Mass Temperature Coefficients of Cosmic-Ray Components

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The temperature pressure curve from U. S. Navy Raob data was integrated by an empirical formula determined graphically. The resulting average "mass temperature" of various fractions of the atmosphere was correlated by a method of least squares with the vertical cosmic-ray intensity of three components; hard, medium, and soft, shielded by 30.5, 20.5, and 0.5 cm of lead, respectively. The temperature coefficient was found to be an increasing function of the fraction of the atmosphere for larger fractions.

The temperature coefficients obtained for the fractions 1, 3/4, 6/10, and 1/4 are for the hard component: -0.25 ± 0.06, -0.18 ± 0.04, -0.15 ± 0.04, and -0.17 ± 0.04 percent per degree centigrade, respectively. The results for the medium component are: -0.61 ± 0.11, -0.43 ± 0.07, -0.34 ± 0.08, and -0.39 ± 0.08 percent per degree centigrade, respectively. Comparative results for data from a Millikan-Neher electroscope shielded by 12 cm of lead are: -0.25 ± 0.05, -0.17 ± 0.03, -0.16 ± 0.03, and -0.17 ± 0.03 percent per degree centigrade for the respective fractions listed above. The results for the soft component correlated so poorly that no values are quoted.

I. INTRODUCTION

INVESTIGATION into the correlation of cosmic-ray intensity with average "mass" temperatures of the atmosphere has been stimulated by Blackett's[1] theory on the temperature effect and the subsequent discovery of the "air-mass" effect by Loughridge and Gast.[2] Benedetto, Hess, and Altman,[3] using a counter telescope arrangement with an aperture of 0.297 steradian and shielded by approximately 22 cm of lead, have contributed to this study.

The authors have made a similar study employing a counter telescope of 18° zenith angle differentiating the hard, medium, and soft components shielded by 30.5, 20.5, and 0.5 cm of lead, respectively. This instrument has been described at length by Smith.[4] A "Millikan-Neher"[5] recording electroscope shielded by approximately 12 cm of lead was operated, but not concurrently with the telescope, to study the temperature coefficient for radiation from all directions. This instrument is described by Millikan and Neher.

II. MASS TEMPERATURES

Hess and Benedetto[3] suggested using an average temperature in which the temperature was weighted proportionally to the mass of air of a given temperature rather than to the space occupied by the band. They accomplished this by integrating the atmospheric pressure *vs.* temperature curve and dividing by the pressure over the particular fraction desired.

For the purpose of obtaining the mass temperature of various fractions for the present work, Raobs (radio observation, balloon soundings) taken at Sand Point Naval Air Station, Seattle, were obtained from the U. S. Weather Bureau, Seattle, Washington.

Atmospheric temperatures as a function of pressure of standard levels were plotted for arbitrarily chosen data scattered through the year as shown in Fig. 1. The pressure levels 1000, 850, 700, 500, 300, 200, and 100 are designated "standard" millibar levels whose temperatures are reported on all Raobs provided there is no equipment failure. To accomplish the graphical integration of the pressure-temperature curve, it was divided into pressure intervals such that the temperature of the standard millibar pressure

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[1] P. M. A. Blackett, Phys. Rev. **54**, 973 (1938).

[2] D. H. Loughridge and P. F. Gast, Phys. Rev. **58**, 583 (1940).

[3] V. F. Hess and F. A. Benedetto, Phys. Rev. **60**, 610 (1941); F. A. Benedetto, G. O. Altman, and V. F. Hess, Phys. Rev. **61**, 266 (1942).

[4] L. E. Smith, Jr., *Intensity Coefficients of Cosmic Ray Components* (University of Washington Thesis, 1945). H. T. Stetson, Sci. Mo. **58**, 207 (1944); Electronic Industries **94**, January (1944).

[5] R. A. Millikan and H. V. Neher, Phys. Rev. **50**, 15 (1936).

levels gave a fair representation of the average for that interval, as shown in Fig. 1 where the standard levels are indicated by S.L. The same division appeared satisfactory throughout the year. A weight (in terms of multiples of 50 millibars) was assigned to each standard millibar level and the following formulae derived for the average mass temperatures of the various fractions of the atmosphere:

$$T_{1/4}=[3(T_{850})+2(T_{1000})]/5,$$
$$T_{6/10}=[3(T_{700}+T_{850})+2(T_{1000})+4T_{500}]/12,$$
$$T_{3/4}=[3(T_{300}+T_{7000}+T_{850})+2(T_{1000})+4T_{500}]/15,$$
$$T_{1}=[3(T_{100}+T_{300}+T_{700}+T_{850})+2(T_{200}+T_{1000})+4T_{500}]/20.$$

$T_{1/4}$, $T_{6/10}$, $T_{3/4}$, and T_1 represent the mean mass temperatures of the lower 1/4, 6/10, 3/4, and 1 fractions of the atmosphere, respectively. T_{100}, T_{200}, T_{300}, etc., are the temperatures of the 100, 200, 300, etc., millibar levels as given on the Raob reports. The mean mass temperatures were determined for each day from the morning (0800) and evening (1800) reports. The values used in correlating were the average of these two taken to represent the daily mean centered about noon (1200).

The Raob balloons do not always reach the higher standard levels. Missing upper air data were filled in by interpolation and through reference to data from the Tatoosh station 150 miles NW of Seattle. The difference between the upper air at these two stations is usually small and changes slowly. A consideration of the formula for T_1, for example, will reveal that an error of five degrees in estimating T_{100} will cause an error in T_1 of but about three percent under average atmospheric conditions.

III. TREATMENT OF DATA

The data from the telescope and the electroscope were divided into periods for analysis as given in Table I.

The temperatures used represent the daily means centered about noon. The counts for each hour of the day were reduced to a standard barometer of 30 inches of mercury, using barometric coefficients as given by Smith[4] and Gast[6]

TABLE I. Periods for analysis of data.

Period	Cosmic-ray telescope	Number of days
I	July 4–July 24, 1946	21
II	Aug. 21–Sept. 7, 1946	18
III	Sept. 12–Oct. 2, 1946	21
IV	Oct. 3–Oct. 22, 1946	20
V	Oct. 26–Nov. 17, 1946	23
	Millikan-Neher Meter	
I	Jan. 4–Jan. 31, 1947	28
II	Feb. 2–Feb. 28, 1947	27
III	March 1–March 30, 1947	30

for the telescope and electroscope, respectively. These values are:

telescope:

- hard— -3.81 percent per inch of mercury,
- medium— -8.64 percent per inch of mercury,
- soft— -9.65 percent per inch of mercury,
- Millikan-Neher— -4.32 percent per inch of mercury.

The counts were then averaged about noon for each 24-hour period. It was these averages which were correlated with the temperature.

The correlations were by a method of least squares as given by Forsythe[7] and Doan.[8] The essential relations are given below:

(a). X = the deviation of the temperature from some value near the mean.

(b). Y = the deviation of the counts from some value near the mean.

(c). n = the number of days in the period.

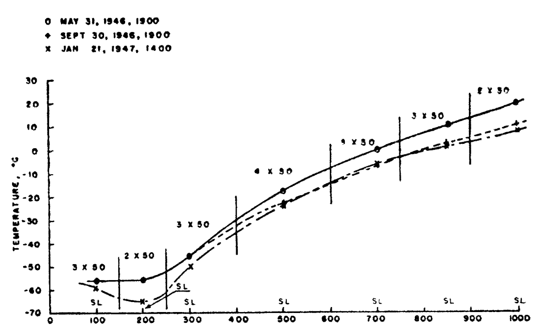

FIG. 1. Temperature-pressure curves from Raob reports.

[6] P. F. Gast, *Various Factors Affecting Cosmic Ray Intensity* (University of Washington Thesis, 1941).

[7] C. H. Forsythe, *Mathematical Analysis of Statistics* (John Wiley and Sons, Inc., New York, 1924), pp. 223.

[8] R. L. Doan, Phys. Rev. **49**, 107 (1936).

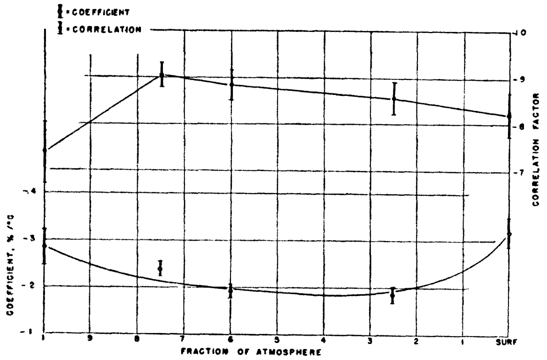

FIG. 2. Temperature coefficient *vs.* fraction of atmosphere, hard, Period I.

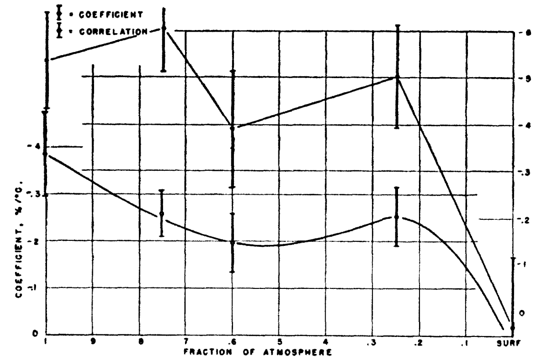

FIG. 4. Temperature coefficient *vs.* fraction of atmosphere, hard, Period III.

(d). $\bar{y}$ = the average of the daily counts being correlated.

(e). $\sigma_x = [(\Sigma X^2/n) - h^2]^{\frac{1}{2}}$ = standard deviation of temperature.

(f). $\sigma_y = [(\Sigma Y^2/n) - k^2]^{\frac{1}{2}}$ = standard deviation of counts.

(g). $\nu_{xy} = (\Sigma XY/n) - hk$ = product-moment.

(h). $r = \nu_{xy}/\sigma_x\sigma_y$ = correlation factor.

(i). $P_r = \pm 0.675(1-r^2)/(n)^{\frac{1}{2}}$ = probable error in r.

(j). $\alpha = 100\nu_{xy}/\sigma_x{}^2\bar{y}$ = temperature coefficient in percent per degree C.

(k). $P_\alpha = \pm 0.675(1-r^2)^{\frac{1}{2}}\sigma_y 100/(n)^{\frac{1}{2}}\sigma_x\bar{y}$ = probable error in α in percent per degree C.

(l). $h = \Sigma X/n$ = the displacement of the origin of the deviations from the mean.

(m). $k = \Sigma Y/n$ = the displacement of the origin of the deviations from the mean.

IV. EXPERIMENTAL RESULTS

On the basis of this work the values given below are the most probable values of the temperature coefficient for the fractions 1, 3/4, 6/10, and 1/4, respectively, for the hard component: -0.25 ± 0.06, -0.18 ± 0.04, -0.15 ± 0.04, and -0.17 ± 0.04 percent per degree centigrade, respectively. The values for the medium component are -0.61 ± 0.11, -0.43 ± 0.07, -0.34 ± 0.08, and -0.39 ± 0.08, percent per degree centigrade, respectively. The Millikan-Neher meter data led to -0.25 ± 0.05, -0.17 ± 0.03, -0.16 ± 0.08, and -0.17 ± 0.03 percent per degree centigrade for the respective fractions listed above. The correlations for the soft component were very low both positive and negative. It is felt that the results for the soft component should not be quoted pending further study.

Figures 2–7 show the variation in temperature coefficient with fraction of the atmosphere for various components and periods as indicated. In all cases the temperature coefficient is an increasing function of fraction of the atmosphere for higher fractions. For small fractions approaching surface temperature it is not possible

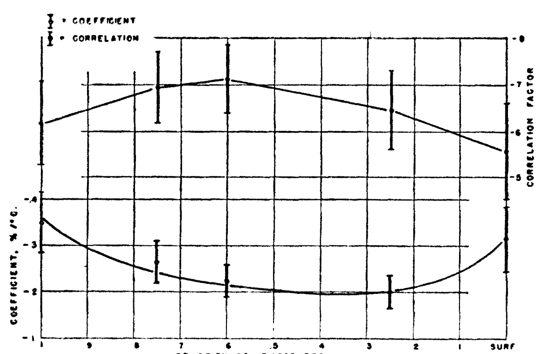

FIG. 3. Temperature coefficient *vs.* fraction of atmosphere, medium, Period I.

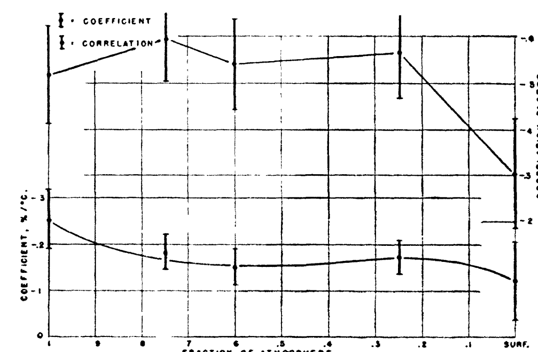

FIG. 5. Temperature coefficient *vs.* fraction of atmosphere, hard, average of Periods I, II, III, and V.

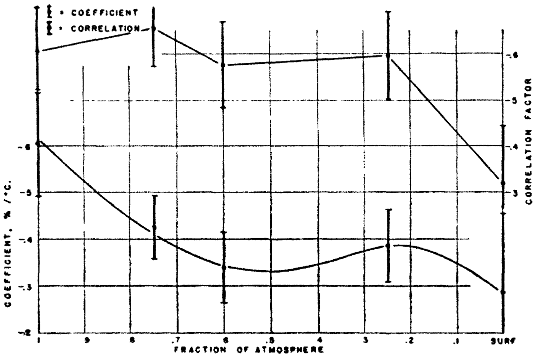

FIG. 6. Temperature coefficients *vs.* fraction of atmosphere, medium, average of Periods I, II, III, and V.

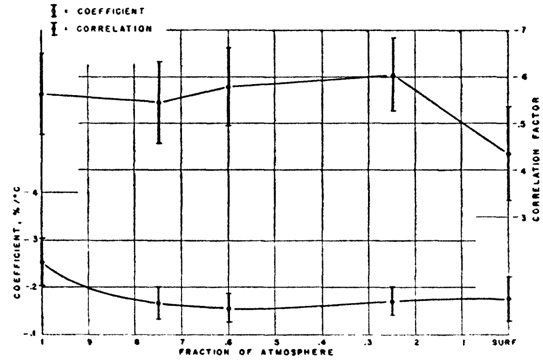

FIG. 7. Temperature coefficient *vs.* fraction of atmosphere, Millikan-Neher meter, average of Periods I, II, and III.

to predict whether the coefficient will rise or continue to fall. Figures 2–4 show cases of both possibilities. It appears certain, however, that the correlation falls off as the surface temperature is approached. Surface temperature is influenced by the local terrain more than the upper air. One would not expect surface temperature to correlate the same as upper air temperatures. The results for the Millikan-Neher meter vary in a manner similar to those from the telescope.

V. THEORETICAL DISCUSSION

Benedetto[3] made the assumption that an increase of intensity is linearly proportional to a downward shift of air mass. It then follows that:

$$dI/I = -dZ/L \tag{1}$$

where

I = mesotron intensity,
Z = height of mesotron production level, and
L = mesotron mean free path.

The temperature coefficient may be defined as:

$$\alpha = (-1/I)\cdot(dI/dT) = (-1/I)\cdot(dI/dZ)\cdot(dZ/dT), \tag{2}$$

and putting (1) in (2):

$$\alpha = +(1/L)\cdot(dZ/dT). \tag{3}$$

Equation (3) then indicates that α is proportional to dZ/dT. In calculating the coefficient, the mass temperature of a certain fraction of the atmosphere was correlated with cosmic-ray intensity. So from Eq. (3), α is proportional to av.$[\partial Z/\partial T]_f$·Av.$[\partial Z/\partial T]_f$ is equal to dZ/dT averaged over the fraction in question.

It was remarked that Benedetto assumed that an increase in cosmic-ray intensity was linearly proportional to a downward shift in the mesotron producing layer. Assume that the layer is located at a height $Z = L$, the average path length above the earth's surface. The shape of the path-length distribution curve is complicated by the fact that the mesotrons have a finite lifetime. However, if the distribution curve does not differ markedly from a Gaussian curve, the mean free path length, L will intersect the distribution curve at a point where the slope is not changing rapidly. A number of independent experiments fix at least one mesotron producing level in the vicinity of the 100 millibar, 16 kilometer, level. As will be shown later in this section, this level rises but about 60 meters per degree centigrade. So for a normal temperature change of a few degrees, the change in height of the level is a small percentage of the height of the level. In view of this, the section of the distribution curve over which L varies will be nearly a straight line. Thus it appears that Benedetto's assumption of the linear proportionality between the number of particles reaching the surface and the height of the producing layer is a reasonable one.

It was decided to check the proportionality

TABLE II. Change in height of atmospheric layers.

Fraction	$[\partial Z/\partial T]_f$ m/°C	av. $[\partial Z/\partial T]_f$ m/°C
1/5	6.89	3.29
2/5	15.4	6.79
3/5	26.8	11.8
4/5	47.0	17.6

between the coefficients obtained in the present work and av.$[\partial Z/\partial T]_f$. The "law of atmospheres" is given by:

$$p = p_0 e^{-Mgh/RT} \tag{4}$$

where p = pressure at height h, p_0 = pressure at $h=0$, M = molecular weight of air, R = gas constant, g = acceleration due to gravity, and T = absolute temperature of the air. Solving Eq. (4) for h,

$$h = (-RT/Mg)\ln(p/p_0). \tag{5}$$

If T is taken as the average mass temperature of the layer $(p_0 - p)$, Eq. (5) gives the height of the top of the layer. Taking small increments in h and T,

$$[\Delta h/\Delta T]_f = (-R/Mg)\ln(p/p_0), \tag{6}$$

where $[\Delta h/\Delta T]_f = [\partial Z/\partial T]_f$ = the change in the height of the top of the layer f with respect to temperature. Equation (6) may be integrated with respect to p to find the area under the $[\Delta h/\Delta T]_f$ *vs.* p curve as follows:

$$\text{Area}_f = \int_{p_f}^{p_0} [\Delta h/\Delta T] dp = -\int_{p_0}^{p_f} [\Delta h/\Delta T] dp. \tag{7}$$

Now, putting Eq. (6) in (7),

$$\text{Area}_f = (R/Mg)\int_{p_0}^{p_f} \ln[p/p_0] dp$$
$$= (R/Mg)[p\ln(p/p_0) - p]_{p_0}^{p_f}.$$

If the proper p limits are used and the area obtained divided by the interval in pressure, the result is the av.$[\partial Z/\partial T]_f$ for the fraction in question. The upper limits of the fractions 1/5, 2/5, 3/5, and 4/5 were taken as the 800, 600, 400, and 200 millibar levels, respectively. p_0 was taken as 1013. Equations (6) and (8) were used to compute $[\partial Z/\partial T]_f$ and av.$[\partial Z/\partial T]_f$ for each of these fractions. The results are given in Table II.

The ratio of av.$[\partial Z/\partial T]_{4/5}$ to av.$[\partial Z/\partial T]_{1/5}$ is 17.6/3.29. Reference to Benedetto's work shows a ratio of 24/17. It appears likely that the upper limits of his fractions are not those used in this paper. However, regardless, it is obvious that the two independent determinations of av.$[\partial Z/\partial T]_f$ are not in agreement. When one compares the values of av.$[\partial Z/\partial T]_f$ given in Table II with the values of the coefficients obtained in this work, it can be said that the temperature coefficient and av.$[\partial Z/\partial T]_f$ are both increasing functions of fraction of the atmosphere for higher fractions.

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