centrifugal force $m\omega_H^2 r$, is balanced by the Lorentz force $+\mu_0 e\omega_H rH$.

Now let us take a slice of thickness d in this beam, and insert it between the fields of our previous chargeless wave, making the whole structure move with a velocity W along the z axis, as shown on Fig. 2. We obtain continuity of the radial field in the boundary planes (at $t=0$, these planes are located at $z=0$ and $z=d$) when we take

$$\tfrac{1}{2}Aj(\omega/W)=\tfrac{1}{4}(\mu_0 II)^2(e/m), \qquad (4)$$
$$A=-\tfrac{1}{2}(eW/m\omega)j(\mu_0 II)^2.$$

The radial fields are matched across the boundary planes, when the beam is fine enough, and $KR\ll 1$.

The *composite wave* represented in Fig. 2 is *stable*, and propagates along the z axis with the velocity W. Its wave-length is

$$\Lambda'=\Lambda+d, \qquad (5)$$

and its total current averages

$$I=\rho W(d/\Lambda')\pi R^2. \qquad (6)$$

The magnitude II of the stabilizing magnetic field determines the value of the electric field in the chargeless region and also the space-charge density in the charged slices. The current can be adjusted by changing the thickness d of the space-charge slices. Its maximum value is, of course,

$$I_{\max}=\rho W\pi R^2. \qquad (7)$$

The composite wave thus found has discontinuities in the derivative of the longitudinal field distribution (Fig. 2B) and represents a sort of *electromagnetic shock wave*. It yields a rigorous solution of the wave equation with space-charge and represents a generalization of the solutions obtained for the two extreme special cases of the traveling wave amplifier or the linear accelerator. The solution obtained here should correspond to the final stage of complete bunching of the particles by the wave, after oscillations of the particles about their equilibrium positions (the circles of Fig. 1) have died out, and the field rearrangements have been completed.

Of course, the solution is valid only at a certain distance of the boundary of the wave guide, and would be seriously distorted in the neighborhood of the boundary, according to the type of guide structure used to slow down the waves.

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On the Neutron-Proton Force*

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IT has been pointed out[1] that neutron-proton scattering experiments at energies below about 10 Mev do not give any *detailed* information about the neutron-proton force. They only determine the strength and range of the force, not its exact distance dependence.

J. Schwinger[2] has developed a powerful variational approach to scattering theory. This method, applied to the case of the scattering of neutrons by protons, leads to great simplifications in the analysis of the experiment. Schwinger was able to prove rigorously that to a good approximation the shape of the well does not matter for the scattering. The well is described phenomenologically by the scattering length[3] to which it gives rise (evaluated at zero energy) and by an *equivalent range*, r. This equivalent range depends both upon the width and the depth of the well, becoming smaller as the well is made deeper. For a square well of range b, the equivalent range r equals b if the depth of the well is such as to give a resonance level in the scattering exactly at zero energy. For the triplet state, therefore, $r<b$, for the singlet state, $r>b$. It is the great advantage of this analysis, how-

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[1] J. Smorodinsky, J. Phys. U.S.S.R. **8**, 219 (1944); **11**, 195 (1947).

[2] J. Schwinger, Phys. Rev. **72**, 742 (1947).

[3] E. Fermi and L. Marshall, Phys. Rev. **71**, 66 (1947).

ever, that we are *not* required to refer everything back to a square well. On the contrary, we can analyze the scattering experiments in order to find the equivalent ranges in the triplet and singlet states, without knowing the well shape. Of course, if we have reason to suspect (from some other evidence) that a particular well shape is the one realized in nature, then we can very easily find the depth and range of this particular well necessary to give the observed scattering length and equivalent range. The present work is based upon this method. It will be reported more fully following publication of Schwinger's basic work.

Schwinger's analysis shows that the phase-shift is given by

$$k \cot\delta = -a^{-1} + (\tfrac{1}{2}) r k^2 + \text{terms of order } (r^3 k^4). \quad (1)$$

The notation is as follows:

$k = Mv/2h =$ wave number of the neutron in the center-of-gravity system,
$a =$ scattering length,[3] (positive for a bound state) evaluated at zero energy,
$r =$ effective range.

In the *triplet state*, there exists a relation between the scattering length a_t and the effective triplet range r_t since we know the binding energy of the deuteron. This relation is

$$(a_t)^{-1} = \alpha[1 - (\tfrac{1}{2}) r_t \alpha + \text{terms of order } (r_t \alpha)^3], \quad (2)$$

where $\alpha = 2.29 \times 10^{12}$ cm^{-1} is the reciprocal "radius of the deuteron." The effective triplet range r_t depends upon the range and depth of the nuclear force in the triplet state; the latter two are related through the binding energy of the deuteron. Figure 1 shows the resulting relation between the range b of the force and the effective triplet range r_t if a square well shape is assumed for the distance dependence of the force. It will be seen that b/r_t approaches unity as r_t approaches zero.

As regards the *singlet state*, we know that the singlet scattering length a_s is negative (virtual state) and the effective singlet range r_s is intrinsically positive. For a square well shape we have the relation

$$b \cong r_s[1 + (4/\pi^2)(r_s/a_s)]. \quad (3)$$

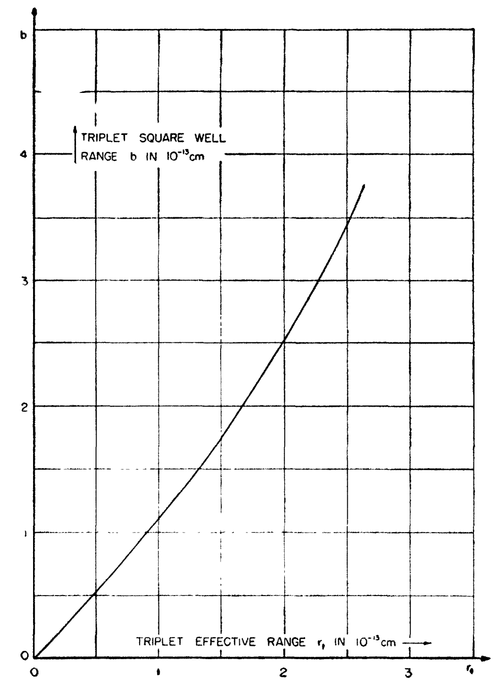

FIG. 1. Abscissa: effective range in the triplet state. Ordinate: the square-well range corresponding to this effective range. The depth of the square well is adjusted for each range to give the observed binding energy of the deuteron.

This is a small correction (about 5 percent for $r_s = 2.6 \times 10^{-13}$ cm) in the negative direction (a_s is negative).

The magnitude of the higher terms in (1) and (2) depends upon the shape of the well. The terms are small if the potential falls off sharply outside its range, large if there is a long tail.

If we neglect the higher order terms, the neutron-proton cross section implied by (1) is

$$\sigma = \tfrac{3}{4}(4\pi)[(-a_t^{-1} + \tfrac{1}{2} r_t k^2)^2 + k^2]^{-1} + \tfrac{1}{4}(4\pi)[(-a_s^{-1} + \tfrac{1}{2} r_s k^2)^2 + k^2]^{-1}. \quad (4)$$

$r_t r_s$ and a_s are independent constants; a_t is given by (2).

The experiments[4] are not sufficiently accurate to test the validity of Eq. (4). In particular, the assertion of Bohm and Richman[5] that a long-tailed well is necessary to fit the data is true only if one insists (as they did) upon assuming the neutron-proton range to equal the proton-proton range in both spin states. There seems to

[4] C. L. Bailey, W. E. Bennet, T. Bergstrahl, R. G. Nucholls, H. T. Richards, and J. H. Williams, Phys. Rev. **70**, 583 (1946); D. Frisch, Phys. Rev. **70**, 589 (1946).
[5] D. Bohm and C. Richman, Phys. Rev. **71**, 567 (1947).

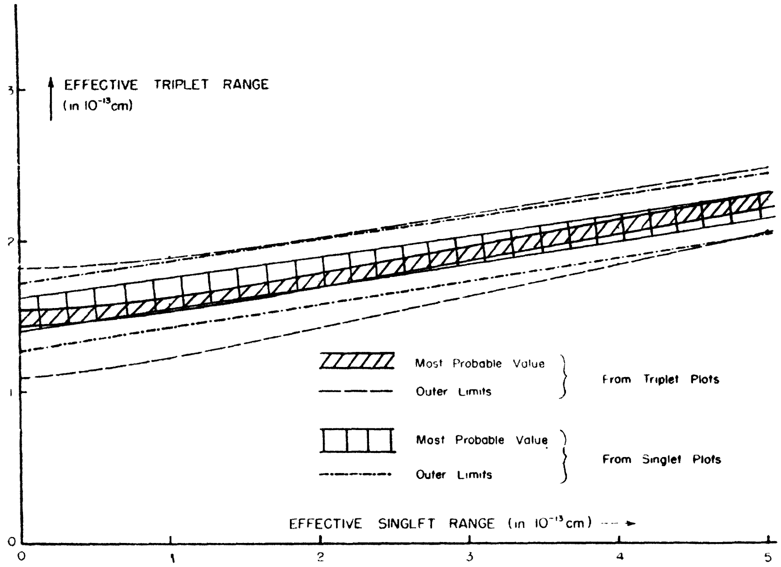

FIG. 2. Abscissa: effective range in the singlet state. Ordinate: effective range in the triplet state. The curves define the regions of best fit to the neutron-proton scattering data at energies below 6 Mev. The singlet plots use energies below about 2 Mev, the triplet plots use energies between 1 Mev and 6 Mev, approximately.

be no cogent reason for this assumption, and the present analysis did not use it. On the contrary, *we assumed that the potential falls off sufficiently sharply so that the higher terms in (1) can be neglected.* (They are completely negligible for a square well shape at energies below 10 Mev.)

Our analysis of the scattering data proceeds as follows:

I. Singlet plots.—Assume a value for r_t, compute a_t from (2), use the experimental cross section to compute $(-a_s^{-1}+\frac{1}{2}r_sk^2)$ from (4), plot against k^2. This is feasible for energies below about 2 Mev.

II. Triplet plots.—Assume values for a_s and r_s, use the experimental cross section to compute $(-a_t^{-1}+\frac{1}{2}r_tk^2)$ from (4), plot against k^2. This is feasible for energies above about 1 Mev.

I gives a_s and r_s as functions of r_t (incidentally, the value of a_s depends practically exclusively upon the epithermal cross section, and r_t enters only insofar as it affects the triplet scattering length a_t and, through it, the triplet epithermal cross section). We can use the results of the singlet plots to draw a curve of mutually compatible values of r_s and r_t in the r_s-r_t plane.

II gives r_t as a function of a_s and r_s. Since r_t is mostly determined from the high energy points, it is only slightly dependent upon the choice of a_s, as long as this choice is kept reasonable. Hence we can use the results of the triplet plots to draw another (independent) curve of mutually compatible values of r_s and r_t in the r_s-r_t plane. The curves are independent because entirely different energy ranges are used in their determination.

The two curves should cross somewhere in the r_s-r_t plane, their intersection point determining both effective ranges. Since the data are not infinitely accurate, each "curve" is really a region of compatibility, and the intersection of these two regions would then determine a region of values of r_s and r_t consistent with all the scattering data. This region would narrow down to a point as the data become more accurate.

Such a plot is shown in Fig. 2. It is seen that the regions of compatibility determined from the singlet and triplet plots overlap completely. (This is an unlucky accident, and wouldn't have happened if the singlet state were real, for example.) It seems out of the question to increase the accuracy of the experiments to the point where *these* two regions are narrowed down until they give a unique point of intersection. *Hence, scattering experiments alone are not sufficient to determine even the effective ranges of neutron-proton forces in the two spin-states.* They only give a functional relation between r_s and r_t.

It might be interesting to see whether the scattering data alone are consistent with the assumption of a square well potential with the *same* range b in both spin states. If we try $b=2.8\times10^{-13}$ cm, the value initially assumed by

Bohm and Richman, Fig. 1 gives $r_t = 2.15 \times 10^{-13}$ cm and formula (3) yields $r_s = 2.94 \times 10^{-13}$ cm. The corresponding point on Fig. 2 is right on the outer limit of the allowed region, and is therefore a very poor fit to the data. On the other hand, if we assume $b = 2.6 \times 10^{-13}$ cm, the effective ranges are $r_t = 2.03 \times 10^{-13}$ cm and $r_s = 2.72 \times 10^{-13}$ cm. The corresponding point on Fig. 2 is somewhat above the shaded (best) region, but well inside the outer limits. Hence the scattering data alone do not exclude this hypothesis. We might remark here that if one takes only the proton-proton scattering data of Herb, Kerst, Parkinson, and Plain[6] and does not use the data of Hafstad, Heydenberg, and Tuve,[7] then $b = 2.6 \times 10^{-13}$ cm is in good agreement with the proton-proton scattering data.

More information can be obtained from a direct measurement of the *coherent neutron-proton scattering amplitude at low energies*, f:

$$f = 2(\tfrac{3}{4}a_t + \tfrac{1}{4}a_s). \tag{5}$$

There are two methods for measuring f: scattering of neutrons by parahydrogen and scattering of neutrons by crystals containing hydrogen atoms. The results are:

$f = -(3.95 \pm 0.12) \times 10^{-13}$ cm parahydrogen,[8]
$f = -(4.72 \pm 0.40) \times 10^{-13}$ cm crystals.[9]

The parahydrogen value is subject to many systematic errors (admixture of orthohydrogen, uncertainty in the capture cross section, etc.), so the quoted statistical error may be misleading. It should be noticed that these two determinations of f are inconsistent. The stated experimental errors do not overlap.

The epithermal cross section σ_0 is given from (4):

$$\sigma_0 = \tfrac{3}{4}(4\pi a_t^2) + \tfrac{1}{4}(4\pi a_s^2). \tag{6}$$

σ_0 has been determined by the scattering of epithermal neutrons from crystals and by the use of neutron velocity spectrometers. The values are:

$\sigma_0 = (20.2 \pm ?) \times 10^{-24}$ cm^2, crystals;[10]
$\sigma_0 = (20.6 \pm 1.0) \times 10^{-24}$ cm^2, velocity spectrometer;[11]
$\sigma_0 = (20.0 \pm 0.15) \times 10^{-24}$ cm^2, velocity spectrometer.[12]

Some crystal measurements have been made with neutrons from the indium resonance. This energy (1.44 ev) is low enough to introduce a systematic error into σ_0, tending to make the measured value too high. (The author wishes to thank Dr. McDaniel and Dr. Jones for pointing out this source of error.)

Knowing f and σ_0, we can determine a_s and a_t. The value of a_t then gives the effective triplet range r_t by Eq. (2). In Fig. 3 we show curves of constant r_t in the $\sigma_0 - f$ plane.

We have also indicated some of the experimental results on Fig. 3. The top three horizontal lines give the crystal measurement of f (the

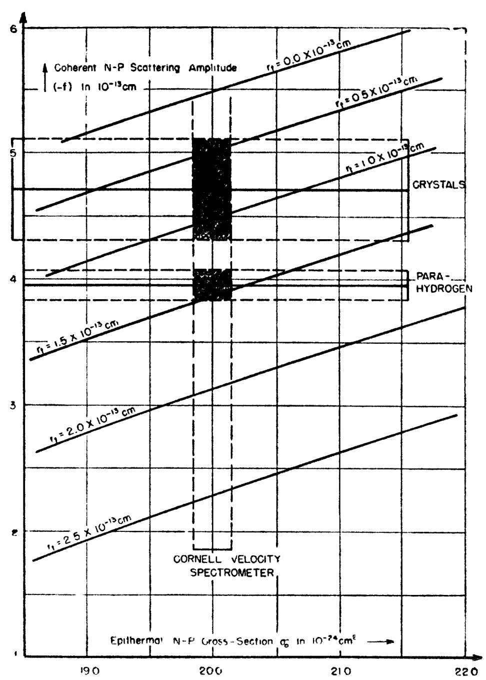

FIG. 3. Abscissa: epithermal N–P scattering cross section, σ_0. Ordinate: (negative) coherent epithermal N–P scattering amplitude, $(-f)$. The slanted curves give the theoretical relation between σ_0 and $(-f)$ for assumed values of the effective range r_t in the triplet state. The horizontal lines indicate measured values of $(-f)$, the vertical lines indicate the measured value of σ_0. (In each case the center line is the most probable value, the dashed lines on either side represent the claimed experimental error.)

[6] R. G. Herb, D. W. Kerst, D. B. Parkinson, and G. J. Plain, Phys. Rev. **55**, 998 (1939).
[7] N. P. Heydenberg, L. R. Hafstad, and M. A. Tuve, Phys. Rev. **56**, 1078 (1939).
[8] R. B. Sutton, T. Hall, E. E. Anderson, H. S. Bridge, J. W. deWire, L. S. Lavatelli, E. A. Long, T. Snyder, and R. W. Williams, Phys. Rev. **72**, 1147 (1947).
[9] G. C. Shull, E. O. Wollan, G. A. Morton, and W. L. Davidson, Phys. Rev. **73**, 262 (1948).
[10] J. Marshall, Phys. Rev. **70**, 107A (1946).
[11] W. W. Havens, L. J. Rainwater, and C. S. Wu, Bull. Am. Phys. Soc. **23**, No. 2, 37 (1948).
[12] B. D. McDaniel and W. Jones, private communication.

center line corresponds to the most probable value, the dashed lines to the limits of error), the bottom three horizontal lines give the parahydrogen measurement of f. The three vertical lines correspond to the Cornell measurement of σ_0.

Consider first what happens if we accept the crystal measurement of f. Figure 3 shows that then r_t is less than 1.2×10^{-13} cm, and may be as low as 0.35×10^{-13} cm. On the other hand, Fig. 2 cannot be reconciled with a triplet range less than 1.2×10^{-13} cm. There is no overlap here. *The crystal measurement of f, in conjunction with the Cornell measurement of σ_0, is inconsistent with the scattering data.*

The situation is somewhat better if we accept the parahydrogen determination of f. Figure 3 then shows that r_t must lie between 1.30×10^{-13} cm and 1.55×10^{-13} cm. Figure 1 shows that this implies a square well range b in the triplet state between 1.48×10^{-13} cm and 1.83×10^{-13} cm. This is considerably shorter than has been commonly assumed. There is an argument about the range of the nuclear forces in the triplet state based on the finite value of the quadrupole moment of the deuteron.[13] This consideration would exclude such a short triplet range. It must be recognized, however, that the argument in question depends crucially upon the assumption of the *same* distance dependence for the central force and the tensor force. There seems to be little *a priori* reason for believing this. Hence a square well triplet range b of less than 1.83×10^{-13} cm, while surprising, cannot be excluded on theoretical grounds.

However, if r_t is less than 1.55×10^{-13} cm, Fig. 2 shows that the *effective singlet range r_s is very likely less than 1.2×10^{-13} cm and certainly less than 2.0×10^{-13} cm. This result is in direct contradiction to the commonly used hypothesis of the charge-independence of nuclear forces in the singlet state.* It might be pointed out that the argument from the mirror nuclei is not relevant here. In mirror nuclei we can compare the effects of neutron-neutron and proton-proton forces. Here we are comparing neutron-proton and proton-proton forces.

It is, of course, possible that the real neutron-proton well shape is of the long-tail kind so that our whole analysis is invalid. Estimates show that the well would have to be very long-tailed indeed to make the present experimental data consistent. It is premature, however, to consider this possibility seriously until the value of f is beyond doubt.

In view of the importance of the measurement of f for our understanding of the nature of nuclear forces, it is strongly suggested that it be redetermined with a view toward narrowing down the limits of error (as well as eliminating possible systematic errors). Figure 3 gives an indication of the accuracy necessary to narrow down the effective triplet range within reasonable limits. Here is a case where *a gain of a factor 5 (or even 2 or 3) in the accuracy of the experiments would make a lot of difference.* In view of an unexpectedly large absolute value of the coherent scattering amplitude f, it is very encouraging that there are two completely independent ways of measuring this quantity.

The author thanks Professor J. Schwinger and Professor V. Weisskopf for some valuable discussions concerning this problem and Professor McDaniel and Dr. Jones for permission to quote their measurement of σ_0.

Note Added In Proof.—After this manuscript had been sent off, a detailed paper on crystal diffraction studies has come out from the Oak Ridge Laboratory.[14] According to this paper, the best value for the coherent neutron-proton scattering amplitude f is $f=(3.96\pm0.20)10^{-13}$ cm. The difference between this value and the one reported earlier is due to the fact that the thermal motion of the protons in the crystal lattice had been taken into account incorrectly at first.

This result changes the conclusion of this note somewhat. We may now consider the value of f reasonably well established (even though a better accuracy would be very desirable) and we can therefore state the conclusion in a more definite way:

If the nuclear potential is not "long-tailed," the effective triplet range is between 1.30 and 1.55 $\times10^{-13}$ cm, and the effective singlet range is less than 2.0×10^{-13} cm (in contradiction to the hypothesis of the charge-independence of nuclear forces).

An analysis of the data with arbitrary well-shapes is now in progress.

[13] J. Schwinger, Phys. Rev. **60**, 164 (1941).

[14] Shull, Wollan, Morton and Davidson, Phys. Rev. **73**, 842 (1948).

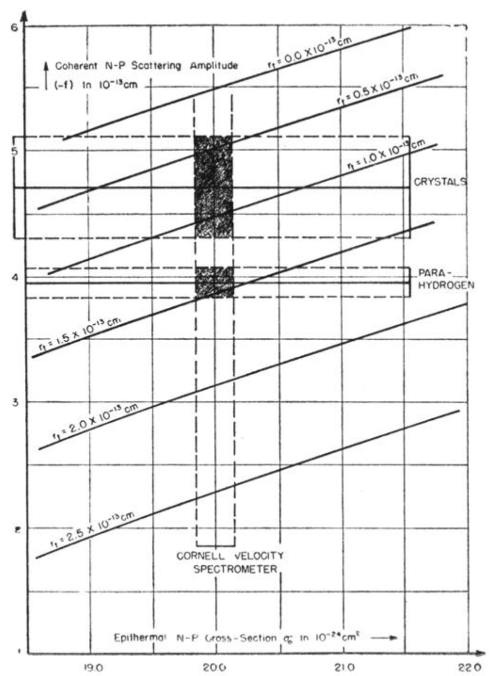

FIG. 3. Abscissa: epithermal N–P scattering cross section, σ_0. Ordinate: (negative) coherent epithermal N–P scattering amplitude, $(-f)$. The slanted curves give the theoretical relation between σ_0 and $(-f)$ for assumed values of the effective range r_t in the triplet state. The horizontal lines indicate measured values of $(-f)$, the vertical lines indicate the measured value of σ_0. (In each case the center line is the most probable value, the dashed lines on either side represent the claimed experimental error.)