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On the β-Decay of Mesons

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The β-decay of cosmic-ray mesons is discussed. The possible process of decay into electron-neutrino plus a neutral particle is treated in some detail. This process is found to fit in well with the familiar formalism of β-decay of atomic nuclei. A comparison is made with present experimental data on mesons.

1. INTRODUCTION

RECENT cloud-chamber investigations of the β-decay of cosmic-ray mesons performed by Anderson *et al.*[1] and by Thompson[2] lead to values of the energy of the electron emitted lying predominantly in the neighborhood of 25 Mev. This result shows that the hypothesis of decay of the meson into an electron and a neutrino can hardly be upheld. Moreover, the experiments do not seem to exclude the possibility of a continuous spectrum of the energy of the electron, as would be the consequence of the introduction of a neutral recoil particle, of mass roughly half the mass of the decaying meson.

It is natural to consider the possibility that the neutral particle introduced in this way is the same as the neutral meson emitted in the μ-decay of the π-meson. The rather uncertain estimates of the mass of the neutral meson are so far in agreement with an assumption of this kind.

On the other hand, the spread in the value of the ratio $m_\pi/m_\mu = 1.65 \pm 0.15$[3] could just permit a decay process in which the μ-meson decayed into an electron and a neutral meson, without emission of a neutrino. The latter process demands a definite energy of the electron, and a mass of the μ-meson $\cong 175 m_e$, if $m_\pi = 313 m_e$.

In the present paper we shall discuss the former alternative, i.e,

$$\mu^\pm \rightarrow \mu^0 + e^\pm + \nu. \quad (1)$$

A process of this kind may have interesting consequences as regards the interpretation of the β-decay of atomic nuclei. In fact, Klein[4] has suggested that the decay of the μ-meson should be regarded as the prototype of all β-decay processes.

Klein's proposal is primarily based on the following comparison of the decay constants; the product $f \cdot t$ of the half-life and the Fermi integral of the distribution in energy of the electron and the neutrino, as evaluated for a maximum energy of the electron of 2×25 Mev$\cong 100 m_e$ (we put

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[1] C. D. Anderson, R. V. Adams, P. E. Lloyd, and R. R. Ran, Phys. Rev. **72**, 724 (1947).

[2] B. Rossi, Pocono Conference, March 30, 1948; see further E. C. Fowler, R. H. Cool, and J. C. Street, Phys. Rev. **74**, 101 (1948); J. L. Zar, J. Hershkowite, and E. Berezin, Phys. Rev. **74**, 111 (1948).

[3] C. F. Powell and G. P. S. Occhialini, Nature **161**, 551 (1948); C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Proc. Phys. Soc. **61**, 173 (1948).

[4] O. Klein, Nature **161**, 897 (1948). We are much indebted to Professor Klein for having communicated his manuscript to us before publication.

TABLE I.

	n	H^3	He^6
t	20^m	12^y	0.85^s
E (in units of m_e)	2.48	0.022	7.85
Uncertainty in E	0.03	0.004	1.00
$f\cdot t$	1700	200	800
Uncertainty in $f\cdot t$	?	a factor 2	a factor 2

$c=1$), and $t=1.5\cdot 10^{-6}$ sec., is equal to

$$f\cdot t = E^5/30\cdot t \cong 500. \qquad (2)$$

This value of $f\cdot t$ is just of the same order of magnitude as in the case of β-decay of the lightest nuclei.

In this connection it will be of importance to discuss the values of the decay constants of the lightest nuclei, and in particular the decay constant of H^3. The maximum energy of the electron in the β-decay of H^3 is so small that it becomes necessary to take into account the effect of the Coulomb field of the recoil nucleus. The best estimate of the half-life and the maximum energy seems to be $t=12$ years and $E^{\max}=11$ kev. We then find $f\cdot t=200$. The value of $f\cdot t$ is very sensitive to variations in the maximum energy, however. If the maximum energy is changed by 2 kev, $f\cdot t$ will change by a factor ~ 2. This calculation corresponds to zero mass of the neutrino. It was pointed out by Konopinski[5] that a non-vanishing mass of the neutrino can lead to a large increase in the value of f for H^3, and thus the $f\cdot t$ value calculated for zero mass of the neutrino may be an underestimate. Though the upper limit to the neutrino mass was recently found to be as low as 25 kev,[6] still the effect from a finite mass cannot definitely be ruled out.

On the other hand, arguments in favor of a larger value of $f\cdot t$ for H^3, as put forward on the basis of a comparison with $f\cdot t$ values of other elements, can hardly be considered as compelling, primarily because no exhaustive explanation of the variation of the $f\cdot t$ values has been found so far. But one should bear in mind also that H^3 represents an extreme case, where corrections on the Fermi theory of β-decay may be necessary because of the low maximum energy.

In Table I we shall use, with all due reservation, the tabulated $f\cdot t$ values for the lightest elements as a basis for a comparison with the β-decay of the meson.

The half-life of the neutron, $t\cong 20^m$, as found in recent experiments,[7] leads to a comparatively high $f\cdot t$ value, $f\cdot t\cong 1700$.

2. THE FERMI INTEGRAL AND THE ENERGY DISTRIBUTION

The estimate of the Fermi integral made above in the case of β-decay of the μ-meson may be improved because the neglect of the kinetic energy carried off by the recoil particle is by no means a satisfactory approximation. We shall find that a more correct evaluation of the Fermi integral leads to values which are considerably smaller than those found with neglect of the recoil.

Consider a β-decay process where a particle of mass M is transformed into a particle of mass m, an electron, and a neutrino. The available energy can for the present purpose be assumed to be so large that the extreme relativistic approximation holds for the electron and the neutrino. As to the recoil particle, however, the exact relativistic energy formula must be used. On these assumptions the maximum energy of the electron is found to be

$$E^{\max}=(M/2)(1-m^2/M^2). \qquad (3)$$

The maximum energy of the electron is thus somewhat smaller than the mass difference $M-m$.

When the recoil energy is taken into account the Fermi integral can be written as

$$f_\alpha = \tfrac{1}{2}\int E_e^2 dE_e \int E_\nu^2 dE_\nu \times \int d\cos\vartheta\cdot(1+\alpha\cos\vartheta)\cdot\delta(M-E_e-E_\nu - [m^2+E_e^2+E_\nu^2+2E_eE_\nu\cos\vartheta]^{\frac{1}{2}}), \qquad (4)$$

in the extreme relativistic approximation for the electron. Here ϑ is the angle between the directions of emission of the electron and the neutrino, and E_e and E_ν denote the energies of the electron

[5] E. J. Konopinski, Phys. Rev. 72, 518 (1947).
[6] D. J. Hughes and C. Eggler, Phys. Rev. 73, 809 (1948).
[7] Mr. Aage Bohr has kindly informed us of these measurements.

and the neutrino, respectively. In the factor $(1+\alpha\cos\vartheta)$ one has a choice between several values of α, all compatible with the invariance conditions imposed on the coupling between the heavy particles and the electron-neutrino field.[8] The value $\alpha=1$ corresponds to Fermi's original choice. It is noteworthy that the term $\alpha\cos\vartheta$ gives a considerable contribution to f and also affects the form of the energy spectrum of the electron, which is not the case for β-emitting nuclei, at least not for allowed transitions.

Besides the factor $(1+\alpha\cos\vartheta)$ in (4), similar expressions involving the recoil particle may be introduced as a consequence of the above-mentioned invariance conditions. The precise form of such expressions will depend on the character of the wave equation to be ascribed to the μ-meson and the neutral meson. Fortunately, it can be foreseen that these unknown terms will be of minor importance and may be neglected in the present approximation. Such a neglect is permissible if the velocity of the recoil particle is not too close to the velocity of light. This is illustrated by the following example: if the neutral meson and the μ-meson follow the Dirac wave equation, one has to introduce in (4) the extra factor $\frac{1}{2}(1+m/E_m)$, where E_m is the total energy of the recoil particle. If $m\sim\frac{1}{2}M$ the resulting correction on the Fermi integral will only amount to a few percent.

The integrations in (4) are easily performed, and lead to the formula

$$f_\alpha=(M^5/384)\{(1+\alpha)[(7/10)-5\beta+2\beta^2+2\beta^3+\tfrac{1}{2}\beta^4-\tfrac{1}{5}\beta^5+6\beta^2\log(1/\beta)]-\alpha[1-8\beta+8\beta^3-\beta^4+12\beta^2\log(1/\beta)]\},\quad (5)$$

with $\beta=m^2/M^2$. Equation (5) takes a very simple form when $\alpha=1$. Using (3) we find in this case

$$f_{+1}=E^{\max 5}/30. \quad (6)$$

The formula (6) is quite similar to the familiar extreme relativistic Fermi integral, where the recoil energy is neglected. Other values of α lead to somewhat more involved expressions for f_α. In the extreme case $\alpha=-1$, we put

$$f_{-1}=\varphi(\beta)\cdot f_{+1}, \quad (7)$$

where $\varphi(\beta)$ can be found by a comparison of (5) and (6). In Fig. 1, φ is plotted as a function of β, and is in general somewhat larger than 1. For intermediate values of α the Fermi integral f_α can be found from an interpolation in Fig. 1, together with formula (6). It appears that if the masses are given, the choice $\alpha=1$ will lead to the lowest value of the Fermi integral.

In order to make a comparison with the observed β-energies one must find the form of the energy spectrum of the electron. It turns out that the spectrum is not symmetric with respect to $E^{\max}/2$, but slightly shifted towards higher energies, the shift being least pronounced for $\alpha=1$. The reason why the shift comes in is simply that the energy shared by the electron and the neutrino is not always equal to the maximum energy. If these two particles are not emitted in the same direction the sum of the energies of the electron and the neutrino will be larger than the maximum energy of the electron.

The differential contribution df_α to the Fermi integral from the energy interval E_e, E_e+dE_e is proportional to the probability of emission of an electron in this energy interval. From Eq. (4) we calculate df_α, and thus obtain the energy spec-

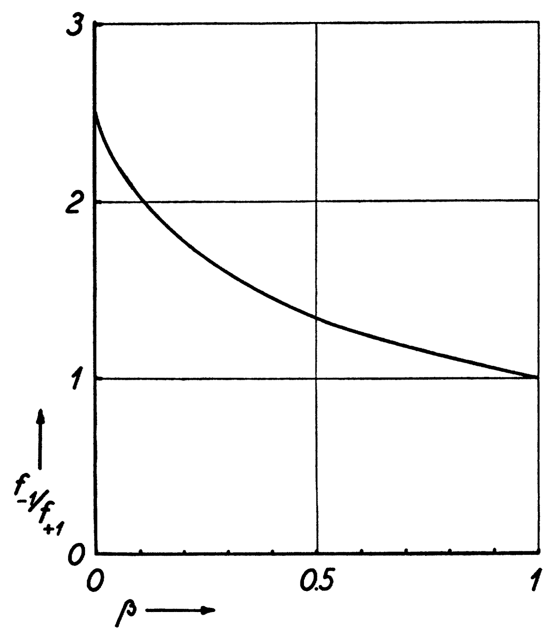

FIG. 1. The ratio $\varphi=f_{-1}/f_{+1}$ is plotted as a function of $\beta=m^2/M^2$. It is seen that $f_{-1}\approx f_{+1}$ when $\beta\approx1$, i.e., when recoil effects can be neglected.

[8] Compare, e.g., D. R. Hamilton, Phys. Rev. **71**, 456 (1947).

TABLE II.

$\langle E\rangle$	α	E^{max}	M	m	m_π/M	$f\cdot t$
55	+1	98	216	67	1.45	450
55	−1	88	207	79	1.51	500
50	+1	90	208	77	1.50	300
50	−1	82	201	87	1.56	340
45	+1	82	201	87	1.56	190
45	−1	75	196	94	1.60	200

trum of the electron,

$$df_\alpha = dE_e \frac{E_e^2(E^{max}-E_e)^2}{6M(M-2E_e)^3} \times\{(1+\alpha)[3M^4+3M^2m^2 - E_e(12M^3+6Mm^2) + E_e^2(14M^2+4m^2) - 4ME_e^3] - \alpha[3M^4+3M^2m^2 - E_e(10M^3+2Mm^2) + 8M^2E_e^2]\}. \quad (8)$$

The asymmetry of the β-spectrum is illustrated in Fig. 2, where the ratio of the average energy, $\langle E\rangle$, to the maximum energy, E^{max}, is plotted as a function of $\beta = m^2/M^2$, in the two extreme cases $\alpha = \pm 1$.

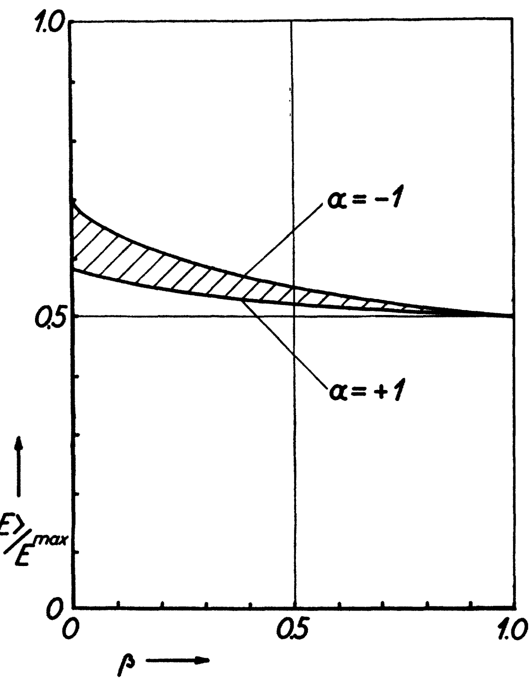

FIG. 2. The curves show the dependence of $\langle E\rangle/E^{max}$ on β, in the two extreme cases $\alpha = \pm 1$. For intermediate values of α the ratio $\langle E\rangle/E^{max}$ will stay inside the hatched area.

3. COMPARISON WITH EXPERIMENTAL DATA

The observations on the β-decay of the cosmic-ray meson suggest a value of the average energy close to $50m_e$. From the average energy we can find the masses M, m if some further relations between the masses are known. As the supplementary relations we shall use the mass of the π-meson, $m_\pi = 313m_e$, as measured in the artificial production of mesons in Berkeley,[9] together with the value of the kinetic energy of the μ-meson resulting from the decay of a π-meson, given by Powell and co-workers.[10] Together, these data give approximately the sum of the masses, $M+m$. We assume here that the heavy meson observed in the Berkeley experiments is the same as the π-meson found by the Bristol group. This assumption, though plausible, has not been confirmed beyond all doubt.

The values of M, m, E^{max}, m_π/M, and $f\cdot t$, as resulting from a given average energy, are listed in Table II in the two cases $\alpha = \pm 1$. In order to show the dependence of these quantities on the average energy we have used three different values of the average energy, $\langle E\rangle = 45$, 50, and $55m_e$.

We can compare the tabulated values of m_π/M with Powell's ratio,[3] $m_\pi/m_\mu = 1.65 \pm 0.15$. Except for the case of $\langle E\rangle = 55\cdot m_e$ the values agree fairly well with this mass ratio. For each energy the two possibilities $\alpha = \pm 1$ lead to rather different mass values, while the product $f\cdot t$ remains less sensitive to the choice of α. The $f\cdot t$ values are in good accordance with those of the lightest elements, as given in Table I.

It seems probable that accurate determinations of the masses of mesons will be made in the near future. Then, the validity of the process discussed here may be tested directly, since the maximum energy is determined from the mass values, and the latitude left in the expression for the average energy as different values of α are assumed is then directly found from the two limiting curves for $\langle E\rangle/E^{max}$, as shown in Fig. 2.

[9] E. Gardner and C. M. G. Lattes, Science, March 12 (1948).

[10] C. M. G. Lattes, G. P. S. Occhialini, and C. F. Powell, Nature **160**, 453 and 486 (1947).

Another way of testing the process (1) is to compare the theoretical expectations as regards the energy spectrum with the energy values actually found for the electron. It is found from (8) that the mean deviation from the average energy is, approximately,

$$\Delta E \equiv \{\langle E^2\rangle - \langle E\rangle^2\}^{\frac{1}{2}} \cong 0.2 \cdot E^{\max}, \qquad (9)$$

independent of the mass values and of the choice of α. This result contrasts strongly with the discrete energy value expected in the alternative decay process with emission of two particles instead of three.

In the case of decay of negative mesons an effect of a special kind can come in. If a slow meson is captured in an s-orbit around an atomic nucleus, the uncertainty of the momentum of the meson will be $\Delta p \sim m_\mu Ze^2/\hbar$. The uncertainty of momentum gives rise to a smearing out of the energy spectrum of the electron emitted, and the maximum energy available to the electron is increased by $\sim E^{\max} \cdot Z/137$. The change of the average energy is very small, however. In the same way, the Fermi integral and consequently the lifetime is approximately unchanged for a meson bound in medium nuclei; the small change in f tends to increase the half-life.

In this connection it should be remembered that as long as the properties of the different mesons are not known, the influence of the interaction of mesons with nuclei on the β-decay of the μ-meson cannot be appreciated. Accordingly, great caution should be observed in comparisons between experimental results and theoretical expectations as regards β-decay, in those cases where effects of interaction with nuclei may play a role.

4. EFFECT OF THE FINITE RANGE OF INTERACTION

In performing the above calculations we have assumed that the wave-lengths of the particles involved are sufficiently large to allow the transition probability to remain proportional to the volume available in phase space, i.e., the transition probability depends on the Fermi integral. Now, one cannot exclude the possibility of a departure from this simple picture. The wave-length of an electron of energy 25 Mev is $\lambda\!\!\!^{-} = 8 \cdot 10^{-13}$ cm. The three particles emitted in the β-decay will have wave-lengths of this order of magnitude, or even less. In the Fermi theory of β-decay one might expect corrections to the Fermi integral of the order of $a^2/\lambda\!\!\!^{-2}$, where a is a suitably defined range of interference of the wave functions of the particles. The value of a is possibly somewhat larger than 10^{-13} cm. A correction of this kind will probably not be so large as to affect essentially the present approximate evaluations.

On the other hand, if the β-decay is described by an intermediary mesonic coupling between the heavy particles and the electron-neutrino field as in the Yukawa theory, corrections to the Fermi integral will be of the type $a/\lambda\!\!\!^{-}$. More definitely, one obtains as a weight factor in the Fermi integral (4) a term of the type $m_1^2/(m_1 - E_e - E_\nu)^2$, where m_1 is the mass of the meson responsible for the coupling. If $m_1 \sim 300 m_e$ the result will be an increase of the Fermi integral by a factor ~ 2, while the energy spectrum of the electron remains nearly unchanged. Moreover it is seen that the Fermi integral is quite sensitive to the value of the mass m_1, and for small values of m_1 the energy spectrum is changed appreciably. However, the familiar description of the interaction of mesons with nucleons meets with well-known difficulties, and in the present treatment we have omitted corrections of the above kind, even though the corrections will seem to be of importance in mesonic descriptions of β-decay.

Our thanks are due Professor C. Møller for discussions on the subject of the present paper.