repetitions of this process destroyed the HNO_3. The residue, dissolved in 1 ml of HCl, 1 drop of H_2SO_4, and 15 ml of water, was treated with 5 ml of a saturated oxalic acid solution, and boiled for ten to fifteen minutes. After standing for four hours, it was washed with dilute HCl (1:99), filtered with paper pulp, and finally ignited to gold. This procedure was intended to separate the gold from most other metals including platinum, silver, and mercury, calcium, and iron.

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A Note on the Magnetic Moment of the Electron

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Recent experiments seem to require a modification in the g-factor of the electron. It has been suggested that the coupling between the electron and the radiation field is responsible, and Schwinger has calculated the effect on the basis of a general subtraction formalism for the infinities of quantum electrodynamics. It is here shown that the change in magnetic moment may be derived very simply without any reference to an elaborate subtraction formalism.

I.

It has been suggested recently by Schwinger[1] that the coupling between an electron and the radiation field leads to a change in the g-factor of the electron, as seems to be required by experiment.[2] This result was derived on the basis of his (as yet unpublished) general subtraction formalism for the infinities of quantum electrodynamics. It is the purpose of this note to show that the change in magnetic moment may be derived very simply without any reference to an elaborate subtraction formalism.

We may characterize the problem as follows: Given an electron in a homogeneous magnetic field, what is the energy of this electron as a result of interactions with the zero-point vibrations of the quantized radiation field? It is well known that this energy is infinite, the infinities which arise usually being ascribed to changes in the mass and charge of the electron. What we seek are those parts of the energy which do not correspond to the ordinary mass and charge changes, but those which arise because of the presence of the external magnetic field. The problem of subtracting the original infinities is made very simple in this case by the existence of a state for the electron (in a homogeneous magnetic field) which has as an energy simply $E=m$.[3] This is a direct result of the fact that the (unperturbed) Dirac electron has a g-factor of exactly 2. For this state we have the energy of the orbital magnetic moment exactly canceling the energy of the spin magnetic moment. The change in energy of such a state due to a change in mass ($E_{\text{self}}(0)$) and charge of the electron is simply

$$\Delta E = E_{\text{self}}(0), \tag{1}$$

which is independent of the external field. This means that when we calculate the energy and find terms which depend on the external field strength, these terms must represent the true change in the energy, and they must converge. This expectation is borne out by the calculation of the energy to terms linear in H_0 (the external field strength). The coefficient of this latter term gives immediately the alteration of the g-factor of the electron.

II.

We now proceed to an outline of the calculation. For the electron we must use the quantized formalism of the theory of holes[4] and for the

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[1] J. S. Schwinger, Phys. Rev. **73**, 415 (1948); Bull. Am. Phys. Soc. **23**, 15 (April 1948).

[2] See for example: P. Kusch and H. M. Foley, Phys. Rev. **72**, 256 (1947); **73**, 412 (1948); J. Nafe and E. Nelson, Phys. Rev. **73**, 718 (1948).

[3] We shall use throughout natural units, e.g., $\hbar=c=1$. m is the mass of the electron.

[4] Cf. G. Wentzel, *Ein. in die Quant. Theorie der Wellenfelder* (Franz Deuticke, Wien, 1943), pp. 158–91.

radiation field we must use the formalism of quantum electrodynamics.[5]

The Hamiltonian is then given by

$$H=H_0+H_1+H_{\text{static}}, \tag{2}$$

where

$$H_0=i\int(-\psi^*(\alpha\cdot\nabla-ieA_0)\psi-im\psi^*\beta\psi)d^3x+(1/8\pi)\int(E^2+H^2)d^3x,$$

putting

$$\psi=\sum_n a_n\psi_n,$$

$$H_0=\sum_n a_n^*a_nE_n+\sum_s k_sc_s^*c_s,$$

$$H_1=-e\int(\psi^*\alpha\psi)\cdot Ad^3x=-\sum_{n,s}(eA_s/\sqrt{2})c_s^*a_n^*a_{n'}V_{nn'}^{(s)}+\text{Herm. conj.},$$

$$H_{\text{static}}=\frac{1}{2}\int\int\frac{\rho(x)\rho(x')}{|x-x'|}d^3xd^3x'=\frac{e^2}{2}\sum_{n,n',l,l'}a_n^*a_{n'}a_l^*a_{l'}\times\int\int\frac{(\psi_n(x)\psi_{n'}(x))(\psi_l(x')\psi_{l'}(x'))}{|x-x'|}d^3xd^3x'.$$

The following notations have been used:

α is the Dirac vector matrix $(\alpha_x, \alpha_y, \alpha_z)$;

A_0 is the vector potential of the external field;

E_n, ψ_n are the eigenvalues and eigenfunctions of the Dirac equation

$$(\alpha\cdot((1/i)\nabla-eA_0)+m\beta)\psi_n=E_n\psi_n;$$

a_n^*, a_n are, respectively, the creation and destruction operators of an electron in state n. They satisfy $a_n^*a_{n'}+a_{n'}a_n^*=\delta_{nn'}$, etc.;

c_s^*, c_s are, respectively, the creation and destruction operators of a photon in a state with momentum k_s and polarization vector e_s. $c_sc_{s'}^*-c_s^*c_{s'}=\delta_{ss'}$, etc.;

A is the vector potential of the radiation field—

$$A=\sum_s(A_s/\sqrt{2})(c_se^{ik_s\cdot x}+c_s^*e^{-ik_s\cdot x})e_s;$$

$$A_s=(4\pi/k_sG)^{\frac{1}{2}},\quad G=\text{normalization volume};$$

$$V_{nn'}^{(s)}\equiv\int d^3x(\psi_n^*\alpha_s\psi_{n'})e^{-ik_s\cdot x},\quad \alpha_s\equiv e_s\cdot\alpha.$$

We are now interested in calculating the energy to $0(e^2)$. This means that we have to take the mean value of H_{static} for one particle in the state m, and no photons present. This yields:

$$\langle H_{\text{static}}(m)\rangle_{\text{Av}}=\frac{e^2}{2}\sum_{\substack{l\\E_l>0}}\times\int\int\frac{(\psi_m^*(x)\psi_l(x))(\psi_l^*(x')\psi_m(x'))}{|x-x'|}d^3xd^3x'$$
$$+\frac{e^2}{2}\sum_{\substack{n,n\neq m\\E_n>0\\l,E_l<0}}\int\int\frac{(\psi_n^*(x)\psi_l(x))(\psi_l^*(x')\psi_n(x'))}{|x-x'|}d^3xd^3x'$$
$$+e^2\sum_{\substack{l\\E_l<0}}\int\int\frac{(\psi_m^*(x)\psi_m(x))(\psi_l^*(x)\psi_l(x'))}{|x-x'|}d^3xd^3x'.$$

The third term represents the interaction between the vacuum charge density

$$\rho_0(=\sum_{l,E_l<0}e\psi_l^*(x')\psi_l(x'))$$

and the charge density of the particle

$$\rho_m(=e\psi_m^*(x)\psi_m(x)).$$

This term may be dropped (as well as the corresponding term from H_1 for the interaction between the particle current and vacuum current), since it is well known[6] that a homogeneous magnetic field gives rise to no polarization of the vacuum effects. From this value we must subtract the corresponding value when only the vacuum is present (negative energy states filled). This gives:

$$\langle H_{\text{static}}(\text{vac})\rangle_{\text{Av}}=\frac{e^2}{2}\sum_{\substack{n,l\\E_n>0\\E_l<0}}\times\int\int\frac{(\psi_n^*(x)\psi_l(x))(\psi_l^*(x')\psi_n(x')}{|x-x'|}d^3xa^3x'.$$

Defining now the electrostatic energy as

$$\langle H_{\text{static}}(m)\rangle_{\text{Av}}-\langle H_{\text{static}}(\text{vac})\rangle_{\text{Av}}$$

[5] See reference 4, pp. 107–133.

[6] V. F. Weisskopf, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **14**, 6 (1936).

we get

$$\Delta E_{\text{static}} = \frac{e^2}{2} \sum_l \delta_l \times \int\int \frac{(\psi_m^*(x)\psi_l(x))(\psi_l^*(x')\psi_m(x'))}{|x-x'|} d^3x d^3x', \quad (6)$$

where

$$\delta_l = +1, \quad E_l > 0,$$
$$= -1, \quad E_l < 0.$$

To calculate the contribution from $H_1(\Delta E_{\text{dynamic}})$, one must use second-order perturbation theory. When this is done, the vacuum value and polarization of the vacuum terms removed, we are left with

$$\Delta E_{\text{dynamic}} = (-2\pi e^2/G) \sum_s \frac{1}{k_s} \sum_l \frac{|V_{ln}^{(s)}|^2}{\delta_l k_s + E_l - E_m}. \quad (7)$$

The total energy will be given by

$$\Delta E = \Delta E_{\text{static}} + \Delta E_{\text{dynamic}}.$$

We now have to sum the series (6) and (7), using the exact eigenfunctions and eigenvalues of the Dirac equation. These are given in the appendix. The state m in question is that with $n = p_2 = p_3 = 0$, $E_m = m$, and

$$\psi_m = (1/G^{\frac{1}{2}}) \begin{bmatrix} 0 \\ v(x) \\ 0 \\ 0 \end{bmatrix}, \quad (8)$$

$$v(x) = \exp - [(eH_0)x^2/2](eH_0/\pi)^{\frac{1}{4}}.$$

Let us first evaluate the electrostatic energy. Substituting the above value for ψ_m and the values of the eigenfunctions from the appendix, we get:

$$Q \equiv \sum_l \delta_l (\psi_m^*(x)\psi_l(x))(\psi_l^*(x')\psi_m(x'))$$
$$= (m/G^{4/3}) \sum_{p_2,p_3} \sum_{n=0}^{\infty} (1/E_n)(v(x)v^{(n)}(x) \times \exp i(p_2y+p_3z))(v(x')v^{(n)}(x') \times \exp -i(p_2y+p_3z))$$

where

$$E_n = (m^2 + p_3^2 + 2eH_0 m)^{\frac{1}{2}}$$
$$v^{(n)}(x) = \exp[-\xi^2/2] H_n(\xi) \times [(eH_0)^{\frac{1}{4}}/\pi^{\frac{1}{4}} 2^{n/2} (n!)^{\frac{1}{2}}]$$
$$\xi = (eH_0)^{\frac{1}{2}} [x - (p_2/eH_0)].$$

Therefore,

$$\Delta E_{\text{static}} = \frac{e^2}{2} \int\int \frac{Q}{|x-x'|} d^3x d^3x'$$
$$= \frac{mc^2}{2G^{4/3}} \sum_{p_2,p_3} \sum_n \frac{1}{E_n} \int\int \frac{(v(x)v^{(n)}(x) \exp i(p_2y+p_3z))(v(x')v^{(n)}(x') \exp -i(p_2y'+p_3z'))}{|x-x'|} d^3x d^3x'.$$

It is convenient to go over to momentum space in order to evaluate these integrals. Define

$$\rho_n(x) \equiv v(x)v^{(n)}(x) \exp[i(p_2y+p_3z)],$$

and

$$\rho_n(k) \equiv \int \rho_n(x) \exp[-ik\cdot x] d^3x.$$

Then, as is well known,

$$\int\int [\rho_n(x)\rho_n^*(x')/|x-x'|] d^3x d^3x' = (4\pi/G) \sum_k [\rho_n(k)\rho_n^*(k)/k^2].$$

Therefore

$$\Delta E_{\text{static}} = (2\pi e^2 m/G^{7/3}) \sum_{p_2,p_3} \sum_{n=0}^{\infty} (1/E_n) \times \sum_k (1/k^2)\rho_n(k)\rho_n^*(k).$$

The integrals giving $\rho_n(k)$ are easily evaluated:

$$\rho_n(k) = G^{2/3} \delta_{k_2 p_2} \delta_{k_3 p_3} \frac{(-)^n}{2^{n/2}(n!)^{\frac{1}{2}}} \left(\frac{p_2 + ik_1}{(eH_0)^{\frac{1}{2}}} \right)^n \times \exp\left[-\frac{p_2^2 + k_1^2 + 2ip_2k_1}{4eH_0} \right],$$

or

$$\Delta E_{\text{static}}=\frac{2\pi e^2 m}{G}\sum_{p_2,p_3}\sum_{n=0}^{\infty}\frac{1}{E_n}\sum_{k_1}\frac{1}{(k_1^2+p_2^2+p_3^2)}\times\frac{\left(\frac{p_2^2+k_1^2}{eH_0}\right)^n}{2^n n!}\exp[-(p_2^2+k_1^2)/2eH_0]$$

$$=\frac{e^2m}{4\pi^2}\sum_{n=0}^{\infty}\frac{1}{n!}\frac{1}{E_n}\iiint_{-\infty}^{\infty}dk_1dp_2dp_3\times\frac{1}{(k_1^2+p_2^2+p_3^2)}\left(\frac{p_2^2+k_1^2}{2eH_0}\right)\times\exp[-(p_2^2+k_1^2)/2eH_0].^{**}$$

Going over to polar coordinates for the variable k_1, p_2 we get (writing $r^2=k_1^2+p_2^2$ and $p_3=p$)

$$\Delta E_{\text{static}}=\frac{e^2m}{2\pi}\sum_{n=0}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}dp\int_0^{\infty}rdr\frac{1}{r^2+p^2}\times\left(\frac{r^2}{2eH_0}\right)^n e^{-r^2/2eH_0}\frac{1}{\sqrt{m^2+p^2+2eH_0n}}$$

$$=\frac{e^2m}{2\pi}\int_{-\infty}^{\infty}dp\int_0^{\infty}rdr\frac{1}{(r^2+p^2)}f$$

$$f\equiv\sum_{n=0}^{\infty}\frac{\eta^n}{n!(m^2+p^2+2eH_0n)^{\frac{1}{2}}}e^{-\eta},\quad \eta\equiv\frac{r^2}{2eH_0}.$$

Now we need f for vanishingly small values of H_0, since it is the linear term (in H_0) of ΔE_{static} which gives the change in g-factor of the electron. Therefore, we seek to sum this series for $H_0\rightarrow 0$, i.e., for $\eta\rightarrow\infty$. The function $\eta^n/n!$ for η very large has a sharp maximum at $n=\eta$. On the other hand the function

$$g(n)=(m^2+p^2+2eH_0n)^{-\frac{1}{2}}$$

is, under the same conditions, a very slowly varying function of n. We therefore expand it about the point $n=\eta$, and get

$$g(n)=g(\eta)+(n-\eta)g'(\eta)+\frac{(n-\eta)^2}{2!}g''(\eta)+\cdots$$

or

$$f=e^{-\eta}\sum_{n=0}^{\infty}\left(g(\eta)+(n-\eta)g'(\eta)+\frac{(n-\eta)^2}{2!}g''(\eta)+\cdots\right)(\eta^n/n!)$$

$$=g(\eta)+[\eta g''(\eta)/2]+\cdots$$

$$=[1/(m^2+p^2+r^2)^{\frac{1}{2}}]+\tfrac{3}{4}[(eH_0)r^2/(m^2+p^2+r^2)^{5/2}]+\cdots.$$

Therefore,

$$\Delta E_{\text{static}}=\frac{e^2m}{2\pi}\int_{-\infty}^{\infty}dp\int_0^{\infty}rdr\frac{1}{(r^2+p^2)(m^2+p^2+r^2)^{\frac{1}{2}}}\times\left[1+\frac{3}{4}\frac{(eH_0)r^2}{(m^2+p^2+r^2)^2}+\cdots\right]$$

$$=\Delta E_{\text{static}}(0)+\frac{3e^2m}{8\pi}\left(\int_{-\infty}^{\infty}dp\int_0^{\infty}rdr\times\frac{r^2}{m(r^2+p^2)(m^2+p^2+r^2)^{5/2}}\right)(eH_0)+\cdots$$

$$=\Delta E_{\text{static}}(0)+(e^2/3\pi)(e/2m)H_0+\cdots.\qquad(8)$$

The first term $\Delta E_{\text{static}}(0)$ is simply the electrostatic self-energy of an electron at rest.[7] The remaining term converges, and is to be interpreted as the change of the g-factor of the electron due to its interaction with longitudinal photons.

Proceeding now to the expression for the dynamic self-energy, we find (after carrying out the integration in $V_{mn}^{(s)}$ several obvious summations, and simplifying):

$$\Delta E_{\text{dynamic}}=F(m)-F(-m),$$

where

$$F(m)\equiv\sum_{n=0}^{\infty}(e^2/4\pi)\int_{-\infty}^{\infty}dp\int_0^{\infty}\frac{\eta^n e^{-\eta}}{n!}\times\left\{\frac{1+(p^2/k^2)}{E_{n+1}(k+E_{n+1}-m)}+\frac{1-(p^2/k^2)}{E_n(k+E_n-m)}\right\}rdr,$$

$$k^2\equiv p^2+r^2.$$

Exactly the same method as before may be used to carry out the summation over n. We

** Here we have replaced the sums by integrals

$$\sum_{k_1}\rightarrow(G^{\frac{1}{3}}/2\pi)\int dk_1,\text{ etc.}$$

[7] V. F. Weisskopf, Phys. Rev. 56, 72 (1939).

shall not enter into the details of this straightforward calculation, but shall only quote the result:

$$\Delta E_{\text{dynamic}}=\Delta E_{\text{dynamic}}(0) -(5e^2/6\pi)(e/2m)H_0+\cdots. \quad (9)$$

The first term is simply the dynamic self-energy of an electron at rest (cf. reference 7). What remains converges and is the contribution of the transverse photons to the magnetic moment of the electron. We thus get for the energy of an electron in the state m,

$$\Delta E=\Delta E_{\text{static}}(0)+\Delta E_{\text{dynamic}}(0) -(e^2/2\pi)(e/2m)H_0 \quad (10)$$

(dropping higher powers of H_0). The sum of the first two terms of (10) is simply $E_{\text{self}}(0)$, and according to the ideas of the first section this is to be subtracted. Therefore, we are left with

$$\Delta E_{\text{true}}=-(e^2/2\pi)(e/2m)H_0. \quad (11)^{***}$$

(11) is to be regarded as the true change in energy of an electron in an external magnetic field, due to coupling with the radiation field. To interpret this in terms of a change in g-factor we note that if the electron had a g-factor of $g=2(1+\delta)$, then in the state m its energy would be

$$E=eH_0/2m(1-\tfrac{1}{2}g)=-(eH_0/2m)\delta.$$

Equating E and ΔE_{true} we get

$$\delta=(1/2\pi)e^2,$$

or $\delta=(1/2\pi)(e^2/\hbar c)$ in conventional units. Therefore

$$g=2(1+(1/2\pi)(e^2/\hbar c)). \quad (12)$$

This corresponds exactly to the result of Schwinger.

In conclusion I should like to thank Professor W. Pauli and Doctor Res Jost for many valuable and stimulating discussions.

APPENDIX

The eigenfunctions fall into four classes, corresponding to the two different spins and signs of energy. They are

$$E_1=+[m^2+p_3^2+2eH_0(n+1)]^{\frac{1}{2}},$$

$$\psi_1(n,p_2,p_3)=\exp i(p_2y+p_3z)/[2(E_1^2+mE_1)]^{\frac{1}{2}}\begin{bmatrix}(E_1+m)v^{(n)}\\ 0\\ p_3v^{(n)}\\ -(eH_0(2)(n+1))^{\frac{1}{2}}v^{(n+1)}\end{bmatrix}(1/G^{\frac{1}{2}}),$$

$$E_2=+[m^2+p_3^2+2eH_0(n)]^{\frac{1}{2}},$$

$$\psi_2(n,p_2,p_3)=\exp i(p_2y+p_3z)/[2(E_2^2+mE_2)]^{\frac{1}{2}}\begin{bmatrix}0\\ (E_2+m)v^{(n)}\\ -(2eH_0n)^{\frac{1}{2}}v^{(n-1)}\\ -p_3v^{(n)}\end{bmatrix}(1/G^{\frac{1}{2}}),$$

$$E_3=-[m^2+p_3^2+2eH_0(n+1)]^{\frac{1}{2}},$$
$$=-E_1,$$

$$\psi_3(n,p_2,p_3)=\exp i(p_2y+p_3z)/[2(E_1^2-mE_1)]^{\frac{1}{2}}\begin{bmatrix}v^{(n)}(-E_1+m)\\ 0\\ p_3v^{(n)}\\ -[eH_0(2)(n+1)]^{\frac{1}{2}}v^{(n+1)}\end{bmatrix}(1/G^{\frac{1}{2}}),$$

*** It should be mentioned that in reality nothing is subtracted from the magnetic moment term. One could have defined $M\equiv-\left.\frac{\partial E}{\partial H_0}\right|_{H_0=0}$, which would have given a finite result automatically. This consideration shows that we can expect our result to be relativistically correct, since we have used a Lorentz invariant formalism throughout.

$$E_4 = -[m^2+p_3^2+2eH_0(n)]^{\frac{1}{2}},$$
$$= -E_2,$$

$$\psi_4(n, p_2, p_3) = \exp i(p_2y+p_3z)/[2(E_2^2-mE_2)]^{\frac{1}{2}} \begin{pmatrix} 0 \\ v^{(n)}(-E_2+m) \\ -v^{(n-1)}(2eH_0n)^{\frac{1}{2}} \\ -p_3v^{(n)} \end{pmatrix} (1/G^{\frac{1}{2}}),$$

where

$$v^{(n)} = e^{-\xi^2/2}H_n(\xi)[(eH_0)^{\frac{1}{4}}/\pi^{\frac{1}{4}}2^{n/2}(n!)^{\frac{1}{2}}], \quad \xi = (eH_0)^{\frac{1}{2}}[x-(p_2/eH_0)].$$

H_n are the ordinary Hermite polynomials.

The usual representations of the Dirac matrices have been used, with the exception that α_y and α_x have been interchanged.

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On the Life of Self-Quenching Counters*

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A Geiger-Müller counter filled with argon and ethyl acetate was connected to a mass spectrometer, and spectra were obtained after predetermined numbers of counts. Results showed that factors influencing the life of the counter were the disappearance of the quenching vapor and the formation of non-quenching vapors. An argon-methane counter, studied by the same method, showed that contamination of the insides of the counter, by the dissociation products, limited its life. This process occurred before any appreciable fraction of the methane was consumed.

INTRODUCTION

THE purpose of this investigation was to examine the factors influencing the life of self-quenching Geiger-Müller counters. The life of a counter is usually defined to be the number of counts that the counter is capable of detecting before becoming inoperative as a result of internal failure for any reason. Observed lives are known to vary from 10^7 counts for a methane counter to 10^{10} counts for an argon-alcohol counter.

PRESENT THEORIES FOR THE OBSERVED LIFETIME

The finite life of self-quenching counters is explained by theories proposed by S. A. Korff and R. D. Present,[1] by the Montgomerys,[2] and by Stever.[3] The explanation lies in the quenching mechanism of the counter. We first note that there are three essential quenching mechanisms. (a) Absorption of photons from the avalanche, (b) quenching of secondary emission when the positive ions reach the cathode, and (c) electrostatic quenching. The Korff-Present theory deals with mechanism (a) and (b). They showed that the polyatomic gas in a self-quenching counter has two functions: (*i*) To quench the ultraviolet photons that are emitted by the excited states of the inert gas and (*ii*) to quench secondary emission by positive ions reaching the cathode. The authors point out that the characteristic property of a polyatomic molecule which is of importance

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[1] S. A. Korff and R. D. Present, Phys. Rev. **65**, 274 (1944).

[2] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **57**, 1030 (1940).

[3] H. G. Stever, Phys. Rev. **61**, 38 (1942).