PHYSICAL REVIEW VOLUME 74, NUMBER 1 JULY 1, 1948

Radiation from Electrons Accelerated in a Synchrotron

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(Received March 15, 1948)

High energy electrons subjected to large radial accelerations radiate considerable energy in the optical spectrum. The distribution of energy in the light from a synchrotron beam has been measured and compared with theory at several electron energies up to 80 Mev. The results indicate reasonable agreement with theory. Measurement of total light output allowed an estimate of electron current in the beam. High speed photography of the light permitted observation of the size and motion of the beam within the accelerator tube.

1. INTRODUCTION

DURING the early operation of a 70-Mev synchrotron[1] in this laboratory, intense visible radiation from the high energy electrons was observed[2] through the glass wall of the accelerating tube. Almost fifty years ago Lienard[3] pointed out that electrons rotating in a circle should radiate energy and he gave a formula for the rate of radiation from the centripetal acceleration of an electron. The classical theory was developed considerably by G. A. Schott[4] in connection with early models of the atom. More recently it has been shown that radiative energy losses have an important effect on the operation of high energy electron accelerators,[5] and the theory has been further discussed by several authors.[6]

In betatrons and synchrotrons, in which the radiation almost entirely results from continuous centripetal accelerations of the order of 10^{19} cm/sec./sec., the energy loss per electron is proportional to the fourth power of the electron energy in the relativistic region. The radiation is emitted in a narrow cone around the instantaneous direction of electron motion. Experiments with Polaroid confirm that the light is polarized with the electric vector in the plane of the electron orbit. In our machine the beam is first visible at about 30 Mev as a dull red spot. At 80 Mev, the present peak energy, the light is very brilliant and a bluish-white color. The spectral distribution of the radiation has been determined during the acceleration of electrons to several different levels of peak energy and in this paper is compared with the theory.

2. THEORY

The energy radiated by an electron is distributed over many harmonics of the fundamental rotation frequency of the electron in its orbit (in our machine 163.5 megacycles). Except for the microwave region, the radiation is incoherent and the total radiation from the beam is just the power radiated by one electron times the number of electrons undergoing acceleration.

Professor J. S. Schwinger of Harvard has calculated the distribution of the energy radiated, and has kindly sent us his results (expressions (1) through (4)).

For an electron of constant energy

$$P(\omega)d\omega=(3\sqrt{3}/4\pi)\omega_0(e^2/R)(E/mc^2)^4 \times\left[\int_{\omega/\omega_c}^{\infty}K_{5/3}(x)dx\right](\omega/\omega_c)d\omega, \quad (1)$$

where $P(\omega)d\omega$ is the power radiated by one electron at the circular frequency ω in the range $d\omega$. R is the radius of the orbit in cm; ω_0 the angular velocity of the electron, V/R; e the electron charge in e.s.u.; E the total electron energy; and $K_{5/3}$ a cylinder function as defined in Watson's treatise on Bessel Functions. $\omega_c=\frac{3}{2}\omega_0(E/mc^2)^3$. ω_c is a critical frequency which roughly measures the upper limit of the spectrum. The expression for the total power ra-

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[6] L. I. Schiff, Rev. Sci. Inst. **17**, 6 (1946); J. S. Schwinger, Phys. Rev. **70**, 798 (1946); L. Arzimovich and I. Pomeranchuk, J. Phys. USSR **9**, 267 (1945).

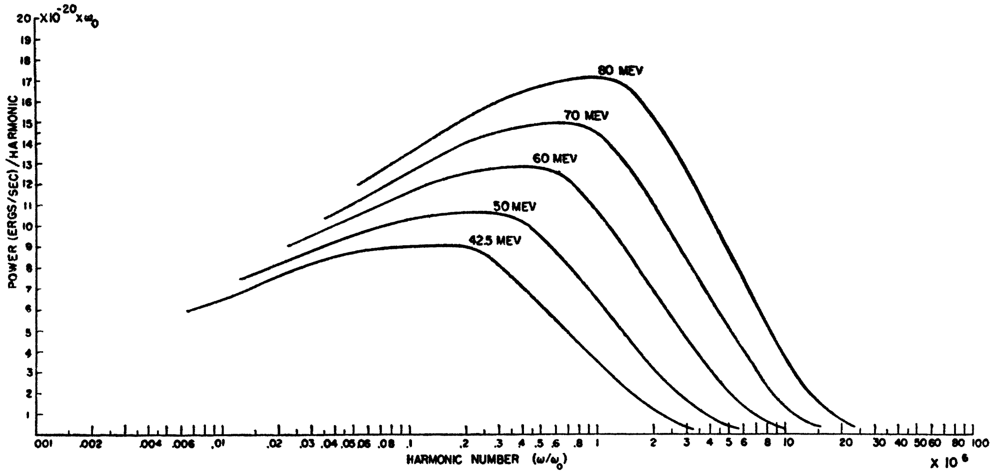


FIG. 1. Radiation loss per harmonic of an electron sinusoidally accelerated during $\frac{1}{4}$-cycle *vs.* harmonic nomber.

diated is:

$$P=\int_0^\infty P(\omega)d\omega=\tfrac{2}{3}\omega_0(e^2/R)(E/mc^2)^4. \qquad (2)$$

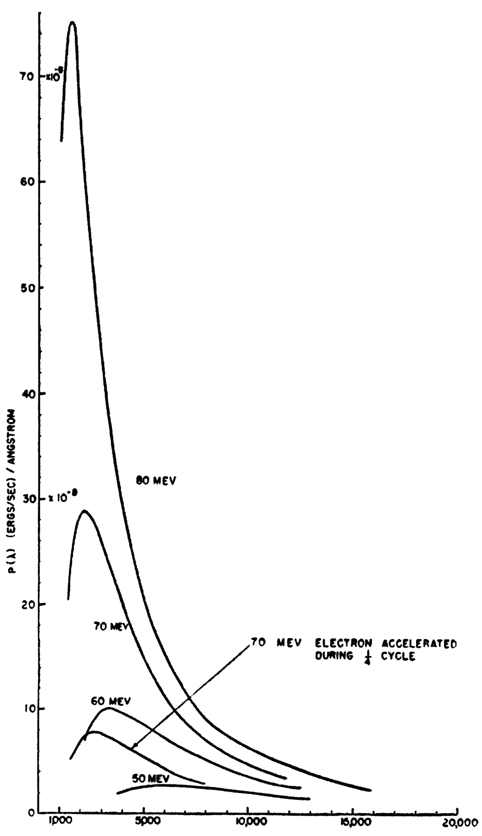


FIG. 2. Radiation loss per angstrom of a constant energy electron *vs.* wave-length.

This is essentially the expression first derived by Lienard. Most of the energy is emitted in the region of the critical frequency and is contained in a cone of $\frac{1}{2}$-angle approximately equal to mc^2/E about the direction of electron motion.

The actual energy distribution observed with the synchrotron, however, is that of an electron of energy E, which varies sinusoidally, i.e., the average of (1) over an acceleration interval. If the magnetic field increases sinusoidally to its maximum value in the time T, $E=E_{\max}\sin(\pi/2T)t$ and

$$\omega_c=\tfrac{3}{2}\omega_0(E_{\max}/mc^2)^3\sin^3(\pi/2T)t=\omega_m\sin^3(\pi/2T)t.$$

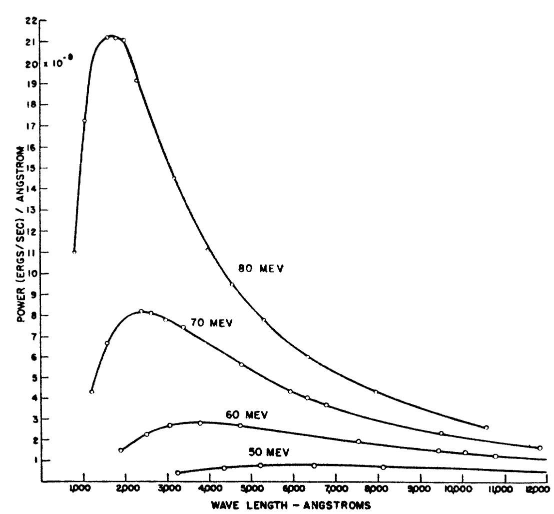


FIG. 3. Radiation loss per angstrom of an electron sinusoidally accelerated during $\frac{1}{4}$-cycle *vs.* wave-length.

It is convenient to use

$$\tau=\omega_m/\omega_c=\sin^{-3}(\pi/2T)t.$$

The desired average power,

$$\bar{P}(\omega)d\omega, \text{ is } (1/T)\int_0^t dtP(\omega)d\omega$$

$$\bar{P}(\omega)d\omega=(3\sqrt{3}/2\pi^2)\omega_0(e^2/R)(E_{\max}/mc^2)^4(\omega/\omega_m)^2$$
$$\times\left[\int_1^\infty(\tau^{\frac{2}{3}}-1)^{\frac{1}{2}}K_{5/3}[(\omega/\omega_m)\tau]d\tau\right](d\omega/\omega_m). \quad (3)$$

The total average power is

$$\bar{P}=\int_0^\infty\bar{P}(\omega)d\omega=(1/4)\omega_0(e^2/R)(E_{\max}/mc^2)^4. \quad (4)$$

The integral in Eq. (3) has been evaluated by graphical integration using the relations

$$K_{5/3}(x)=(4/3x)K_{2/3}(x)+K_{1/3}(x),$$
$$K_{1/3}(x)=(\pi/\sqrt{3})(i^{1/3}J_{-1/3}(ix)-i^{-1/3}J_{1/3}(ix)), \quad (5)$$
$$K_{2/3}(x)=(\pi/\sqrt{3})(i^{2/3}J_{-2/3}(ix)-i^{-2/3}J_{2/3}(ix)),$$

and the tables given by Jahnke and Emde.[7]

The total power per harmonic, which an electron at constant energy radiates, has been calculated from (1), and the power radiated by a sinusoidally accelerated electron calculated from (3). In Fig. 1 the latter result is plotted for an orbit radius of 29.2 cm, that of the Schenectady synchrotron. In Figs. 2 and 3 the theoretical results are plotted with the power per wave-length expressed as a function of wave-length. It is evident from these curves that the distribution of light energy is very different from that of the more conventional light sources.

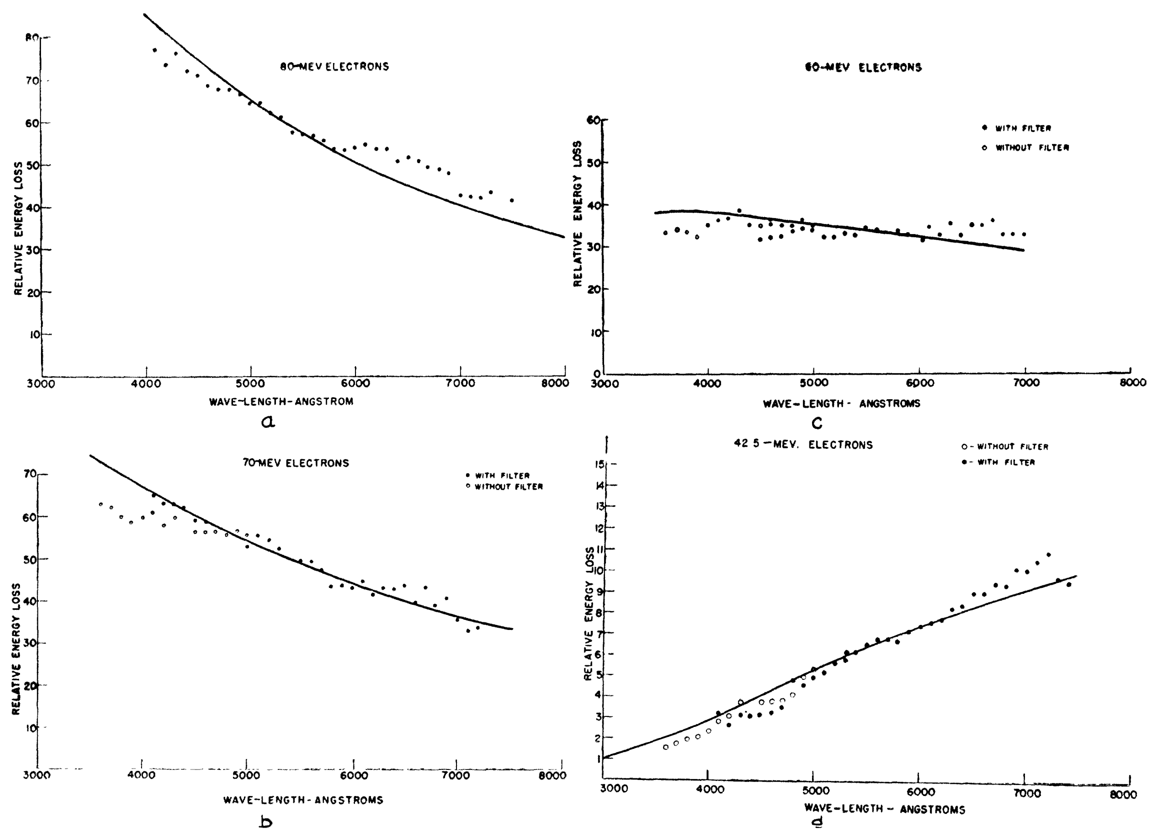


FIG. 4. Comparison of measured energy distribution of the electron light with the theoretical spectral curves. Electrons accelerated to peak energies of (a) 80 Mev, (b) 70 Mev, (c) 60 Mev, and (d) 42.5 Mev.

[7] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York), 4th edition, p. 235.

3. EXPERIMENTAL

To verify that the spectral distribution of the radiation from the electron beam agrees with the theory, we used a recording spectroradiometer developed by Dr. Frank J. Studer of this laboratory. Light from the electrons leaves the accelerating tube through a quartz window and enters the spectroradiometer through a slit which is centered in and perpendicular to the plane of the orbit. In this instrument the light is reflected from a front-surfaced mirror on to a Wood grating of 15,000 lines per inch, which concentrates the reflected light into the first-order spectrum. An arrangement of gears rocks the grating and simultaneously traverses the spectrum with a 1P22 photo-multiplier tube. The photo-tube output is amplified and fed into a curve-drawing meter. The drives for the traverse and the recording meter are synchronized so that the two are in step at all times. The spectroradiometer was calibrated by making records with a standard tungsten lamp of color temperature 2848°K. From this record the sensitivity of the instrument was obtained throughout the wave-length range 3500 to 7000 angstroms. To prevent the second-order spectrum of short wave-length light from reaching the 1P22, a filter which cut off at 4200 angstroms was used in front of the slit during both the experimental runs and calibration in the region above this wave-length.

When using the synchrotron for x-ray production the radiofrequency acceleration voltage is usually turned off when the electrons are at desired energy and the orbit permitted to expand or contract to hit suitably placed targets. However, for these optical experiments, the electrons were accelerated to maximum energy and then decelerated to a low energy before turning off the r.f. accelerating voltage. Different peak energies were obtained by changing the excitation of the magnet. For each excitation level the peak magnetic field, which determines the electron energy, was measured by means of a search coil and an R-C integrating circuit. The machine is driven at 60 cycles/sec.

The agreement of theory and experiment is shown in Fig. 4. The theoretical curves obtained from Eq. (3) were arbitrarily matched with the experimental data at 5000 angstroms. The scatter of the points is probably due to fluctuations of the electron beam current during a run. We also took several spectrograms with a Hilger quartz spectrograph. The plates indicated the continuous character of the spectrum and showed the expected change in distribution of intensity with peak electron energy. However, the variation of plate sensitivity with wave-length made this method seem less attractive than the use of the spectroradiometer.

A measurement of the electron beam current, using Eq. (4), was made in the following way. A shielded Kipp thermopile, located 23 in. from

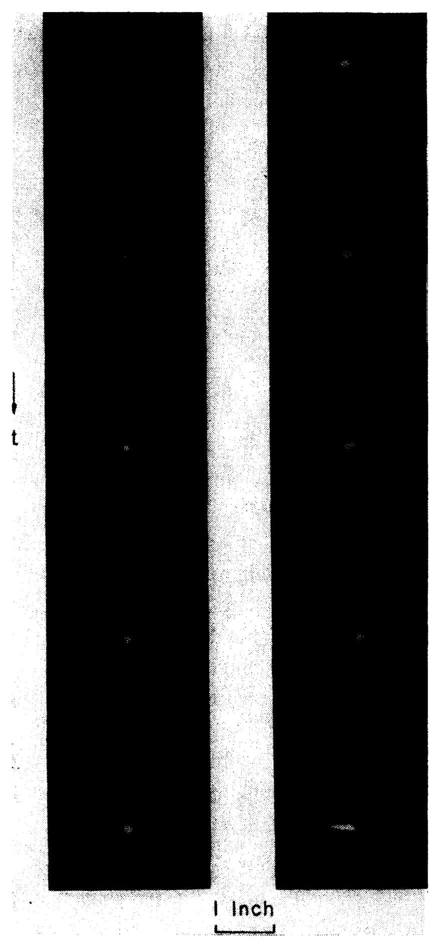


FIG. 5. Consecutive frames taken with Fastex Camera located in the plane of the orbit. The center of the orbit is to the right of the spot. The r-f was turned off before peak magnetic field (after frame 8). The orbit contracts (frame 9) and then expands, moving to the left in the final frame.

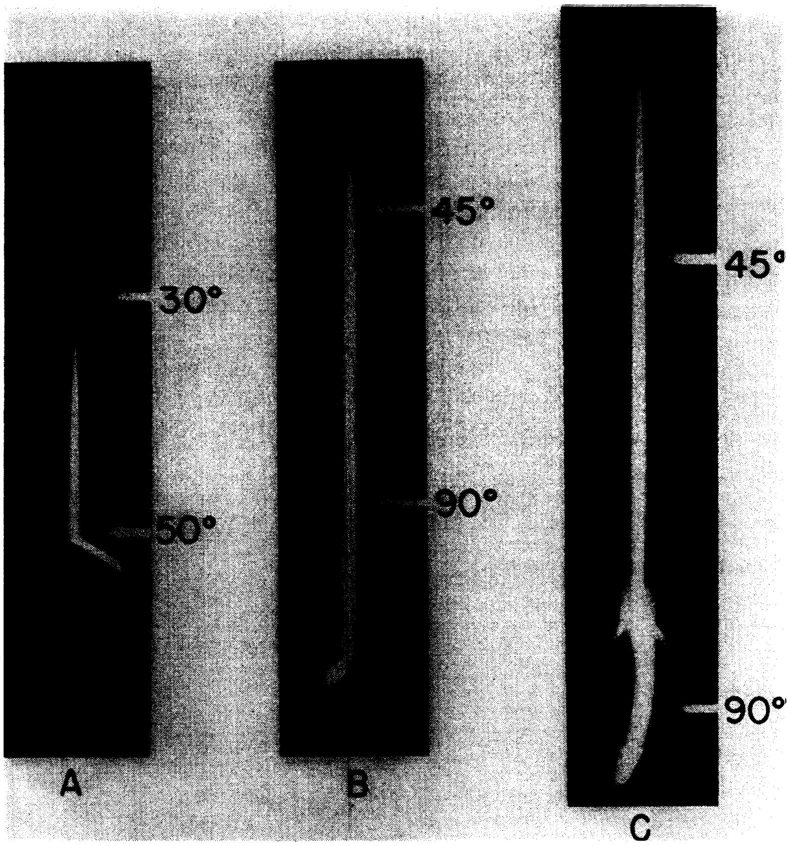


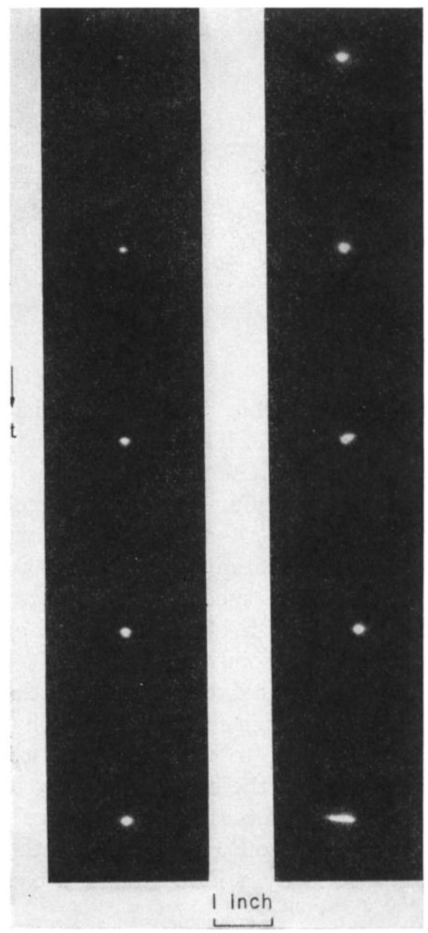
FIG. 6. Photographs of beam taken with a continuous drive camera. The numbers indicate electrical degrees after zero magnetic field. (a) Beam contracted at approximately 55 Mev. (b) Beam expanded by turning off r-f after peak magnetic field. (c) Radiofrequency turned off just before peak magnetic field corresponding to 70 Mev. Beam first contracts slightly and then expands to target. Some electrons are lost from the beam at the time of r-f turn off.

the center of the orbit, received 1.8 microwatts on its active surface. This corresponded to 3.3 milliwatts around the entire orbit, which is the power radiated by about 10^8 electrons accelerated each cycle. The 70-Mev x-ray output with the same injection conditions was about 300 roentgens per minute, as measured in a Victoreen *r* thimble mounted in a $\frac{1}{4}$-inch thick lead cylinder located one meter from the target.

The high intensity of the optical radiation from the beam permits high speed photography of the beam. It is possible to determine that the high energy diameter of the beam is about $\frac{1}{8}$ in., and to follow its movements after the r-f accelerating voltage was turned off. Figure 5 shows the beam as photographed by a Fastex camera operating at 3000 frames per second. A different display of the beam is shown in Fig. 6. These pictures were taken with a General Radio continuous drive camera in which the film moved perpendicular to the plane of the orbit with a uniform speed of 40 ft./sec.

ACKNOWLEDGMENTS

This work has been supported by the Office of Naval Research. We are very grateful to Professor J. S. Schwinger of Harvard University for sending us the equations quoted in this paper. Dr. E. A. Lewis of M.I.T. and Dr. H. Levine of Harvard have also sent us some of their theoretical results. Mrs. R. Wells of this laboratory assisted with the numerical calculations.



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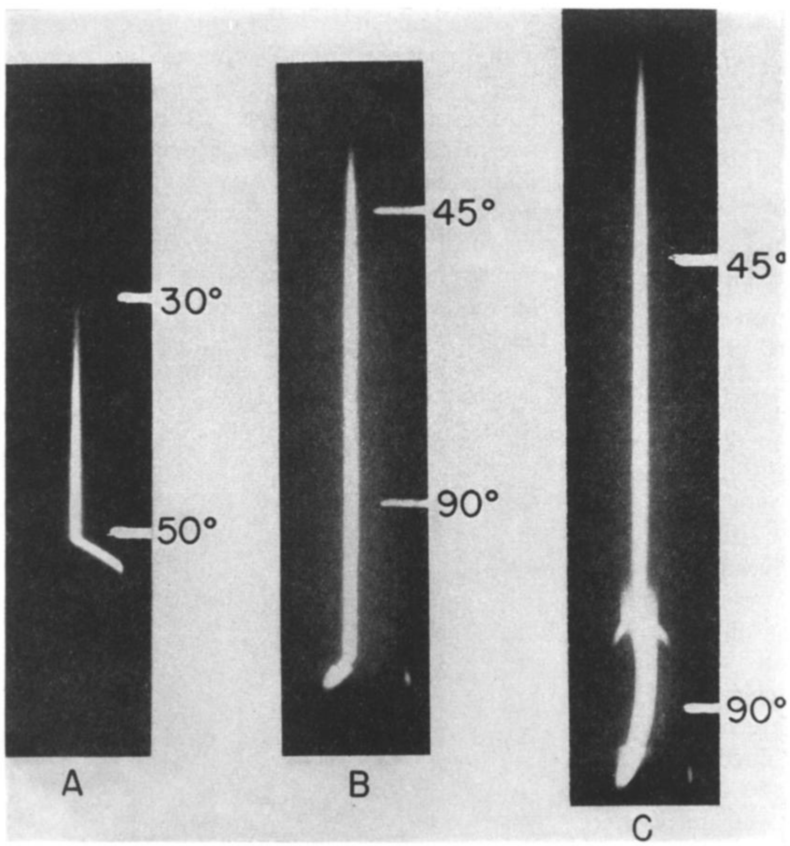

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