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Polarization and Direction of Propagation of Successive Quanta*

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The correlation between the direction of propagation of one quantum and the polarization of another quantum, when the two are emitted successively (in any order) by a radiating system, is investigated theoretically. This direction-polarization correlation is found to be capable of determining the relative parities of all three levels of the radiating system. The correlation is simply related to the already investigated correlation between the directions of propagation of two successive quanta. Results are given explicitly for the possible combinations of dipole and quadrupole radiation. The discussion applies also to certain types of resonance radiation.

I. INTRODUCTION

THE experimental observation[1] of the predicted[2,3] correlation in the directions of propagation of two successive gamma-rays emitted by a radiating nucleus, and the usefulness of this correlation in providing information on gamma-ray multipolarity and on the spins of the relevant nuclear levels, have stimulated interest in further correlations involving the polarizations of two such successive quanta. It has been pointed out by Falkoff[4] that while only the orders of multipolarity of the gamma-quanta enter into the directional correlation, the electric or magnetic nature of the radiating multipole of given order influences the polarization correlation. Falkoff has calculated the polarization correlation for two quanta emitted at an angle of 180° with each other. He finds, in particular, that when the two transitions involve one electric and one magnetic multipole, the correlation differs from that to be expected when both of a given pair of multipoles are electric or both magnetic; the experiment thus determines the relative parity of the initial and final levels of the three levels involved. Along somewhat different lines the present author has treated[5] the case where successive electric dipole quanta are emitted at an arbitrary angle to each other. However, this treatment, which may easily be generalized to arbitrary combinations of magnetic and electric dipoles, was a by-product of a discussion of resonance radiation, and hence specific results are given only for transitions in which the initial and final levels have the same angular momenta.

The present discussion provides the theoretical background for an experiment in which one measures the correlation between the direction of propagation of one quantum and the direction of polarization of another quantum emitted at an angle θ to the first. Just as there is a close relation between resonance radiation and the successive emission of two quanta from a single system, the existence of the correlation under discussion is to be expected by analogy with the polarization of the resonance radiation excited by unpolarized light.

For applications to gamma-radiation, the elimination of one of the polarization-measuring counters is an obvious advantage since polarization-sensitive counters for gamma-radiation[6] involve counting the gamma-quantum after a scattering process. In addition to this saving of one polarization measurement, the present arrangement in many cases allows one to determine enough about the electric or magnetic nature of the two successive transitions to specify the relative parity of all three nuclear levels involved. This is possible in all cases if by some means one may discriminate in favor of detecting coincidences in which the first quantum goes to an arbitrarily chosen, particular one of the counters.

II. THEORY AND CALCULATIONS

It will be noted immediately that since the two successive quanta are not detected in iden-

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[1] E. L. Brady and M. Deutsch, Phys. Rev. **72**, 870 (1947).
[2] D. R. Hamilton, Phys. Rev. **58**, 122 (1940).
[3] G. Goertzel, Phys. Rev. **70**, 897 (1946).
[4] D. L. Falkoff, Phys. Rev. **73**, 518 (1948).
[5] D. R. Hamilton, Astrophys. J., **106**, 457 (1947).

[6] H. S. Snyder, S. Pasternack, and J. Hornbostel, Phys. Rev. **73**, 440 (1948).

tical counters, the experiment under discussion possesses a dissymmetry in comparison to the 180° polarization-polarization correlation experiment of reference 4. One might thus expect the results to depend on whether the first or the second quantum is detected in the polarization-insensitive counter. Such a dependence in fact arises in exactly half the cases considered; this situation can be covered, however, by a simple generalization from the case where the first quantum is assumed to go to the polarization-insensitive counter. Therefore, this assumption will be the basis of discussion until further notice.

The details of the calculations follow very closely reference 2, with which a familiarity is assumed. There the directional correlation of the two quanta is specified in terms of the probability, $W(\theta)$, that the second quantum will come off with a direction of propagation making an angle θ with that of the first quantum. Formally, the direction of propagation of the first quantum is taken as the axis of quantization; $W(\theta)$ is then the angular distribution of the second quantum with respect to this axis, and the discussion is formulated in terms of the transitions between the various magnetic sub-states A_l, B_n, C_p of the initial, intermediate, and final levels (A, B, C) of the radiating system. With the above choice of axis of quantization and with the first quantum unpolarized, the phases of the intermediate states B_n are random; the specific multipoles which radiate the second quantum in the transitions B_nC_p are randomly phased and therefore radiate independently, with an intensity proportional to the (unequal) populations of the states B_n and to the B_nC_p transition probabilities. The details of the radiation of the second quantum from this assemblage of multipoles will now be summarized.

Given the electric or magnetic multipole associated with the line BC, the polarization of the quantum emitted in a given direction in the transition B_nC_p is determined by $\Delta m \equiv p-n$. The polarization is (for dipole and quadrupole and presumably for higher multipoles) elliptical with the principal axes of the ellipse parallel to the unit vectors $\boldsymbol{\theta}_0$ and $\boldsymbol{\phi}_0$ which in turn lie along the directions of increasing θ and φ in the usual spherical coordinate system. With the transitions B_nC_p and $B_{-n}C_{-p}$ there are associated equal transition probabilities, and polarization ellipses identical in shape and orientation but opposite in sense of traversal. Since the multipoles associated with B_nC_p and $B_{-n}C_{-p}$ are randomly phased (for the conditions assumed here, i.e., no knowledge of the polarization of first quantum) these two randomly phased ellipses are equivalent to two randomly phased waves linearly polarized along $\boldsymbol{\theta}_0$ and $\boldsymbol{\phi}_0$. Thus all the information about the polarization of the second quantum may be expressed by stating the intensities J_θ and J_φ in the linear polarizations $\boldsymbol{\theta}_0$ and $\boldsymbol{\phi}_0$.

In a notation analogous to that of reference 2, the relative values of the J_θ and J_φ associated with a given Δm may be denoted by $f_{|\Delta m|\theta}$ and $f_{|\Delta m|\varphi}$ for electric dipole radiation and by $g_{|\Delta m|\theta}$ and $g_{|\Delta m|\varphi}$ for electric quadrupole radiation; similarly, $f_{|\Delta m|} \equiv f_{|\Delta m|\theta}+f_{|\Delta m|\varphi}$ and $g_{|\Delta m|} \equiv g_{|\Delta m|\theta}+g_{|\Delta m|\varphi}$. The dependence of these functions on the angle θ between the axis of quantization and the direction of propagation of the quantum in question is given[7] by

$$\begin{array}{lll}
f_{1\theta}=\frac{1}{2}\cos^2\theta & f_{1\varphi}=\frac{1}{2} & f_1=\frac{1}{2}(1+\cos^2\theta) \\
f_0\theta=1-\cos^2\theta & f_{0\varphi}=0 & f_0=1-\cos^2\theta \\
g_{2\theta}=\frac{1}{2}(\cos^2\theta-\cos^4\theta) & g_{2\varphi}=\frac{1}{2}(1-\cos^2\theta) & g_2=\frac{1}{2}(1-\cos^4\theta) \\
g_{1\theta}=\frac{1}{2}(4\cos^4\theta-4\cos^2\theta+1) & g_{1\varphi}=\frac{1}{2}\cos^2\theta & g_1=\frac{1}{2}(4\cos^4\theta-3\cos^2\theta+1) \\
g_{0\theta}=3(\cos^2\theta-\cos^4\theta) & g_{0\varphi}=0 & g_0=3(\cos^2\theta-\cos^4\theta).
\end{array} \tag{1}$$

It will be noted that

$$2f_{1\theta}+f_{0\theta}=2f_{1\varphi}+f_{0\varphi}=2g_{2\theta}+2g_{1\theta}+g_{0\theta}$$
$$=2g_{2\varphi}+2g_{1\varphi}+g_{0\varphi}=2,$$

corresponding to the fact that if transitions with the various Δm occur with equal intensity, the resulting radiation is isotropic and unpolarized.

[7] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1935); for example, Chap. IV.

The analogous equations for magnetic dipole and magnetic quadrupole radiation are obtained from Eq. (1) by interchanging θ and φ where they occur in the subscripts. This corresponds to the fact that the radiation fields of the electric and magnetic multipoles associated with a given Δm differ only by the interchange of E and H; their angular distributions of radiation are identical, their polarizations orthogonal. Thus, for any given line BC with given multipole order one will have $(J_\theta)_{\text{elec}}=(J_\varphi)_{\text{mag}}$ and $(J_\varphi)_{\text{elec}}=(J_\theta)_{\text{mag}}$.

If the relative number of transitions in the various Δm is denoted by $\beta_{|\Delta m|}$ for dipole radiation and by $\gamma_{|\Delta m|}$ for quadrupole radiation, then the total radiated intensity (all polarizations) is given for dipole radiation by

$$W(\theta)=\beta_0 f_0+2\beta_1 f_1, \tag{2}$$

and for quadrupole radiation by

$$W(\theta)=\gamma_0 g_0+2\gamma_1 g_1+2\gamma_2 g_2.$$

In terms of the foregoing β and γ, the J's are given for dipole radiation by

$$\left.\begin{aligned}(J_\theta)_{\text{elec}}=(J_\varphi)_{\text{mag}}=\beta_0 f_{0\theta}+2\beta_1 f_{1\theta},\\ (J_\varphi)_{\text{elec}}=(J_\theta)_{\text{mag}}=\beta_0 f_{0\varphi}+2\beta_1 f_{1\varphi},\end{aligned}\right\}\quad \text{(dipole)} \tag{4}$$

and for quadrupole radiation by

$$\left.\begin{aligned}(J_\theta)_{\text{elec}}=(J_\varphi)_{\text{mag}}=\gamma_0 g_{0\theta}+2\gamma_1 g_{1\theta}+2\gamma_2 g_{2\theta},\\ (J_\varphi)_{\text{elec}}=(J_\theta)_{\text{mag}}=\gamma_0 g_{0\varphi}+2\gamma_1 g_{1\varphi}+2\gamma_2 g_{2\varphi}.\end{aligned}\right\}\quad \text{(quadrupole)} \tag{5}$$

The function $W(\theta)$ of Eqs. (2) and (3) is, of course, simply the angular correlation function of reference 2, there expressed as a power series in $\cos^2\theta$:

$$W(\theta)=1+\alpha_2\cos^2\theta+\alpha_4\cos^4\theta. \tag{6}$$

(Here α_2 and α_4 are the quantities R/Q and S/Q of the earlier work.) From Eqs. (1), (2), (3), and (6) the values of the β and γ may be found directly in terms of the α_2 and α_4 which have already been calculated in reference 2. The $W(\theta)$ of Eq. (6) has no particular normalization. However, in the later discussion of the case where the two transitions have different multipole order it is useful to have $W(\theta)$, when expressed in the form of Eqs. (2) or (3), normalized in some way, e.g.,

$$\int W(\theta)d(\text{solid angle}) =\int(J_\theta+J_\varphi)d(\text{solid angle})=8\pi.$$

With this condition, the β and γ are found from Eqs. (1), (2), (3), and (6) to be given by

$$\left.\begin{aligned}(3+\alpha_2)\beta_0=3(1-\alpha_2),\\ (3+\alpha_2)\beta_1=3(1+\alpha_2),\end{aligned}\right. \tag{7a}$$

$$\left.\begin{aligned}(15+5\alpha_2+3\alpha_4)\gamma_0=5(3+5\alpha_2+3\alpha_4),\\ (15+5\alpha_2+3\alpha_4)\gamma_1=15(1+\alpha_2+\alpha_4),\\ (15+5\alpha_2+3\alpha_4)\gamma_2=15(1-\alpha_2-\alpha_4).\end{aligned}\right. \tag{7b}$$

From Eqs. (1), (4), (5), and (7) we then find directly for dipole radiation

$$\left.\begin{aligned}(3+\alpha_2)(J_\theta)_{\text{elec}}=3(1+\alpha_2\cos 2\theta),\\ (3+\alpha_2)(J_\varphi)_{\text{elec}}=3(1+\alpha_2),\end{aligned}\right\}\quad \text{(dipole)} \tag{8a}$$

and for quadrupole radiation

$$\left.\begin{aligned}(15+5\alpha_2+3\alpha_4)(J_\theta)_{\text{elec}} =15[(1+\alpha_2+\alpha_4)-\tfrac{1}{2}\alpha_4\sin^2 2\theta],\\ (15+5\alpha_2+3\alpha_4)(J_\varphi)_{\text{elec}} =15[1+(\alpha_2+\alpha_4)\cos 2\theta].\end{aligned}\right\}\quad \text{(quadrupole)} \tag{8b}$$

It will be recalled now that the "dipole" and "quadrupole" labels in Eq. (8) refer to the second quantum only, corresponding to the assumption that the first quantum goes to the polarization-insensitive detector; the properties of this first quantum enter Eq. (8) only through their influence on α_2 and α_4. One might conceivably, although with some difficulty, arrange an experiment to correspond to this assumption; but, in general, one will want to know what happens when each counter is able to detect either the first or the second quantum, but not with the same efficiency. As a necessary preliminary to this more general question it turns out that we must first consider a question complementary to that answered by Eq. (8): What happens when the only observed coincidences are those in which the second quantum goes to the polarization-insensitive detector?

A proposition very relevant to this question concerns the directional correlation function

$W(\theta)$ discussed in reference 2. In that reference it is shown that if two pairs of transitions are each other's inverses—i.e., differ by an interchange of initial and final angular momenta and of first and second radiation processes—then $W(\theta)$ has the same dependence on θ for both pairs of transitions. To state this more formally for purposes of extension, let $W_{ABC}(\boldsymbol{\kappa}_1\mathbf{e}_1\boldsymbol{\kappa}_2\mathbf{e}_2)$ be the probability of emission of a quantum with unit propagation and polarization vectors $\boldsymbol{\kappa}_1$ and $\mathbf{e}_1$ in the line AB, followed by a quantum $\boldsymbol{\kappa}_2\mathbf{e}_2$ in the line BC; then a statement from p. 128 of reference 2 may be expressed in the form

$$\sum_{e'e''} W_{ABC}(\boldsymbol{\kappa}'\mathbf{e}'\boldsymbol{\kappa}''\mathbf{e}'') = \sum_{e'e''} W_{CBA}(\boldsymbol{\kappa}''\mathbf{e}''\boldsymbol{\kappa}'\mathbf{e}') = W(\theta),$$

where θ is, of course, the angle between $\boldsymbol{\kappa}'$ and $\boldsymbol{\kappa}''$. Here the state A is alternately the initial and final state, but in both cases has the same angular momentum and is connected to the state B by a radiative transition of the same nature. If the reasoning used in obtaining the above theorem is interrupted short of the above result, one finds that it is also true that

$$W_{ABC}(\boldsymbol{\kappa}'\mathbf{e}'\boldsymbol{\kappa}''\mathbf{e}'') = W_{CBA}(\boldsymbol{\kappa}''\mathbf{e}''\boldsymbol{\kappa}'\mathbf{e}'). \quad (9)$$

Now, the values of J_φ when the first or the second quantum goes along the z-axis to the polarization-insensitive detector are proportional, respectively, to

$$\sum_{e'} W_{ABC}(\mathbf{k}\mathbf{e}'\boldsymbol{\kappa}''\boldsymbol{\phi}_0) \quad \text{and} \quad \sum_{e'} W_{ABC}(\boldsymbol{\kappa}''\boldsymbol{\phi}_0\mathbf{k}\mathbf{e}');$$

and it is apparent from Eq. (9) that

$$\sum_{e'} W_{ABC}(\boldsymbol{\kappa}''\boldsymbol{\phi}_0\mathbf{k}\mathbf{e}') = \sum_{e'} W_{CBA}(\mathbf{k}\mathbf{e}'\boldsymbol{\kappa}''\boldsymbol{\phi}_0). \quad (10)$$

This says, in words, that to find J_φ when the second quantum, instead of the first, goes to the polarization-insensitive detector one finds what J_φ would be if the first quantum of the inverse process $C\rightarrow B\rightarrow A$ were to go to this detector. A similar statement, of course, holds for J_θ.

Let us now return to the more general situation in which neither detector can distinguish completely between the first and second quanta. We may denote by a_{BC} the relative over-all efficiency for detection of the first (AB) quantum in the unpolarized detector and the second (BC) in the polarization-sensitive detector (a_{BC} will thus be simply the product of the individual counter efficiencies for these quanta); and let a_{AB} denote relative over-all efficiency for the converse process (second quantum to unpolarized detector). Similarly, let $J_{\theta BC}$ be the expected value of J_θ when $a_{BC}=1$ and $a_{AB}=0$ (the case to which Eq. (8) corresponds), and $J_{\theta AB}$ the same for $a_{BC}=0$ and $a_{AB}=1$. In the notation of Eq. (10),

$$\begin{aligned} J_{\theta BC} &\sim \sum_{e'} W_{ABC}(\mathbf{k}\mathbf{e}'\boldsymbol{\kappa}''\boldsymbol{\theta}_0), \\ J_{\theta AB} &\sim \sum_{e'} W_{ABC}(\boldsymbol{\kappa}''\boldsymbol{\theta}_0\mathbf{k}\mathbf{e}') \\ &= \sum_{e'} W_{CBA}(\mathbf{k}\mathbf{e}'\boldsymbol{\kappa}''\boldsymbol{\theta}_0) \end{aligned} \quad (11a)$$

If the total relative intensities observed in the $\boldsymbol{\theta}_0$ and $\boldsymbol{\phi}_0$ polarizations are I_θ and I_φ, then

$$\begin{aligned} I_\theta &= a_{AB}J_{\theta AB} + a_{BC}J_{\theta BC}, \\ I_\varphi &= a_{AB}J_{\varphi AB} + a_{BC}J_{\varphi BC}. \end{aligned} \quad (11b)$$

A notable simplification may be made when both the transitions AB and BC have the same multipole order. Recalling that the $W(\theta)$ of Eq. (4) is always the same for the sequence ABC and its inverse, CBA, it is apparent that the β and γ of Eqs. (2) and (3) must be the same for ABC and its inverse. If now the transitions AB and BC not only have the same multipole order but also are both electric or both magnetic, the J_θ and J_φ of Eqs. (4) and (5) must be the same for ABC and its inverse, i.e., $J_{\theta AB}=J_{\theta BC}$, $J_{\varphi AB}=J_{\varphi BC}$; and the total I_θ and I_φ are therefore the same as the J_θ and J_φ of Eqs. (4) and (5), no matter which quantum goes to which counter and *entirely independently of counter efficiencies:*

$$\begin{aligned} (I_\theta)_{ED,ED} &= (I_\varphi)_{MD,MD} = 1+\alpha_2\cos 2\theta, \\ (I_\varphi)_{ED,ED} &= (I_\theta)_{MD,MD} = 1+\alpha_2, \end{aligned} \quad (12a)$$

$$\begin{aligned} (I_\theta)_{EQ,EQ} &= (I_\varphi)_{MQ,MQ} \\ &= 1+\alpha_2+\alpha_4-\tfrac{1}{2}\alpha_4\sin^2 2\theta, \quad (12b) \\ (I_\varphi)_{EQ,EQ} &= (I_\theta)_{MQ,MQ} = 1+(\alpha_2+\alpha_4)\cos 2\theta. \end{aligned}$$

Here and in Eqs. (13), (14), and (15) the first and second pairs of subscripts indicate, respectively, the multipole nature of the first and second quanta—i.e., ED for electric dipole, etc.

The next convenient category to consider comprises those pairs of transitions for which the multipole orders are different (i.e., one dipole, one quadrupole) and for which one multipole is electric, one magnetic. It is shown in reference 2 that with one dipole and one quadrupole transition there is no $\cos^4\theta$ term in $W(\theta)$—i.e., $\alpha_4=0$. It will be noted that in this case the functions of Eq. (8b) become very similar to those of Eq. (8a). This latter fact has the happy consequence that when the values of I_θ and I_φ are worked out in accordance with Eq. (11b), the relative efficiencies enter I_θ and I_φ only through a multiplicative function of α_2 which is independent of θ and is the same for I_θ and I_φ and hence irrelevant; and we have, *independently of the counter efficiencies*,

$$\begin{aligned}(I_\varphi)_{MD,EQ}&=(I_\varphi)_{EQ,MD}=(I_\theta)_{ED,MQ}\\&=(I_\theta)_{MQ,ED}=1+\alpha_2\cos 2\theta,\\(I_\theta)_{MD,EQ}&=(I_\theta)_{EQ,MD}=(I_\varphi)_{ED,MQ}\\&=(I_\varphi)_{MQ,ED}=1+\alpha_2.\end{aligned}\tag{13}$$

An analogous simplification does not result when Eq. (11b) is applied to pairs of transitions in which one transition is dipole, one quadrupole, but both are magnetic or both electric. In this case we have

$$\begin{aligned}(I_\theta)_{MD,MQ}&=(I_\theta)_{MQ,MD}=(I_\varphi)_{ED,EQ}=(I_\varphi)_{EQ,ED}\\&=a_D(1+\alpha_2)+a_Q(1+\alpha_2\cos 2\theta),\\(I_\varphi)_{MD,MQ}&=(I_\varphi)_{MQ,MD}=(I_\theta)_{ED,EQ}=(I_\theta)_{EQ,ED}\\&=a_D(1+\alpha_2\cos 2\theta)+a_Q(1+\alpha_2).\end{aligned}\tag{14}$$

Here a_D is the earlier a_{AB} or a_{BC}, according as the quantum AB or BC is the dipole quantum; and similarly for a_Q, with Q for "quadrupole."

The only remaining case to be covered is that of both quanta having the same multipole order, with one multipole electric and one magnetic. Here we find

$$\begin{aligned}(I_\theta)_{ED,MD}&=(I_\theta)_{MD,ED}\\&=a_E(1+\alpha_2\cos 2\theta)+a_M(1+\alpha_2),\\(I_\varphi)_{ED,MD}&=(I_\varphi)_{MD,ED}\\&=a_E(1+\alpha_2)+a_M(1+\alpha_2\cos 2\theta),\\(I_\theta)_{EQ,MQ}&=(I_\theta)_{MQ,EQ}\\&=a_E\left(1+\alpha_2+\alpha_4-\frac{\alpha_4}{2}\sin^2 2\theta\right)\\&\quad+a_M[1+(\alpha_2+\alpha_4)\cos 2\theta],\\(I_\varphi)_{EQ,MQ}&=(I_\varphi)_{MQ,EQ}\\&=a_E[1+(\alpha_2+\alpha_4)\cos 2\theta]\\&\quad+a_M[1+\alpha_2+\alpha_4-\tfrac{1}{2}\alpha_4\sin^2 2\theta].\end{aligned}\tag{15}$$

Here a_E is the earlier a_{AB} or a_{BC}, according as the quantum AB or BC is emitted by the electric multipole; the same holds for a_M. It will be observed that when $a_E=a_M$, then for all the pairs of transitions considered in this paragraph $I_\theta=I_\varphi\sim W(\theta)$.

III. DISCUSSION

Equations (12)–(15) cover all the possible sixteen combinations of dipole and quadrupole transitions. These equations simply extend the results on directional correlation[2] to include the effects of observing the polarization of one quantum; hence for all the cases considered it will be noted that $I_\theta+I_\varphi\sim W(\theta)$, independently of the efficiencies a_{AB} and a_{BC}. The most information is extracted from a given experiment by observing I_θ and I_φ as functions of θ; but for exploratory work it may be noted that in every case the ratio I_θ/I_φ has its maximum deviation from unity at $\theta=90°$, and is always unity at 180°.

For the eight cases covered by Eqs. (12) and (13) the relative counter efficiencies are completely irrelevant as far as any effect on the functional form of the results is concerned. Of these eight cases the sequences *EQ,EQ* and *MQ,MQ* may be identified individually by their characteristic dependences of I_θ and I_φ on θ; however, the experiment makes no distinction (for a given value of α_2) within the groups (*ED,ED*; *ED,MQ*; *MQ,ED*) and (*MD,MD*; *MD,EQ*; *EQ,MD*). It will, of course, often turn out in such an indeterminate case that knowledge of the spin of the ground state, or some auxiliary data such as lifetimes or internal conversion coefficients, will make a given value of α_2 consistent with only one of the three possibilities. But even if this were not so, it should be noted that within each of these groups the changes in the parity of the nuclear states are identical. We may recall that the parity relations for these various transitions are as follows:

ED: yes,	*MD*: no,
EQ: no,	*MQ*: yes,

in which "yes" indicates that the parity of the nuclear state must change in the radiative transition, and analogously for "no." Then for the group (*ED,ED*; *ED,MQ*; *MQ,ED*) (yes, yes) states A and C have the same parity, B a parity different from A and C. For the group (MD,MD; MD,EQ; EQ,MD) (no, no) states A, B, and C all have the same parity. For many purposes (as, for example, determining certain selection rules in beta-decay) knowledge of these parity relations is more important than being able to make a choice within the group.

In this same connection, it will be noted that for the eight cases covered by Eqs. (14) and (15) the parities of states A and C are different, with state B varying. It will also be noted that the counter efficiencies do enter these latter equations, in contrast to the case just discussed. It will be recalled that the a's occurring in Eqs. (14) and (15) are products of the individual counter efficiencies; if both quanta have the same energy, or if both counters have the same dependence of efficiency on energy, then the a's are equal. In Eqs. (14) and (15), equal a's make $I_\theta = I_\varphi \sim W(\theta)$, affording no differentiation between the eight sequences covered by these equations; thus with equal a's, unless auxiliary evidence is available, one is not able for these eight cases to measure the parity of the intermediate state B relative to states A and C.

Suppose, however, that discrimination exists and that one knows which of the two possible types of coincidence is being discriminated against —e.g., knows that the relative coincidence-counting efficiency is higher when the first quantum goes to the polarization-sensitive counter, which would correspond to $a_{AB} > a_{BC}$. The inequality of the a's means that $I_\theta \neq I_\varphi$. Here, as before, the sequences EQ,MQ and MQ,EQ will have a characteristic dependence on θ which identifies them individually. The other six sequences fall into the two groups (MQ,MD; ED,EQ; ED,MD) and (MD,MQ; EQ,ED; MD,ED) within each of which no further distinction may be made; at $\theta = 90°$ one of these groups will have $(I_\theta/I_\varphi) > 1$, one group $(I_\theta/I_\varphi) < 1$, and which is which depends on which a is greater. Unless one of the a's is zero (a rather unlikely situation) the deviation of I_θ/I_φ from unity will be less (for a given α_2) than is the case for Eqs. (12) and (13). But the most relevant comment about the above two groups of sequences is that all the sequences of the first group correspond to states B and C having different parity from A, while the sequences of the second group have states A and B differing from C in parity.

Assuming the existence of some means for even partial discrimination, the procedure for identification of successive transitions of dipole and quadrupole character may be summarized as follows:

(*i*) A directional correlation experiment distinguishes[2] between the 12 sequences (dipole-dipole and dipole-quadrupole) for which $\alpha_4 = 0$ and the four quadrupole-quadrupole sequences for which $\alpha_4 \neq 0$; it also provides definite values of α_2 and α_4 which are necessary for use in further identification of sequences in what follows.

(*ii*) When neither are the a's equal nor is one of them zero, any one of the four quadrupole-quadrupole sequences may be identified.

(*iii*) The two groups (ED,ED; ED,MQ; MQ,ED) and (MD,MD; MD,EQ;EQ,MD) have, independently of counter efficiencies, (I_θ/I_φ) at $\theta = 90°$ equal to $(1-\alpha_2)/(1+\alpha_2)$ and $(1+\alpha_2)/(1-\alpha_2)$, respectively. To each of these groups corresponds a unique relative parity assignment of states A, B, C.

(*iv*) For the two groups (MQ,MD; ED,EQ; ED,MD) and (MD,MQ; EQ,ED;MD,ED) I_θ/I_φ deviates from unity less than for the previous two groups, this deviation depending on the coincidence-counting efficiency and providing a means of identifying within which of these two groups a sequence falls. To each of these groups corresponds a unique relative parity assignment of the states A, B, C.

The assignment of relative parities of states A, B, and C is thus unique; but no more light is shed on assignments of multipole orders alone than is available from a directional correlation experiment.

The basis of the present discussion actually does not depend on both quanta being emitted quanta; the discussion applies equally well to the polarization and intensity distribution of resonance radiation excited by unidirectional unpolarized or circularly polarized light, or to the angular variation of intensity (but not polarization) of the resonance radiation excited by linearly polarized light. These cases will correspond to one of the a's being unity, one zero.

Goertzel's discussion[3] of the effect of a magnetic field upon correlation experiments may be extended[8] to the present case. It is found that when no polarization measurements are made, a strong enough magnetic field parallel to the direction of propagation of either quantum should preserve the angular correlation against the perturbing effect of other fields such as those arising from the atomic electrons; a magnetic field in any other direction will to a varying degree wash out the correlation, the effect being complete when a strong enough field is perpendicular to the plane determined by the directions of the two quanta. When, on the other hand, a polarization measurement is made on one quantum, the correlation between this polarization and the direction of propagation of the other quantum is preserved only when the strong magnetic field is parallel to the direction of propagation of the unpolarized quantum. Since this statement holds true no matter whether the unpolarized quantum is the first or second to be emitted, magnetic field effects should not provide a means of discriminating between the first and second quanta.

[8] Private communication from Dr. Goertzel.

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The Use of Phase Space in Classical and Quantum Theory

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It is suggested that the symmetry between position and momentum space in classical and quantum mechanics may be extended by closer consideration of the concept of phase space. The discussion is limited to the kinematics of a single particle in phase space, to which is associated a metric which describes both the gravitational and the electromagnetic fields acting on the particle. The formulation is applicable only to a charged particle, and implies that for such a particle in an electromagnetic field the Poisson bracket of two position coordinates does not vanish in general, so that in quantum theory these coordinates do not commute. A fundamental constant is introduced as the ratio of the natural and Gaussian units of electromagnetic field strengths and, expressing this constant in terms of $\hbar$ and a length l, the theory satisfies a correspondence principle with present theory in the limit $l\rightarrow 0$. By postulating that $l\sim 10^{-13}$ cm, one is led to the basis of a theory which is indistinguishable from present theory for field strengths small compared with $(137)^{\frac{1}{2}}el^{-2}$, but which leads to essential modifications for the interaction of charged particles separated by distances of the order of nuclear dimensions.

I. CLASSICAL FORMULATION

SOME of the difficulties of present quantum theory appear to arise in the process of quantization, while others trace their origin to the classical theory. Therefore, attempts have been made to modify classical theory, but, unfortunately, such modifications suffer from the defect that they are not gauge invariant,[1] or not unique,[2] or difficult to quantize,[3] or possess some other disadvantage. Dirac's theory of the electron represents a close approximation to the truth, provided that the wave-lengths involved are large compared with $l\sim r_e=e^2/mc^2$. The possibility it admits of extremely large, even infinite energies and frequencies is clearly incorrect. In fact, it has been suggested by Bethe and Oppenheimer[4] that a new constant exists of the dimensions of a frequency, so that for frequencies below

[1] For example, G. Mie, Ann. d. Physik **37**, 511 (1912); **39**, 1 (1912); **40**, 1 (1913).
[2] M. Born and L. Infeld, Proc. Roy. Soc. **A144**, 425 (1934).
[3] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. **17**, 157 (1945).
[4] H. Bethe and J. R. Oppenheimer, Phys. Rev. **70**, 451 (1946).