tion between $^2P_{3/2}$ ground state and the $^2P_{\frac{1}{2}}$ state. Unfortunately, Caldirola has chosen the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ states as eigenstates of the same potential well. Consequently, the eigenfunctions are orthogonal, and the photo-magnetic effect vanishes. Caldirola apparently did not notice this and, using approximations, obtained a non-zero value for a vanishing integral.

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Notes on Feenberg's Series—Rearrangements*

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THIS note concerns a simplified discussion of the infinite series—rearrangement procedures recently employed by Feenberg.[1] The essential point in the simplification is that, by introducing specialized notation, the expressions involved can be handled in closed form. We shall consider only the algebraic aspects of the rearrangement procedure; questions of convergence cannot be adequately handled at this level of generality.[2]

If W_{ij} is any matrix and α any list of possible index values, introduce the following notation:

$$W_{ab}{}^{n+1}(\alpha)=\sum_{i_1\cdots i_n\neq\alpha} W_{ai_1}W_{i_1i_2}\cdots W_{i_{n-1}i_n}W_{i_nb}\quad (n\geqslant 1). \quad (1)$$

Briefly, (1) defines $W_{ab}{}^k(\alpha)$ as the (ab) element of the matrix W^k *except* that, in all sums involved in forming W^k, the index values listed in α are omitted. It is also convenient to extend (1) to include the trivial cases where no sums are involved.

$$W_{ab}{}^1(\alpha)=W_{ab};\quad W_{ab}{}^0(\alpha)=\delta_{ab}. \quad (1a)$$

In Feenberg's notation, the basic matrix involved in the manipulations to be discussed is $W_{ab}=V_{ab}/(a)$. (His restriction $V_{ii}=0$ is of course not needed and will not be used here.)

Now choose an index value q (not already listed in α) and regroup terms on the right of (1) in the following way:

$i_1=q$; others $\neq\alpha$ (A)
$i_1\neq q$ or α; $i_2=q$; others $\neq\alpha$ (B)
·
·
·
$i_1\cdots i_{n-1}\neq q$ or α; $i_n=q$ (N-1)
$i_1\cdots i_n\neq q,\ \alpha.$ (N)

We thereby obtain:

$$W_{ab}{}^{n+1}(\alpha)=\sum_{i=0}^{n-1} W_{aq}{}^{i+1}(\alpha q)W_{qb}{}^{n-i}(\alpha)+W_{ab}{}^{n+1}(\alpha q). \quad (2a)$$

Regrouping in the reverse order ($i_n=q$, others $\neq\alpha$, etc.) we find analogously:

$$W_{ab}{}^{n+1}(\alpha)=\sum_{i=0}^{n-1} W_{aq}{}^{n-i}(\alpha)W_{qb}{}^{i+1}(\alpha q)+W_{ab}{}^{n+1}(\alpha q). \quad (2b)$$

Equations (2) are the fundamental relations on which the work below is based; they are the closed forms of [8] and [12] of Feenberg's paper.

In particular, if α does not include a and/or b

$$W_{ab}{}^{n+1}(\alpha)=\sum_{i=0}^{n} W_{ab}{}^{i+1}(\alpha b)W_{bb}{}^{n-i}(\alpha)=\sum_{i=0}^{n} W_{ab}{}^{i+1}(\alpha a)W_{aa}{}^{n-i}(\alpha).$$

Hence, by Cauchy's rule for multiplication of infinite series, if α does not include a and/or b

$$\sum_{n=0}^{\infty} W_{ab}{}^{n+1}(\alpha)=\left[\sum_{n=0}^{\infty} W_{ab}{}^{n+1}(\alpha b)\right]\left[\sum_{n=0}^{\infty} W_{bb}{}^{n}(\alpha)\right]=\left[\sum_0^{\infty} W_{ab}{}^{n+1}(\alpha a)\right]\left[\sum_0^{\infty} W_{aa}{}^{n}(\alpha)\right]. \quad (3)$$

This result immediately establishes[3] Feenberg's [9] and [13]. As a special use of (3), we have

$$\sum_0^{\infty} W_{bb}{}^n(\alpha)=1+\left[\sum_0^{\infty} W_{bb}{}^{n+1}(\alpha b)\right]\left[\sum_0^{\infty} W_{bb}{}^n(\alpha)\right].$$

Thus, either by solving for the quantity on the left or by reapplying this relation indefinitely, we find

$$\sum_0^{\infty} W_{bb}{}^n(\alpha)=\sum_{V=0}^{\infty}\left[\sum_{n=0}^{\infty} W_{bb}{}^{n+1}(\alpha b)\right]^V=1\Big/1-\sum_0^{\infty} W_{bb}{}^{n+1}(\alpha b) \quad (4)$$

which is Feenberg's formal identity [16].

All of the previous formulae break down if the list α already includes the index value to be "removed." The following devices may, however, be employed when α includes both of the matrix indices.

By definitions (1), (1a):

$$\sum_0^{\infty} W_{xa}{}^{n+1}(\alpha)=W_{xa}+\sum_{b\neq\alpha} W_{xb}\sum_0^{\infty} W_{ba}{}^{n+1}(\alpha)$$

by (3):

$$=W_{xa}+\sum_{b\neq\alpha} W_{xb}\left[\sum_0^{\infty} W_{bb}{}^n(\alpha)\right]\left[\sum_0^{\infty} W_{ba}{}^{n+1}(\alpha b)\right].$$

If, at this point, we use (4), we get

$$\sum_0^{\infty} W_{xa}{}^{n+1}(\alpha)=W_{xa}+\sum_{b\neq\alpha}\left\{W_{xb}\sum_0^{\infty} W_{ba}{}^{n+1}(\alpha b)\Big/1-\sum_0^{\infty} W_{bb}{}^{n+1}(\alpha b)\right\} \quad (5)$$

which is Feenberg's [29] (last half) and [30]. Alternatively, we may repeat the process on the last factor of the last term above.

$$\sum_0^{\infty} W_{xa}{}^{n+1}(\alpha)=W_{xa}+\sum_{b\neq\alpha} W_{xb}\sum_0^{\infty} W_{bb}{}^n(\alpha)\times\left\{W_{ba}+\sum_{c\neq\alpha b} W_{bc}\sum_0^{\infty} W_{cc}{}^n(\alpha b)\sum_0^{\infty} W_{ca}{}^{n+1}(\alpha bc)\right\}.$$

Repeating indefinitely, we obtain in the limit:

$$\sum_0^{\infty} W_{xa}{}^{n+1}(\alpha)=W_{xa}+\sum_{b\neq\alpha} W_{xb}W_{ba}\sum_0^{\infty} W_{bb}{}^n(\alpha)+\sum_{\substack{b\neq\alpha\\ c\neq\alpha b}} W_{xb}W_{bc}W_{ca}\sum_0^{\infty} W_{bb}{}^n(\alpha)\sum_0^{\infty} W_{cc}{}^n(\alpha b)+\cdots. \quad (6)$$

Using (4) in (6) we have Feenberg's [21], [24], [29].

Slightly altered analogues to (5) and (6) can be obtained by using the above devices on the last element of each term, instead of the first, thus:

$$\sum_0^\infty W_{xa}{}^{n+1}(\alpha) = W_{xa} + \sum_{b\neq\alpha}\left[\sum_0^\infty W_{xb}{}^{n+1}(\alpha)\right]W_{ba}.$$

Proceeding as before, we find

$$\sum_0^\infty W_{xa}{}^{n+1}(\alpha) = W_{xa} + \sum_{b\neq\alpha}\left\{W_{ba}\sum_0^\infty W_{xb}{}^{n+1}(\alpha b)\Big/1-\sum_0^\infty W_{bb}{}^{n+1}(\alpha b)\right\}, \quad (5a)$$

$$\sum_0^\infty W_{xa}{}^{n+1}(\alpha) = W_{xa} + \sum_{b\neq\alpha} W_{xb}W_{ba}\sum_0^\infty W_{bb}{}^n(\alpha) + \sum_{\substack{b\neq\alpha\\ c=\alpha b}} W_{xc}W_{cb}W_{ba}\sum_0^\infty W_{bb}{}^n(\alpha)\sum_0^\infty W_{cc}{}^n(\alpha b)+\cdots. \quad (6a)$$

The writer is indebted to Professor Feenberg for a pre-publication copy of the paper on which this work is based.

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[1] Eugene Feenberg, "Theory of Scattering Processes," Phys. Rev. **74**, 664 (1948).

[2] It might be pointed out in this connection that one must be careful in attempting to infer the analytic properties of a function from its series representation. Consider for example $f(Z)=\Sigma(1/1-z)^{n+1}$; recalling Laurent series, we might be tempted to suggest that f has an essential singularity at $Z=1$. Actually, $f=-1/z$, which is analytic at $z=1$. The pole on $|z-1|=1$ is duly indicated by failure of convergence, but the series can tell us nothing about regions $|z-1|\leqslant 1$, where it does not converge.

[3] The connections between the present notation and that of Feenberg are:

$$W_{ab}=V_{ab}/(a); \quad S'_{xa}=\sum_0^\infty W_{xa}{}^n$$

$$(\mathfrak{E}_{\alpha p}-E_p)/(p)=\sum_0^\infty W_{pp}{}^{n+1}(\alpha p); \quad \bar{U}_{\alpha;xa}=\sum_0^\infty W_{xa}{}^{n+1}(\alpha).$$

The Emission of Alpha-Particles from the Different Faces of a Radioactive Crystal

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In a letter recently published, K. B. Mather and F. N. D. Kurie[1] state that they can find no significant difference in the α-ray emission of the various faces of a ThSO crystal. This agrees with Merton's result for uranium nitrate,[2] but differs, as they point out, from Muhlestein's experiments.[3]

Preliminary tests have been conducted here on uranium nitrate using the photographic emulsion technique, and would appear to confirm the nul result, but a small effect of the order of, say, one or two percent might not have been detected.

It is perhaps dangerous to apply naive macroscopic concepts to the structure of the nucleus, but if the above experiments had given a positive result, it would seem to imply that

(a) the nucleus in question was not spherically symmetrical but had definite axes of symmetry.
(b) the orientation of the nucleus was controlled by the crystalline forces.

It might be assumed that nuclei possessing nuclear spin and magnetic moment would fulfill the first condition. Elements of even atomic number and mass have zero spin and magnetic moment. Both U^{238} and Th^{232} fall in this class so a nul result for them might be anticipated. U^{235} has, however, a spin of either 5/2 or 7/2,[4] and presumably U^{233} and Pu^{239} would also possess spins greater than zero. It would accordingly seem to be worth while to try the same experiments using one of the above nucleides but, perhaps fortunately, the necessary material seems to be available only in America, as yet.

It may be noted that the amount of U^{235} present in natural uranium could not account for Muhlestein's result. Allowing for the fact that the equilibrium amount of U^{234} will also be present, it can be calculated from the accepted half-lives of U^{238} and U^{235} that only slightly over two percent of the α-rays from natural uranium, free from its decomposition products, would be produced by U^{235}. This could not account for Muhlestein's differences of 15 percent or more between the emission of the various faces of the uranium nitrate crystal.

[1] K. B. Mather and F. N. D. Curie, Phys. Rev. **73**, 1474 (1948).
[2] Merton, Phil. Mag. **38**, 463 (1919).
[3] Muhlestein, Arch-de-So. **2**, 240 (1920).
[4] O. E. Anderson and H. E. White, Phys. Rev. **71**, 911 (1947).

On the Motion of Point Electrons

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It has been shown by Pryce[1] that it is possible to derive finite equations of motion for a point electron, as well as finite electromagnetic self-energy, by subtracting a symmetric non-divergent tensor $A^{\nu\sigma}$ ($\nu, \sigma=0, 1, 2, 3$) from Maxwell's energy-momentum tensor $T^{\nu\sigma}$. In order to assure the non-divergence of $A^{\nu\sigma}$ Pryce assumed

$$A^{\nu\sigma}=\partial K^{\mu\nu\sigma}/\partial x^\mu, \quad (1)$$

where $K^{\mu\nu\sigma}$ is antisymmetrical in μ and ν, and is a function of the variables of the electron and the four coordinates of the point under consideration $x^\mu\cdot K^{\mu\nu\sigma}$ has to be chosen so that $A^{\nu\sigma}$ will be symmetrical and will have the same singularities of the third and fourth order on the world line of the electron as Maxwell's tensor. By use of such a tensor $K^{\mu\nu\sigma}$ Pryce derived the well-known Lorentz-Dirac equations of motion,[2]

$$mv^\sigma-\tfrac{2}{3}e^2(\ddot{v}^\sigma+(\dot{v}, \dot{v})v^\sigma)=ev^\mu F^\sigma{}_{\text{ext}\mu},$$

where $F^\sigma{}_{\text{ext}\mu}$ is the external electromagnetic field.

It has been proved by Bhabha[3] that this result is independent of the particular choice of $A^{\nu\sigma}$, provided Eq. (1) is satisfied.

But as shown by Dirac[2] and Eliezer[4] there are serious difficulties in interpreting the solutions of Eqs. (2) when