Large-Angle Scattering of Na^{24} Gamma-Radiation

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Scattering of the 1.38-Mev and 2.76-Mev gamma-radiation of Na^{24} has been observed at a mean angle of 135°. Mg, Al, Pb, and Hg scatterers were used, and three definite components of scattered radiation were observed. The softest, of energy about 200 kev, is Compton-scattered radiation; the next hardest, of energy 500 kev, is the positron annihilation radiation. The third has an energy between 1.1 and 2.8 Mev. It is due, in part, to bremsstrahlung, but seems to be somewhat too hard to be wholly explained by this process. It is not sharply resonant nuclear scattering since it appears nearly equally in Mg and Al, while only Mg should be sharply resonant. From the lack of sharp resonance with the Mg^{24} level at either 1.38 Mev or 2.76 Mev, it is concluded that these levels must be less than 1/100 ev wide and that the corresponding transitions are of quadrupole or higher order.

I. INTRODUCTION

THE large-angle scattering of gamma-radiation is of interest because the predominant Compton scattering is both low in intensity and long in wave-length. The study therefore reveals the presence of any other processes of interest. It was employed by Chao[1] and Gray and Tarrant[2] and revealed the presence of the 0.5-Mev component resulting from annihilation radiation developed in the scatterer by the positron part of pairs.

The present work was begun with the object of detecting nuclear resonant scattering from Mg^{24} nuclei which are excited by resonant radiation from the radioactive Na^{24} nuclei. These decay by beta-ray emission to Mg^{24} following which two gamma-rays are emitted in cascade.[3] The excitation of Mg^{24} by one of these gamma-rays would therefore be expected.

The experimental observation is not easy because the act of gamma-ray emission lowers the gamma-ray energy resulting from the recoil of the nucleus so that the energy is about 90 ev off resonance. While other factors exist, these in general do not compensate for the recoil effect, and it can be estimated that the expected cross section should be less than 10^{-27} cm². If it is detectable at all, a very carefully conducted experiment is required.

Early study[4] of the large-angle scattering process showed that for lead a hard component exists. There was some evidence for a preferential scattering by Mg as compared with Al. It was therefore decided to construct an automatic scattering apparatus which would permit long periods of counting in order to search for very small effects. The results of these experiments are given here.

While this work was in progress a note by Schiff[5] appeared, calling attention to the potency of the large-angle scattering method. An attempt to observe resonance absorption in lead has been made by Zuber.[6]

II. THEORETICAL CONSIDERATIONS

If the difference of energy of two eigenvalues corresponding to the emission of a quantum is E, then the frequency ν_E of an emitted quantum will be slightly less than the relation $h\nu_E=E$ would assert because momentum has to be given to the recoiling atom. If M is the mass of the atom the conservation of energy and momentum leads to

$$E-h\nu_E=\frac{h^2\nu_E{}^2}{2Mc^2}. \quad (1)$$

Similarly, if radiation is to excite the nucleus by

* Now at Brookhaven National Laboratory.
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1 C. Y. Chao, Phys. Rev. **36**, 1519 (1930).
2 L. H. Gray and G. T. P. Tarrant, Proc. Roy. Soc. **136**, 662 (1932).
3 L. G. Elliott, M. Deutsch, and A. Roberts, Phys. Rev. **63**, 386 (1943).
4 E. C. Pollard and B. B. Benson, Phys. Rev. **71**, 134(A) (1947).
5 L. I. Schiff, Phys. Rev. **70**, 761 (1946).
6 K. Zuber, Helv. Phys. Acta **16**, 429 (1943).

the amount E, there also has to be supplied the kinetic energy of the atom receiving the momentum of the incident quantum. If ν_A is the frequency of a quantum which precisely fits this condition for absorption, then

$$h\nu_A - E = \frac{h^2\nu_A^2}{2Mc^2}. \quad (2)$$

Combining (1) and (2), we have

$$h\nu_A - h\nu_E = \frac{E^2}{Mc^2}. \quad (3)$$

The inelastic scattering of protons by Mg, as observed by Dicke and Marshall,[7] indicates that the first excited state of Mg^{24} is at 1.32 Mev. For Mg^{24} Eq. (3) gives a difference of 90 ev. Hence, if radiation from a source of Na^{24}, which decays to Mg^{24} and then emits gamma-radiation, is used to excite Mg^{24}, the radiation will be off resonance by 90 ev.

Although there exist factors which can reduce this deviation from resonance, we may first consider the cross section expected for scattering of such radiation. The expression for resonant absorption (which would be followed by emission to give scattered radiation) from Heitler's book[8] (p. 115) is

$$\sigma = \frac{\pi^2 c^2 \gamma^2}{2\pi\nu_0^2\left[(\nu-\nu_0)^2 + \frac{\gamma^2}{4}\right]},$$

where γ is the chance of a transition per atom per second, and ν and ν_0 are the actual and resonant frequencies. Rewriting this in terms of Γ, the line width, E and E_0 the actual and resonant energies, all in electron volts, this becomes

$$\sigma = \frac{2.4\times 10^{-8}\Gamma^2}{E_0^2\left[(E-E_0)^2 + \frac{\Gamma^2}{4}\right]}. \quad (4)$$

For Mg^{24} we have, to a close approximation,

$$\sigma = \frac{1.2\times 10^{-20}\Gamma^2}{90^2}.$$

Thus an actual measurement of σ gives information regarding Γ, the line width.

The value expected for Γ depends on the selection rules for the transition. Thus, if the radiation is dipole the value of Γ, according to a formula given by Helmholz,[9] is roughly 1.3 ev, while if it is quadrupole the value is one thousand times less.

The cross sections expected in the two cases are 10^{-23} cm^2 and 10^{-29} cm^2. The former could be detected, but the latter only with great difficulty. The study of large-angle scattering by Mg and Al can therefore, as a minimum, decide whether the radiation from the first excited state of Mg^{24} is dipole or of higher order.

III. BETA-RAY AND CASCADE GAMMA-RAY MOMENTUM CONSIDERATIONS

There exists a possibility that the gamma-radiation is emitted while the nucleus still has the recoil momentum from the beta-ray. A second possibility is that the first of a cascade gamma-ray emission process gives recoil momentum to the nucleus which is still present when the second gamma-ray emerges. Either of these processes can change the distribution of gamma-ray energies considerably.

The analysis of either case is simple and the following results are obtained:

Case I, gamma-ray emitted at an angle θ to the beta-ray,

$$E - h\nu_E = \frac{E^2}{2Mc^2} + \frac{E}{Mc} p_\beta \cos\theta, \quad (5)$$

where $p_\beta = m_0 c\beta/(1-\beta^2)^{\frac{1}{2}}$ is the momentum of the beta-ray.

Case II, first gamma-ray emitted at an angle φ to the second,

$$E - h\nu_E = \frac{E^2}{2Mc^2} + \frac{EE'}{Mc^2}\cos\varphi, \quad (6)$$

where E' is the energy of the first gamma-quantum and E that of the second.

Both of these equations offer very interesting possibilities if the resonant process is ever detected, since the use of coincidence methods permits a complete knowledge of the process to be

[7] Robert H. Dicke and John Marshall, Jr., Phys. Rev. **63**, 86 (1943).

[8] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1936).

[9] A. C. Helmholz, Phys. Rev. **60**, 415 (1941).

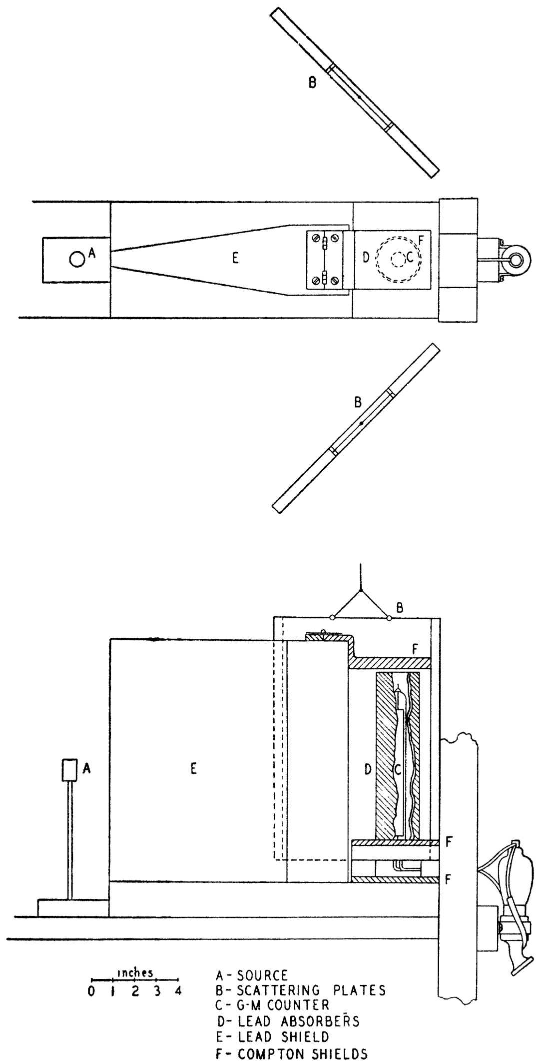

FIG. 1. Top and side views of experimental arrangement for measuring the absorption of large-angle scattered gamma-radiation.

established. The only missing item is the momentum of the neutrino. Experimentally, this can be neglected by selecting only those beta-rays having close to the maximum energy. For resonance to occur in a scattering nucleus the emitted frequency ν_E of Eq. (5) must be equal to the ν_A of Eq. (2). Combining these two we have

$$\frac{E^2}{2Mc^2}+\frac{E}{Mc}p_\beta\cos\theta=-\frac{E^2}{2Mc^2},$$

or

$$\cos\theta=-\frac{E}{p_\beta c}. \qquad (7)$$

In the case of Na^{24} the momentum of the 1.4-Mev beta-ray is 3.8 m_0c. If $E=1.38$ Mev then

$$\cos\theta=-\frac{1.38\times 2m_0c^2}{3.8m_0c^2}=-0.71.$$

Resonance therefore occurs in the scattering Mg^{24} nucleus if the angle between the beta-ray and gamma-quantum emitted by the source nucleus is approximately 135°. The assumption is made that no interactions of the source nucleus with surrounding matter occur in the time between beta-ray and gamma-ray emission events.

This process cannot actually take place in the element studied, since two gamma-rays are emitted in cascade. This destroys the above momentum relationship. It might occur for Fe^{56}, the daughter element of Mn^{56}. In a clear-cut case, Eq. (7) presents the possibility of varying the angle θ and obtaining the contour of the energy level E.

IV. EXPERIMENTAL RESULTS

An attempt was first made to exploit Eq. (7). A "spectrometer" was constructed consisting of

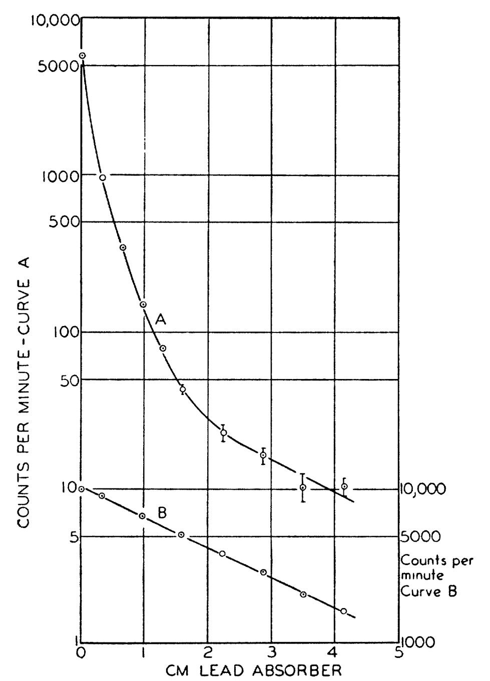

FIG. 2. Curve A, absorption of sodium (24) gamma-radiation scattered by magnesium. Curve B, absorption of direct sodium (24) gamma-radiation.

a source, a beta-ray counter, a scattering block, and a gamma-ray counter shielded from direct radiation but arranged to detect radiation scattered from the block. Double coincidences were counted between beta-rays and scattered gamma-rays. The source used in one case was Na^{24}, the scatterer Mg, in a second case Mn^{56} and Fe^{56} were used as source and scatterer. It was not found possible to overcome the exceedingly high rate of random coincidences. The experiment requires a counter resolution time of the order of 10^{-8} second, whereas the counters used had a resolution time at least one hundred times as great.

Following this unsuccessful attempt it was decided to look for resonant large-angle scattering. Figure 1 shows top and side views of the experimental arrangement. The direct radiation from the source (A) was shielded out by 11 inches of lead (E). A Herbach-Rademan GLC-11 counter (C) was used to detect the scattered radiation. Two scattering plates (B) of up to 1-inch thickness were arranged to give a maximum yield consistent with a range of scattering angles between 120° and 150°. A set of cylindrical lead absorbers (D) could be placed over the counter, out to about 4-cm radial thickness. Lead "Compton" shields (F) were placed above and below the counter to reduce radiation scattered from the floor and ceiling of the room and entering the counter end-on. These effected a substantial reduction in background.

TABLE I. Sample scattering data from one run.

Counter absorption (cm Pb)	Net Mg	Net Al	Ratio
0.00	124 88*	117 95*	1.06 0.93
0.18	37.7 19.1*	33.5 19.1*	1.12 1.00
0.35	35.9 6.6*	31.5 6.6*	1.14 1.00
0.67	5.8 2.8*	3.6 3.1*	1.60 0.90
1.34	0.32 0.72*	0.0 1.65*	— 0.44
2.19	0.31*	0.34*	0.91
3.36	0.92	0.00	—
4.03	0.54	0.37	1.46

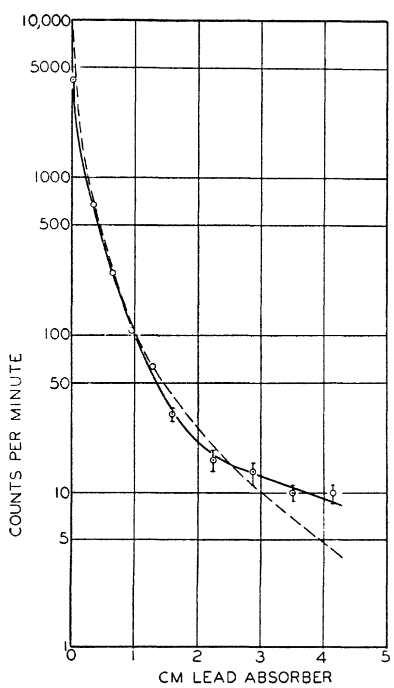

FIG. 3. Absorption of sodium (24) gamma-radiation scattered by aluminum. The dashed curve is a rough theoretical estimate, including Compton-scattered, positron annihilation, and bremsstrahlung components.

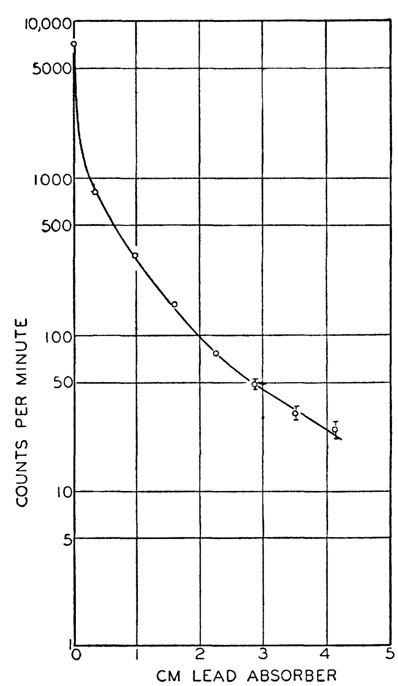

FIG. 4. Absorption of sodium (24) gamma-radiation scattered by lead.

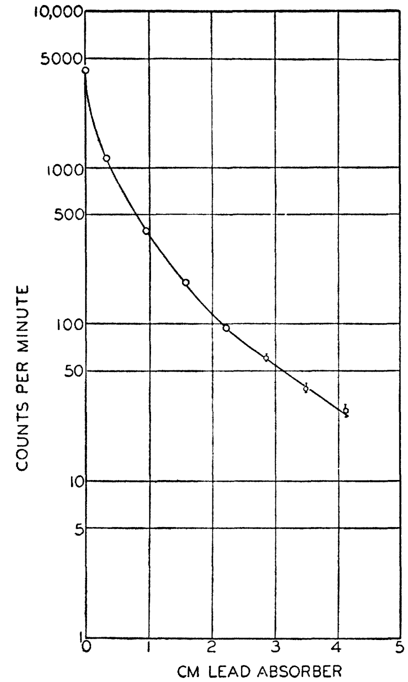

FIG. 5. Absorption of sodium (24) gamma-radiation scattered by mercury.

Four sets of experiments were tried. The first was an attempt to detect resonant scattering in Fe^{56}, using Mn^{56} as the resonant gamma-ray source. The experiment was not very thorough, but it showed that no large (10^{-24} cm^2) scattering cross section exists peculiar to this experiment. An attempt was also made to observe resonant scattering in C^{13}, using N^{13} as the resonant gamma-ray source. This failed because no gamma-rays other than annihilation radiation were detected. A third experiment was carefully carried out aimed at observing resonant scattering by Mg^{24}, using Na^{24} as the resonant source. This can now be described.

The object was the observation of an increase in scattering by Mg over Al at various thicknesses of lead absorber placed over the counter. The resonantly scattered radiation should be more penetrating than Compton scattered radiation and annihilation radiation and accordingly an increase in the ratio of magnesium scattering to aluminum scattering should take place at higher absorption. In order to obtain statistically significant numbers of particles counted, and avoid systematic errors due to source decay and counter variation, an automatic scatterer-changing device was constructed.[10] This changed the scatterers every five minutes in the order MgNoAlAlNoMg, etc., where "No" indicates the absence of a scatterer. Separate recorders and clocks measured the total counts and total time for each scatterer in position and for background over a half-hour or one-hour period at a given absorption. By subtracting out the background counting rate the net scattering yield was obtained for each element. This was done at a number of values of absorption, and for comparison all results were referred to a standard source strength by correcting for decay. The results of one run are given in Table I. The star indicates data taken with a second counter. The number of counts at each point was well over ten thousand, but at the high absorptions the yield of scattered radiation of any kind was so small that the effects of random fluctuations became very large. From Table I it can be seen that the results up to 0.67-cm absorption yield a ratio which has reasonable consistency, but that thereafter the ratio is widely varying, indicating that statistical fluctuations are not removed. The count at 4.03 cm gave the following figures:

Al 13,610 in 211.6 minutes
Mg· 13,880 in 211.9 minutes
No 13,250 in 210.6 minutes.

Thus while there appears to be no doubt that both elements do give a penetrating scattered radiation, it cannot be asserted that one scatters more than the other.

Similar comparisons were made between lead and copper radiators. These showed a very definite yield from lead at high absorptions but not very much from copper. In view of this it was decided to plot absorption curves for Mg, Al, Pb, and Hg scatterers separately. Mg and Al were used for the reasons already given. Pb and Hg were chosen in case Pb showed any specific resonance resulting from the known 2.62-Mev gamma-ray; Hg then served as comparison. The Hg scatterers consisted of sheet-iron containers $\frac{1}{4}$ inch thick filled with liquid mercury. Sources were made by deuteron bombardment of metallic sodium and varied from 20 to 60 millicuries in

[10] D. E. Alburger and E. C. Pollard, Phys. Rev. **72**, 169(A) (1947).

strength. The results are given in Figs. 2–5. These show the absorption of scattered radiation from magnesium, aluminum, lead, and mercury, respectively. It can be seen that for each element there are present several components of scattered radiation and that one of these is definitely too hard to be explained as annihilation radiation or Compton-scattered radiation. Curve B of Fig. 2 shows the absorption of direct radiation for comparing with the hard scattered component and also for furnishing a check on the absorbers and geometry. To obtain this a well-decayed sodium source was dissolved in water and spread uniformly on a piece of cardboard cut to the same length and width as the scattering plates. This large-area source, approximating the geometry of a uniformly illuminated scattering plate, was then placed in the position normally occupied by one of the scatterers and the absorption curve plotted. The absorption coefficient is approximately what one would expect at 2.8 Mev from theoretical curves given by Heitler.

V. DISCUSSION

Three processes which can give scattered quanta are known to occur when gamma-radiation strikes a radiator. These are Compton scattering, pair production and positron annihilation, and bremsstrahlung. The first two can be numerically estimated quite readily. If it is assumed that the number of effective scattering atoms is N, of atomic number Z, that the product of two the solid angle factors is Ω, and that the counter efficiency is ϵ for 250-kv radiation (Compton-scattered) and is proportional to the energy of the gamma-rays,[11] we find

$$\text{Compton-scattered yield} = 2.27\times 10^{-25} ZN\Omega\epsilon,$$
$$\text{Pair-annihilation yield} = 1.87\times 10^{-27} Z^2 N\Omega\epsilon.$$

These are based on the expressions given in Heitler's book[8] (pp. 156 and 200).

For bremsstrahlung the theoretical calculation is difficult. An approximate estimate was made as follows. The bremsstrahlung is due to Compton recoil electrons which can suffer energy loss by radiation. The distribution in energy of these electrons can be inferred from the angular dependence of Compton scattering, since the electron possesses the residual energy lost by the quantum. A rough distribution curve was plotted, and the yield of bremsstrahlung in the four intervals of energy 2.0–2.5, 1.5–2.0, 1.0–1.5, and 0.5–1.0 Mev estimated. The electrons are assumed to obey the Feather range relation,[12] and the cross sections for bremsstrahlung given by Heitler[8] (p. 173) were used. An additional factor is the angular dependence of the radiation with respect to the electron which loses energy. This has been measured by Petrauskas, Van Atta, and Meyers[13] for energies up to 2.35 Mev. At the 135° average angle of scattering used in this work the radiation is about $\frac{1}{10}$ of the maximum. Therefore, we have included a factor of 0.1 to allow for this angular dependence. At lower energies the value is higher, so that this procedure emphasizes relatively the high energy components of the bremsstrahlung. The net result of this rough analysis is to predict a yield $1.87\times 10^{-29} Z^3 N\Omega\epsilon$ with an effective absorption coefficient in lead in the range 0.5 cm to 4.5 cm of 0.63 cm^{-1}.

The shape of the absorption curve predicted for Al is shown as a dashed line in Fig. 3. The curve for Mg is very nearly the same. It can be seen that agreement at the low energy end is quite good. However, there appears to be an added component for both Al and Mg of very nearly the full energy of the gamma-radiation from Na^{24}, as seen from Fig. 2. The existence of a high energy component was found by Gray and Tarrant,[2] but it had a lower energy and probably can be identified with bremsstrahlung. Our results indicate a still harder component of weak intensity and as yet unexplained origin. The expected yield for nuclear Compton scattering is much less than this. It is possible that some general type of scattering process exists, involving mesons in the nucleus. We do not feel that our experimental data are good enough to assert definitely that an added hard component is present. To settle the matter, sources of one hundred times the intensity should be used. Facilities are not available here for this kind of work.

One result is certain. No *specific nuclear* resonance occurs with a cross section greater than 1/500 of the Compton cross section or greater

[11] Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer, Helv. Phys. Acta **19**, 77 (1946).

[12] N. Feather, Proc. Camb. Phil. Soc. **34**, 599 (1938).

[13] A. A. Petrauskas, L. C. Van Atta, and F. E. Meyers, Phys. Rev. **63**, 389 (1943).

than 10^{-27} cm^2. This cross section corresponds to an upper limit to the line width for this 1.4-Mev transition of 1/100 electron volt. The line is thus exceedingly sharp and certainly quadrupole or higher.

We wish to acknowledge considerable help from discussions with Professor E. R. Beringer. A great deal of the early work was performed by Dr. B. B. Benson to whom we are greatly indebted.

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Structure of the ^2D Terms of the Arc Spectrum of Lithium

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Interferometric wave-length measurements of high precision carried out by employing an atomic beam light source have been used for the study of several features of the arc spectrum of Li^7. Due to the great sharpness of the spectral lines produced by this source it was possible to determine the splitting of several ^2D terms. The hyperfine structure of the line 6708A was found in approximate agreement with earlier investigations. Precise wave-length measurements have been carried out relative to krypton standards. Discrepancies between the wave-lengths of this investigation and those of earlier investigators must be ascribed to lack of resolution or inadequate comparison methods employed in earlier investigations. This consideration is substantiated by our measurements carried out with a vacuum lithium arc, the results of which are consistent with the atomic beam values. Isotopic shift (Li^6 and Li^7) of the resonance line 6708A was measured.

I. INTRODUCTION

ALTHOUGH the arc spectrum of lithium has been extensively investigated, the wave-length material available contains many discrepancies and is not sufficiently accurate for reliable conclusions regarding the structure of the ^2D terms.

At first glance the measurements of Datta and Bose[1] seem to be the most reliable ones as far as average wave-lengths are concerned. These authors employed a vacuum arc and concave grating and claim an accuracy of a few thousandths A. However, the fact that with one exception they did not resolve the $^2P_{1/2,3/2}$ structure makes one doubt this accuracy.

The first observation of the fine structure of lithium lines was made by Zeeman[2] who resolved the resonance line 6708A as a doublet.

Without undertaking absolute wave-length measurements, Kent[3] carried out a detailed investigation of the doublet structure of lithium lines and their Zeeman effect. Using a quartz discharge tube as light source and a Michelson transmission echelon as resolving instrument, he succeeded in resolving the resonance line 6708A, the first two members of the sharp series (8126A and 4972A), and the first two members of the diffuse series (6103A and 4603A) as close doublets. The error involved in the measurements of the separations was one to two percent but the results are sufficiently accurate to show a definite difference between the doublet intervals obtained from the principal and sharp series and those obtained from the diffuse series. The average value of the three $^2S^2P$ combinations is 0.338 cm^{-1} and represents the $^2P_{1/2,3/2}$ splitting. The intervals observed with two pairs of the diffuse series are 0.309 cm^{-1} for 6103A and 0.328 cm^{-1} for 4603A. These separations are distinctly smaller than the $^2P_{1/2,3/2}$ splitting. Kent himself did not discuss these results since he was mainly interested in the Zeeman and Paschen-Back effect of these lines, but it is easy to see that these results are to be expected from

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[1] S. Datta and P. C. Bose, Zeits. f. Physik **97**, 321 (1935).
[2] P. Zeeman, Physik. Zeits. **14**, 405 (1913).
[3] N. A. Kent. Astrophys. J. **40**, 337 (1914).