In any event, really close agreement is hardly to be expected since the theory of the hot wire columns assumes that helium atoms behave as Maxwellian molecules, a very rough assumption.

Performance of Cylindrical and Hot Wire Sections in Series

From the data obtained in the previous sections a calculation may be made as to how the plant would operate under continuous draw-off conditions if all three sections are connected in series. Since the rate of draw-off is small and $2AL$ for each of the three sections is large, Eqs.

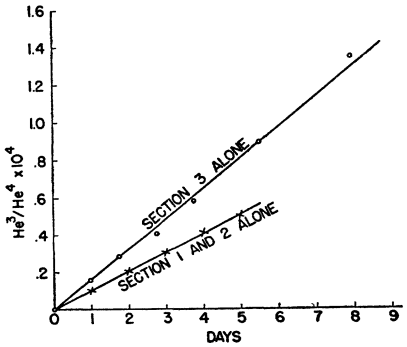

FIG. 2. Increase of He^3/He^4 ratio at tops of cylindrical column (sections 1 and 2) and hot wire column (section 3), when operated individually with no draw-off at top. He^3/He^4 concentration at bottoms maintained constant at 1.5×10^{-7} (well helium). Pressure = 9.7 atmospheres.

(A) and (B) may be assumed to hold for the plant as a whole or the hot wire section alone. Table IV gives the results of such a calculation for the plant operating at a pressure of 7.8 atmospheres and the supply concentration having an average He^3/He^4 ratio of 1.15×10^{-7}. This concentration corresponds to a removal of about 25 percent of the He^3 from the well helium supply.

A test was made with the columns operating at a pressure of 7.8 atmospheres and with a constant draw-off of 14 std. cc/day. After a steady state was reached, data were taken for a week of continuous operation. The concentrations observed at the top of the plant and at the top of section 2 were 2.11×10^{-3} and 1.9×10^{-6}. It is seen that these results are in excellent agreement with the corresponding figures in Table IV. In the course of the experiments a number of other results were obtained. For example, in one test it was found that the hot wire column alone could have a separation factor between its ends of at least 10^4. Since the production capacity of the plant is strictly limited by the H of section 1 (or, in words, the size of section 1), another manner of operation which was employed was to isolate section 3 from 1 and 2 and allow enriched He^3 to accumulate at the top of section 2 for several weeks. If the isolating valves were then opened, the concentration at the top of section 3 would rapidly rise to a high value. By this means it has been possible to produce several hundred standard cc of gas having a He^3/He^4 ratio of as much as 0.0086.

TABLE IV. Calculated performance of entire plant based on measurements on individual sections.

Rate of draw-off, σ std. cc/day	Concentration at top of plant	Concentration at top of section 2	Over-all enrichment	Time required before draw-off begins, days
28	1.05×10^{-3}	1.8×10^{-6}	9,100	7
14	2.10×10^{-3}	1.8×10^{-6}	18,300	14
7	4.20×10^{-3}	1.8×10^{-6}	36,600	29
3.5	8.40×10^{-3}	1.8×10^{-6}	73,000	58

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