Temperature Dependence of Penetration Depth of a Magnetic Field in Superconductors

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IN view of conflicting statements in recent discussions[1,2] of the temperature variation of λ (penetration depth of a magnetic field into a superconductor), it is worth reviewing the available experimental data. The only evidence providing a sensitive test of any proposed law is that from the magnetic properties of the small particles in colloids,[3] which gives essentially $\lambda(T)/\lambda(0)$ (the absolute value of $\lambda(0)$ could not be deduced since the particle size was not accurately known). So far results have been obtained only for mercury and these show (Fig. 1) that

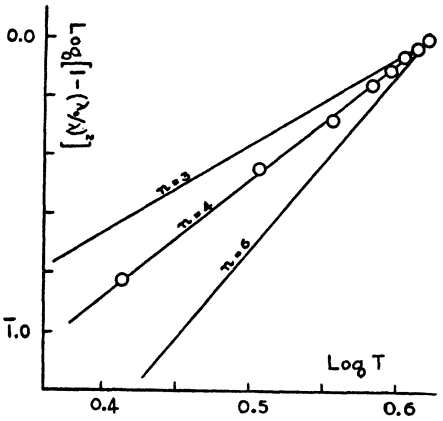

FIG. 1. $\text{Log}[1-(\lambda_0/\lambda)^2]$ *vs.* log$T$ for Hg.

the law

$$[\lambda(0)/\lambda(T)]^2=1-(T/T_c)^n \qquad (1)$$

with $n=4$ is very well obeyed, provided it is assumed (as is probably justified) that the spread in particle size in the colloid used was not sufficient to affect the results. This law is consistent with the Gorter-Casimir theory[4] if it is combined with the London theory[5] and the superconductor is assumed to have a parabolic H_c-T relation or a specific heat proportional to T^3. The relation (1) and this theoretical interpretation were first pointed out by Daunt[1] but owing to a misprint, only recently noticed, the n in (1) appeared as 3 instead of 4; a discussion has also been given by Miller.[6]

Other experiments have been of three kinds, (a) various measurements of $\Delta\lambda=(\lambda(T)-\lambda(T_0))$, with T_0 usually about 2.1°K,[7-10] (b) measurements of r-f resistance[11-13] from which λ can be deduced, but only indirectly and on the basis of an assumed model of the resistive process, and (c) measurements of the critical fields of thin wires[14] and thin films.[15] The measurements of $\Delta\lambda$ are all consistent within experimental error with (1), but they are not sufficiently accurate to prove the truth of (1) or to indicate more than a rough value of n, unless $\lambda(0)$ is already known. It should be noted that the full curves of $\Delta\lambda$ against T for mercury published by Désirant and Shoenberg and by Laurmann and Shoenberg in preliminary notes[8,9] are curves *calculated* from the colloid data (using a value of $\lambda(0)$ deduced by combining the colloid data with the $\Delta\lambda$ data from thin wires[8]); it is therefore impossible to deduce from these curves anything regarding n that is not already contained in the original colloid data. A detailed discussion of experiments on $\Delta\lambda$ was given at the recent conference on metals in Amsterdam.[16] As regards the r-f resistance measurements, it should be emphasized that any derivation of λ from them is only as reliable as the theoretical model of the resistive mechanism assumed; recent measurements at a higher frequency[17] suggest that the frequency dependence is not in accord with the theoretical model used by Pippard to derive values of λ. Thus it is unsafe as yet to use r-f resistance measurements as evidence for any particular law of temperature variation of λ. It may be noticed that the same theoretical model is partially involved also in Pippard's method[7] for $\Delta\lambda$, but since the specific theoretical assumptions enter only in a correction term, the results of this method should be reliable except very close to T_c. The critical field measurements again lead only indirectly to information about λ and, as has been discussed elsewhere,[8] their interpretation must await the development of a detailed theory.

[1] J. G. Daunt, Phys. Rev. **72**, 89 (1947).
[2] de Launay and Steele, Phys. Rev. **73**, 1450 (1948).
[3] D. Shoenberg, Proc. Roy. Soc. **A175**, 49 (1940).
[4] Gorter and Casimir, Physik. Zeits. **35**, 963 (1934).
[5] F. and H. London, Proc. Roy. Soc. **A149**, 71 (1935).
[6] Miller, Australian J. Sci. **10**, 172 (1948).
[7] Pippard, Nature **159**, 434 (1947).
[8] Désirant and Shoenberg, Nature **159**, 201 (1947) and Proc. Phys. Soc. **60**, 413 (1948).
[9] Laurmann and Shoenberg, Nature **160**, 474 (1947).
[10] Shalnikov and Sharvin, J. Exper. Theor. Phys. U.S.S.R. **18**, 102 (1948).
[11] H. London, Proc. Roy. Soc. **A176**, 522 (1940).
[12] Pippard, Proc. Roy. Soc. **A191**, 399 (1947).
[13] Maxwell, Marcus, and Slater, Bull. Am. Phys. Soc. **23**, 40 (1948).
[14] Pontius, Phil. Mag. **24**, 787 (1937).
[15] Appleyard, Bristow, Misener, and H. London, Proc. Roy. Soc. **A170**, 540 (1939).
[16] D. Shoenberg, Communication to the Conference on Metals, Amsterdam (1948).
[17] Pippard, Nature **162**, 68 (1948).

Frictional Electricity

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IN Phys. Rev. **66**, 9–16 (1944) you published a paper by Mr. D. E. Debeau on frictional electricity. He got the remarkable result that the charge produced on small insulating particles sliding down a metal depended very much on the pressure of the air. At very low pressures of the order of 10^{-4} mm of mercury the charge is large; as the pressure is increased up to about 1 mm it gets less and less and thereafter increases again with further pressure increase. The charge at 1 mm was only about 1/7 of that at 10^{-4}. He explains this result as being due to adsorbed layers of gas but to explain the minimum he is driven to the conclusion there must be two adsorbed layers.

A much simpler explanation seems worth investigating. The frictional charges are very large and after the stream has slid some distance over the metal the charge per unit volume is sufficient to create a large potential difference between the particles and the metal. If this gets high enough to produce a discharge through the air round the particles, it will put an automatic limit on the final charge on the particles. In this case we should expect the curve charge against pressure to resemble the well-known curve of sparking potential against pressure, which it does.

Additional confirmation is that, taking his experimental results for the charges at 1 mm pressure, it is easy to calculate that in the stem of the metal funnel down which the particles pass the P.D. between the center and the sides is of the order of 400 volts which is of the order of the minimum sparking potential.

Microwave Magnetic Dispersion in Carbonyl Iron Powder

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MEASUREMENTS have been made of the complex permeability of mixtures of carbonyl iron powder and paraffin wax at microwave frequencies, using the dual impedance wave guide method.[1] Typical results for the modulus of permeability $|\mu|$ and the magnetic loss angle δ_μ of specimens containing 18 percent by volume of iron powder are given in Table I. The modulus of permittivity

TABLE I.

λ (cm)	58.4	39.6	29.9	22.4	15.4	9.1	5.95	3.09
$\|\mu\|$	2.28	2.23	2.17	2.05	1.91	1.70	1.53	1.40
δ_μ	6.8°	9.0°	12.2°	13.3°	17.7°	17.7°	18.1°	21°

$|\epsilon|$ of the mixture and its dielectric loss angle δ_ϵ remained constant, within the experimental error, over the complet wave-length range, at 5.60 (±2 percent) and 1.0° (±0.1°), respectively.

The magnetic dispersion and absorption of iron does not display any of the resonance characteristics, attributable to internal anisotropy fields, observed in the iron oxides.[1,2] This marked difference in behavior is due to the high conductivity of iron, and the consequent skin effect which restricts the penetration of the microwave field into the iron, to a depth of 10^{-4} to 10^{-5} cm. The skin effect masks any *natural* resonance effects resulting from the anisotropy field, although the much more pronounced *induced* resonances can be observed, when the electron spins are aligned by an applied saturation magnetostatic field.[3,4]

Kittel[5] has pointed out that the skin depth in iron at microwave frequencies is only a small fraction of the domain thickness, so that the applied h.f. magnetic field only penetrates a little way down a surface domain boundary. The force on this boundary is consequently less than at low frequencies, so that the domain boundary shift and the resultant magnetization are correspondingly

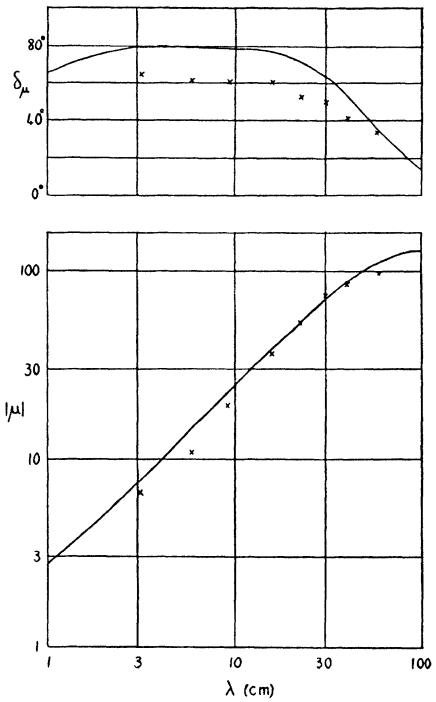

FIG. 1.

reduced, leading to a decrease in permeability with increase in frequency. Kittel has considered a thin film, one domain thick, as a theoretical model, and he has derived and evaluated expressions for its magnetic dispersion.

It is of interest to compare the experimental results with Kittel's theoretical curves. It has been found for carbonyl iron mixtures up to 50 percent concentration, that $|\mu|$ and $\tan\delta_\mu$ obey similar logarithmic and linear concentration laws to those observed for γ-ferric oxide mixtures.[1] Above this concentration, deviations from the laws become apparent, as a result of particle agglomeration. The measurements on mixtures below 50 percent concentration have been extrapolated to zero dilution, and the results are plotted together with the curves for Kittel's film model (see Fig. 1).

The extrapolated permeability values are not in general equivalent to observations on solid iron, since the majority of the particle surfaces responsible for the dispersion disappear into the bulk of the material as the concentration is increased. On the other hand, the extrapolation corresponds much more closely to a thin film, in which the particle surfaces remain exposed to the applied h.f. field. This presumably accounts for the good agreement between the extrapolated experimental data and Kittel's thin film model. The average particle size of the carbonyl iron was 5.5 microns compared with 2.5 microns for Kittel's model.

The results appear to substantiate the validity of the theory, and it is likely that an elaboration of the model, to take account of the eddy currents induced in the material behind the surface film, would lead to better agreement with the observations on solid iron. At wave-