Resonances in $Li^7(p,n)Be^7$

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A qualitative and schematic discussion of the theory of the angular distribution of neutrons emitted in the reaction $Li^7(p,n)Be^7$ is presented. Evidence concerning existence of quasi-stationary states is discussed with the conclusion that there are indications of two such states, one at ~2.2-Mev incident proton energy and another below the reaction threshold. Both levels are probably of odd parity. The existence of the level below threshold is based on less direct evidence than that of the level at ~2.2 Mev. It is assumed that Be^7 is left either in its ground state or another state of the same parity.

I. INTRODUCTION AND NOTATION

IN this section the general reasons for considering the interpretation to be a likely one will first be outlined, and formulas for the angular distribution will also be derived. Considerations of orders of magnitude indicate that the reaction is caused to a considerable degree by the action of s protons. Since the interpretation makes use of a resonance to s protons, the additional reasons for considering them as playing an important part will now be mentioned. Comparison of the observed cross section[1] with those shown in Fig. 10 of Rumbaugh, Roberts, and Hafstad[2] shows that it is in the class of "probable" reactions in the terminology of Goldhaber.[3] An extrapolation of the curve for $Li^7(p,\alpha)He^4$ indicates in fact a cross section of only about 2×10^{-27} cm^2 at 1.86 Mev while the $Li^7(p,n)Be^7$ reaction shows a cross section of about 1.2×10^{-24} cm^2 at 2-Mev proton energy. The order of magnitude is the same as that of $Li^7(d,n)2He^4$, which is one of the most probable reactions of protons with Li. It is generally considered as likely that $Li^7(p,\alpha)He^4$ is caused by p protons and the classification of $Li^7(p,n)Be^7$ among the more probable $L=0$ reactions appears to be a natural one. The comparison made above is sensitive to the incident proton energy, E_p, because the cross section vanishes at the threshold of the reaction. A somewhat more quantitative argument will, therefore, be given. On the hypothesis of s waves for incident protons and ejected neutrons and of direct transitions produced by the deviation of the Hamiltonian from spherical symmetry one obtains the following approximate formula for the expected cross section

$$\sigma\cong E_n^{\frac{1}{2}}(2M)^{\frac{3}{2}}|H_{if}|^2/(2\pi v_p\hbar^4). \qquad (1)$$

All symbols with the exception of the matrix element are explained in the section on notation at the end of this section. The matrix element H_{if} corresponds to wave functions asymptotic to plane waves of unit density for large internuclear distances. The quantity H_{if} will be interpreted as the result of integrating an unknown energy V through a sphere of radius $\hbar/[3(Mm)^{\frac{1}{2}}c]\cong3\times10^{-13}$ cm and the approximate value of $\sigma=0.24\times10^{-24}$ cm^2 will be fitted for the bombarding energy of 2 Mev. One finds by means of Eq. (1)

$$\sigma\cong2.5(E_n/E_p)^{\frac{1}{2}}(V/20mc^2)^2\times10^{-24}\text{ cm}^2, \qquad (1')$$

and hence for $(8/7)E_p=2$ Mev, $(8/7)E_n=2-1.86=0.14$ Mev there follows $V\cong12\ mc^2$. This is a rather large energy for the volume assumed. If the proton and neutron were taken to be in states $L=1$, appreciable barrier penetration effects would enter and the interaction energy V would become unreasonably high. The estimates of effects of barrier penetrability will be made in two ways corresponding to the employment of the regular and irregular functions. Different results are obtained by the two methods, but both indicate that the $L=0$ reaction is the more likely. One way of making the estimate consists in taking the square of the ratio of the regular functions for the neutron in states $L=1$ and

* Assisted by Contract N6ori-44, Task Order XVI, of the Office of Naval Research.
[1] R. Taschek and A. Hemmendinger, Phys. Rev. **74**, 373 (1948).
[2] L. H. Rumbaugh, R. B. Roberts, and L. R. Hafstad, Phys. Rev. **54**, 657 (1938).
[3] M. Goldhaber, Proc. Camb. Phil. Soc. **30**, 561 (1934).

$L=0$ as a measure of relative probabilities of escape. For $E_n=0.35\ mc^2$ the value of kr is ~ 0.26 and the ratio of probabilities of neutron escape is estimated to be $(0.26/3)^2 \cong 1/130$. This should be combined with a factor for proton entry which also favors the $L=0$ process. In view of the relatively high proton energy barrier penetrability is not a simple matter of proton entry. It nevertheless appears safe to claim that the value of V would have to be at least 11 times greater for $L=1$ than for $L=0$, so that the unreasonably large $V\sim 130\ mc^2$ would be required. On the basis of the irregular function the ratio of probabilities of escape is $(0.26)^2=1/14$ and an interaction energy V of $\sim 46\ mc^2$ would be needed. If the assumptions appropriate to the applicability of the irregular function are more nearly applicable than those of the regular one, the possibility of a major part of the reaction occurring through the p waves is perhaps not excluded at $E_n=0.35\ mc^2$ but is nevertheless improbable because 45 mc^2 through a sphere of radius 3×10^{-13} cm is a questionably high value. The combined evidence makes it reasonable to attribute the main part of the reaction for $E_n\sim 0.2$ Mev to the ejection of s neutrons and the entry of s protons.

The experimentally observed angular distribution shows an appreciable component of the $B\cos\Theta$ type even at lower energies than have just been considered. It is natural to interpret the large asymmetry at low energies as an interference effect of s and p neutrons. The relatively large magnitude of the coefficient B is then explained as arising partly through the largeness of the s part of the neutron wave through a cross product between the s and p waves. The large slope of the graph of B as a function of the energy occurs at nearly the same energy as the maximum of A which corresponds to the spherically symmetric part of the angular distribution. This fact also fits in with the explanation of the variation in B being primarily due to an s wave resonance and arising as the result of a rapid variation of the phase of the s wave.

The statistical mixture of the initial states can be represented as

$$\psi_i=[R_0+R_1\Upsilon_1^0+\cdots][\Sigma_{\mu=-2}^{+2}\epsilon_\mu^{(2)}2_\mu+\Sigma_{\mu=-1}^{+1}\epsilon_\mu^{(1)}1_\mu] \quad (2)$$

in terms of symbols explained at the end of this section. The statistical factors ϵ are subject to rules of statistical averaging summarized by

$$\langle\epsilon_\lambda^{(i)*}\epsilon_\mu^{(j)}\rangle=\delta_{ij}\delta_{\lambda\mu}/8, \quad (2.1)$$

which correspond to a unit probability for the incident state. Independent of any detailed hypothesis concerning the mechanism of the reaction one can be sure that any R_02_μ in the incident wave cannot give rise to $R'_01'_{\mu'}$ or to $R'_02'_{\mu'}$ for $\mu'\neq\mu$ but can be responsible for the appearance of $R'_02'_\mu$ in the neutron wave. Similarly, the state R_01_μ cannot give rise to $R'_02'_{\mu'}$ or to $R'_01'_{\mu'}$ for $\mu'\neq\mu$ but can give $R'_01'_\mu$. It is also a general consequence of conservation of angular momentum that the states $R'_02'_\mu$ arising from any R_02_μ have the same coefficient, which is besides independent of μ, and that the states $R'_01'_\mu$ arise from the R_01_μ with an amplitude which is independent of μ, provided the states 1_μ, $1'_\mu$ are defined in the same convention, i.e., provided they transform similarly under rotations. It does not follow, however, either from conservation of angular momentum or other general relations that the states $R'_01'_\mu$ arise from the R_01_μ with the same strength as the $R'_02'_\mu$ arise from the R_02_μ.

Inasmuch as the appearance of the experimental results suggests resonance involving s neutrons, it is natural to suppose that a state of the compound nucleus having total angular momentum with either the value 1 or 2 is involved, and that there is an inequality in the strengths of the transitions leading to the states $R'_01'_\mu$, $R'_02'_\mu$ which have just been discussed. It is not clear from the experimental results on angular distributions that there is a decided preference for attributing an angular momentum of either 1 or 2 to the resonances. The calculations will be made, therefore, without specialization in this particular.

The peak in the coefficient A is an indication that one deals with an s resonance, and the anomalous dispersion-like shape of the curve for the coefficient B suggests that the rapid change of the phase of the s wave with energy is being brought into evidence in the experiments through interference with neutron p waves. The coefficient C can have to do partly with quadratic effects of the p waves or with linear effects of d

waves. A theoretical understanding of the behavior of C is clearly more complicated and less certain than that of A and B. It involves an understanding of the variations of amplitudes of the p and d waves which, on account of the nuclear spins, involve states with various values of total angular momenta. No attempt is made, therefore, to understand the variation of C with energy in a really quantitative manner. The number of available parameters is great enough to make a precise fit to the experiments of little value.

Disregarding temporarily the complexities resulting from the many factors which enter into the variation of C with energy, there are still the following inherent uncertainties of present theories which make a truly quantitative understanding of the coefficients A, B somewhat difficult.

(a) Even the variation of the coefficient A with energy cannot be simply specified by a formula of the dispersion type unless one either calculates the many body problem involved or else has assurance through empirical evidence that one is dealing with a single level resonance case.

On account of the difficulty of ascertaining the actual situation it will be assumed that the variation of the resonant s wave with energy is of the single level with background type.[4]

(b) The p waves enter through the coefficient B. Their variation with energy is hard to predict. It will be assumed below that the variation of the p waves with energy is not important within the range of energies covered by experiment and that the main variations of B with energy result from the presence of cross product terms with the s wave.

(c) Higher levels of even parity giving rise to transitions to neutron p waves have unknown locations. These are influential in determining the p wave background which interferes with the resonant s state.

The many inherent uncertainties of the problem make it desirable to discuss below the way in which the interpretation lacks definiteness rather than try to arrive at a best fit to experiment. The calculations are carried out only schematically in certain respects as will be seen in the next section. The notation used is summarized below.

[4] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936); H. A. Bethe and G. Placzek, *ibid.* **51**, 450 (1937); G. Breit, *ibid.* **58**, 506 (1940); *ibid.*, 1068; *ibid.* **69**, 472 (1946); E. P. Wigner, *ibid.* **70**, 15 (1946); *ibid.* 606; E. P. Wigner and L. Eisenbud, *ibid.* **72**, 29 (1947).

NOTATION

R_L = radial function for relative motion of Li^7 and H^1 with orbital angular momentum $L\hbar$.

Υ_L^M = angular function for relative motion of Li^7 and H^1 with orbital angular momentum $L\hbar$ and projection $M\hbar$ on line of relative motion of H^1 with respect to Li^7. The angular function is normalized to give unity for integration over angles.

2_μ, 1_μ = spin functions of compounded spin of Li^7 and H^1 corresponding, respectively, to resultants $2\hbar$, $\hbar$, and to projections $\mu\hbar$ on line of flight. They are linear combinations of products of spin functions for the incident proton and spin functions for the ground state of Li^7. The latter are solutions of the complete wave equation for the relative motion and spins; it is taken for the ground state.

$\epsilon_\mu^{(2)}$, $\epsilon_\mu^{(1)}$ = statistical factors describing the statistical mixture of spin states 2_μ, 1_μ. The orthogonality conditions for products of these factors and their complex conjugates are stated as Eq. (2.1).

R_L', Υ'^M_L, $2_\mu'$, $1_\mu'$ are defined for Be^7 and n^1 in the same way as corresponding unprimed quantities have been defined for Li^7 and H^1.

E_p = energy of relative motion of Li^7 and H^1.

E_n = energy of relative motion of Be^7 and n^1.

E_0 = resonance energy; $E = E_p - E_0$.

v_p = relative velocity initially.

v_n = relative velocity finally.

m = electronic mass.

r = internuclear distance.

λ = wave-length of relative motion.

$k = 2\pi/\lambda$.

θ = angle between line from the center of mass to proton and the line of flight.

Θ = angle between line from center of mass to neutron and the line of flight.

A, B, C = coefficients in formula $A + B\cos\Theta + C(3\cos^2\Theta - 1)/2$ for the cross section per unit solid angle in the center of mass system.[5]

ψ_i = wave function of initial state ($Li^7 + p$).

ψ_f = wave function of final state ($Be^7 + n$).

M = mass of proton or neutron.

H = Hamiltonian function of the 8 particles.

II. CALCULATIONS

The Li^7 nucleus will be treated schematically as containing only one neutron which matters for the reaction. This neutron will be supposed to be in a central field, and the other particles of Li^7 are supposed to be in closed shells and not to

[5] A, B, C, are equivalent, respectively, to the quantities a_0, a_1, and a_2 in Eq. (16) of reference 1.

influence the p wave background. Similarly, in Be^7 only one proton is supposed to matter. The orbital p wave functions of the internal neutron in Li^7 are denoted by

$$b_m \quad (m=1, 0, -1),$$

while the p wave orbital functions of the proton incident on Li^7 are designated by

$$A_m.$$

The motion of the center of mass is not explicitly considered. Similarly, the proton wave functions in Be^7 are denoted by

$$a_m$$

and the p wave functions of the emerging neutron by

$$B_m.$$

The magnetic quantum number m is defined here with respect to the line of flight, and among the A_m only the function A_0 enters the expressions for the incident wave. In order to simplify the considerations it will be supposed that for the purpose of discussing transitions of the p states it will suffice to consider only the part of the forces of the ordinary and Majorana type, neglecting the spin dependence of nuclear forces and spin-orbit interactions as well. The incident wave and the Li^7 nucleus can be considered to form states of definite orbital angular momentum $L=0$, 1, 2. Transitions from this state to an intermediate state Q of the compound nucleus can take place only if the orbital angular momentum of Q is also L, because of the simplifying assumption concerning dominance of the part of the potential involving coordinates alone. The state Q can disintegrate into states of the Be^7+n system which also have orbital angular momentum L. Since spin dependence of nuclear forces was assumed to have a negligible effect on the p wave, no distinction is made between singlets and triplets in this part of the calculation. The presence of energy differences in the denominators of formulas for the amplitudes of final states brings the position of the states of the compound nucleus into the final result, and the number of adjustable parameters is greatly increased since the positions of the levels are unknown. In the interests of simplicity only two cases will be considered, each of which is admittedly extreme in its assumptions. The first (case I) is that in which the S, P, D states of Q belonging to the same configuration fall in very closely spaced multiplets. The second (case II) corresponds to the idealization of having the P states so far above the S and D states of the same configuration that their presence can be neglected. The S and D states in case II are supposed to fall close together. Case II is intended to represent a condition which is emphasized by Majorana forces. The incident states of definite orbital angular momentum L with magnetic quantum number M are contained among the nine functions

$$X^L{}_M=\Sigma_\mu C^L{}_{\mu,\,M-\mu}A_\mu b_{M-\mu}, \tag{3}$$

where the $C^L{}_{\mu,\,M-\mu}$ are appropriate coefficients for the composition of angular momenta. Similarly, the emerging states are contained among the nine functions

$$Y^L{}_M=\Sigma_\mu C^L{}_{\mu,\,M-\mu}B_\mu a_{M-\mu}. \tag{3.1}$$

The function A_0 is the term $R_1\Upsilon_1{}^0$ in Eq. (2). From Eq. (3) one obtains the relation

$$A_0 b_M=\Sigma_L C^L{}_{0,\,M}X^L{}_M \tag{3.2}$$

by means of which the incident wave can be expressed in terms of the compounded orbital angular momentum functions $X^L{}_M$. By conservation of angular momentum every $X^L{}_M$ gives rise only to a $Y^L{}_M$ with the same L and M. The coefficients with which the $Y^L{}_M$ arise for the same L but different M can also be shown to be the same. On the other hand, the coefficients depend in general on L.

The transitions are pictured as occurring through the intermediate stage of forming a compound nucleus in states $Q^L{}_M$ which in turn give rise to final states $Y^L{}_M$. The matrix elements of the Hamiltonian giving transitions through the $Q^L{}_M$ of a multiplet depend, in general, on the multiplet and on L. Taking into account all of these dependences is somewhat complicated in the general case, and the calculations have been made in the simplified way of neglecting the variations with L except for the omission of the effect of $L=1$ for case II.

One has then as a result of the collision

$$A_0 b_M\rightarrow P\Sigma_L C^L{}_{0,\,M}Y^L{}_M, \tag{3.3}$$

where P is a number independent of L and where terms for $L=1$ are omitted for case II. For case I one finds simply

$$A_0 b_M \rightarrow P a_0 B_M. \quad (3.4)$$

This is a consequence of the fact that the conditions for case I can be obtained by dealing with degenerate intermediate states so that the calculation could be performed by decoupling the proton and neutron in the intermediate state. For case II one obtains instead of Eq. (3.4)

$$\begin{aligned} A_0 b_1 &\rightarrow P(a_0 B_1 + a_1 B_0)/2, \\ A_0 b_0 &\rightarrow P a_0 B_0, \\ A_0 b_{-1} &\rightarrow P(a_0 B_{-1} + a_{-1} B_0)/2. \end{aligned} \quad (3.5)$$

The wave function of Li^7 with magnetic quantum number M will be denoted by Li_M and is

$$Li_M = \Sigma_\mu C^{\frac{3}{2}}_{\mu, M-\mu} \nu_\mu b_{M-\mu}, \quad (3.6)$$

where ν_μ is the spin function of the neutron with projection μ. Similarly, the wave function of Be^7 in its state with total angular momentum $\frac{3}{2}$ will be written as Be_M and is

$$Be_M = \Sigma_\mu C^{\frac{3}{2}}_{\mu, M-\mu} \pi_\mu a_{M-\mu}, \quad (3.7)$$

where π_μ is the spin function of the proton with magnetic quantum number μ. While, energetically, transitions to the excited state of Be^7 are possible, they do not occur with a large enough probability to be observed and they will be, therefore, left out of account. For this reason in Eq. (3.7) only states with spin $i=\frac{3}{2}$ are considered for Be^7. The statistical wave function representing the incident state contains a term

$$\omega_M{}^\mu A_0 \pi_\mu Li_M, \quad (3.8)$$

where $\omega_M{}^\mu$ is a statistical factor analogous to $\epsilon_\lambda{}^{(i)}$ of Eq. (2) but corresponding to the strong field condition of proton and Li.

Substitution of Li_M by means of Eq. (3.6) gives then an expression involving proton and neutron spin functions as well as internal neutron functions b. By means of Eq. (3.3) one obtains the function for the emerging neutron wave which contains internal proton functions a, neutron orbital functions B, as well as the proton and neutron spin functions. According to Eq. (3.7) the scalar product of any $\pi_\mu a_{M-\mu}$ with Be_M is $C^{\frac{3}{2}}_{\mu, M-\mu}$, and the amplitude of the part of the wave function containing Be_M as a factor is, therefore, $C^{\frac{3}{2}}_{\mu, M-\mu}$ to within a factor independent of M. The contributions to the coefficients of the

$$\omega_M{}^\mu Be_{M'} \quad (3.9)$$

containing p waves B_1, B_0, B_{-1} have been obtained in the manner just described. To these contributions there have been added the s waves which arise partly through the supposed resonance.

The contributions to the statistical wave function containing neutron s waves are supposed here to arise as follows. The spins of the incident proton and the Li^7 nucleus compound themselves so as to form states of angular momenta 2 and 1:

$$d^J{}_M = \Sigma_\mu c^J{}_{\mu, M-\mu} \pi_\mu Li_{M-\mu} \quad (J=1, 2) \quad (4)$$

and, similarly, the spins of the emerging neutron and of Be^7 give states

$$e^J{}_M = \Sigma_\mu c^J{}_{\mu, M-\mu} \nu_\mu Be_{M-\mu} \quad (J=1, 2). \quad (4.1)$$

The functions $d^J{}_M$, $e^J{}_M$ have already occurred in Eq. (2) in another notation. The two notations are related by

$$\begin{aligned} d^J{}_M &= 2_M, \ J=2; \quad e^J{}_M = 2'_M, \ J=2; \\ d^J{}_M &= 1_M, \ J=1; \quad e^J{}_M = 1'_M, \ J=1. \end{aligned} \quad (4.1')$$

By means of Eq. (4) the part of the incident statistical wave function with assigned projections of spins of proton and of Li^7 can be expressed as

$$\omega_M{}^\mu \pi_\mu Li_M = \omega_M{}^\mu \Sigma_J c^J{}_{\mu, M} d^J{}_{M+\mu}. \quad (4.2)$$

The state $d^J{}_M$ can give rise to $e^J{}_M$. Since J and M are, respectively, the total angular momentum and its projection, the matrix elements for the transitions are independent of M. They can depend on J, however. The emerging s states arise, therefore, according to the scheme

$$\omega_M{}^\mu \pi_\mu Li_M \rightarrow \omega_M{}^\mu \Sigma_J c^J{}_{\mu, M} S_J e^J{}_{M+\mu}, \quad (4.3)$$

where the S_J depend only on the energy and on J. By means of Eq. (4.1) the emerging wave which is represented by the right side of Eq. (4.3) can be expressed in terms of some coefficients multiplying the expression $\omega_M{}^\mu Be_{M'}$. Combining these contributions with those which arose in the consideration of p waves in connection with Eq. (3.9), one obtains the whole statistical wave function ordered according to spin orientations of Be^7 and of the emerging neutron. Adding the intensities of the waves for each of these terms and remembering that states with different final spin orientations do not interfere with each other,[6] one finds that

$$I_{\mathrm{I}}=\frac{5}{8}|S_2|^2+\frac{3}{8}|S_1|^2+\frac{5}{24}(B_0{}^*S_2+B_0S_2{}^*)-\frac{1}{24}(B_0{}^*S_1+B_0S_1{}^*)+\frac{2}{9}(|B_1|^2+|B_0|^2+|B_{-1}|^2), \quad (5)$$

$$I_{\mathrm{II}}=\frac{5}{8}|S_2|^2+\frac{3}{8}|S_1|^2+\frac{5}{12}(S_2B_0{}^*+S_2{}^*B_0)-\frac{1}{12}(S_1B_0{}^*+S_1{}^*B_0)+\frac{1}{3}|B_0|^2+\frac{1}{18}(|B_1|^2+|B_{-1}|^2), \quad (5.1)$$

where the intensities for cases I and II are distinguished by corresponding subscripts. The factor P has been set equal to unity since no attempt has been made to calculate its magnitude. In Eq. (5) (case I) the quadratic terms in the p waves combine to give a spherically symmetric combination. In Eq. (5.1), however, there is left over a quadratic term which can be confused with a d wave. Changing the scale of the p waves in Eq. (5), one obtains

$$I_{\mathrm{I}}=\frac{5}{8}|S_2|^2+\frac{3}{8}|S_1|^2+\frac{5}{12}(B_0{}'^*S_2+B_0{}'S_2{}^*)-\frac{1}{12}(B_0{}'^*S_1+B_0{}'S_1{}^*)+\frac{8}{9}(|B_1{}'|^2+|B_0{}'|^2+|B_{-1}{}'|^2), \quad (5.2)$$

which is identical in form with Eq. (5.1) except for the quadratic terms in the p waves.

Besides the effect of the s and p waves there is probably also an effect of d waves produced partly as the result of the incident s and d waves. No attempt will be made here to treat the d wave for emerging neutrons quantitatively. It differs appreciably between cases I and II and the uncertainties in the determination of s and p wave parameters are so large that one may consider the experimental value of the coefficient of $P_2(\cos\Theta)$ to be reasonable.

III. DISCUSSION AND CONCLUSIONS

It is impractical to try to predict purely theoretically the energy dependence of all the parameters that enter the resonance theory of nuclear reactions. Since the reaction studied here has a threshold close to the region within which the data lie, it would be wrong to omit the more obvious parts of the dependence on the neutron velocity. The emerging s wave is expected to have an intensity proportional to the neutron velocity v_n, the p wave to $v_n{}^3$, and the cross product terms between s and p waves to $v_n{}^2$. Besides there should be other dependences contained in the usual Γ_n of the denominators of dispersion formulas. Neglecting these, the correction for neutron velocity can be made by dividing the experimental A, B, respectively, by $(E-1.86)^{\frac{1}{2}}$, $E-1.86$. The result of doing so is shown in Fig. 1 in which the quantities

$$A'=0.6A(E-1.86)^{-\frac{1}{2}}, \quad B'=0.36B/(E-1.86)$$

are plotted against E. The values of A and B used here are nearly but not precisely equal to

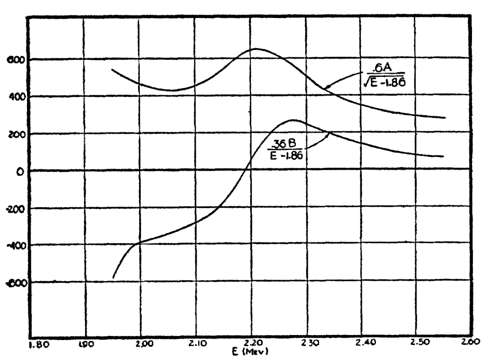

FIG. 1. Experimental angular dependence of neutron intensity plotted against proton energy and corrected for part of the expected effect of neutron velocity (see the beginning of Section III of the text). A and B are determined by the observed neutron intensity

$$I(\Theta)=AP_0(\cos\Theta)+BP_1(\cos\Theta)+CP_2(\cos\Theta).$$

[6] Reference 1 contains a determination of the amplitudes and phase difference of neutron s and p waves which would give the observed neutron intensity in the absence of spin effects.

the values of a_0 and a_1, given by the more systematic analysis of Reines.[1]

It is seen that both A' and B' show a strong variation with energy at $E=1.86$ Mev, suggesting that there might be a resonance below the threshold in addition to the pronounced resonance at about 2.2 Mev. The increase in B' towards the threshold is then explicable as being a consequence of the increase in A' in the same region.

In Fig. 2 there are plotted for comparison graphs of $a+qb$ with

$$a=1/(1+x^2), \quad b=x/(1+x^2).$$

For a simple one-level resonance with background one expects the quantities A', B' to vary with energy approximately as $a+qb$. The quantity

$$x=(E-E_0)/\Gamma,$$

where E_0 is the resonance energy and 2Γ is the half-value breadth. In making the comparison the following considerations enter:

(a) If either S_2 or S_1 is represented by single level resonance with background, i.e., if it is of the form

$$[\alpha/(E-E_0+i\Gamma)]+\beta,$$

then only the part of the right side of Eqs. (5.1), (5.2) containing $|S_2|^2$ or $|S_1|^2$ contains α and β. There is besides, however, an additional background of an additive character in the expected variation of the coefficient A with energy. The comparison of Figs. 1 and 2 has to be made, therefore, by suitably translating the figures up and down before superposing them.

(b) The quantity Γ can be expected to vary with energy. For energies slightly above threshold the variation with energy of the part of Γ having to do with neutron escape is expected to be given by a simple proportionality to v_n. This, however, is the only simple obvious dependence, and deviations from the simple formula can be expected.

(c) The coefficient B' is also expected to be representable only in part by graphs of the type drawn in Fig. 2. The presence of cross product terms involving both S_2, S_1 brings in an additive background term here as well.

(d) In view of the fact that the phases of S_2, S_1, B_0 occurring in Eqs. (5.1), (5.2) have no special relation to each other the coefficient q of $a+qb$ which depends on these phases has to be inferred from experiment.

(e) The resonance level does not, in general, lie[4] at a maximum of either A or B.

(a) Comparison of Figs. 1 and 2 shows that: (a) Experiment is not in agreement with the simple one level with background theory. This is especially true for the marked variations in both A' and B' close to the threshold. It is believed, on the other hand, that the region close to threshold is subject to especially large experimental uncertainties.

(b) For 2.20 Mev$<E<$2.60 Mev there is a marked similarity of the curves of Fig. 2 with those of Fig. 1. The behavior of A' suggests a value of q close to zero. The behavior of B' speaks for $1/q$ close to zero. Such values of q would be obtained if β were small and if there were no phase difference between α and B_0.

(c) If the assignment of q considered in (b) above should be the right one, then subtraction from B' of a negative contribution to the left of the node and representing b of Fig. 2 leaves a part which is appreciable only above the threshold and fits in qualitatively with the increase of A' close to the threshold.

(d) The parts of A', B' to the right of the node of B' fit in reasonably well with the expected variations of a and b. Thus, for example, the maximum of b falls at $x=1$, which should be also

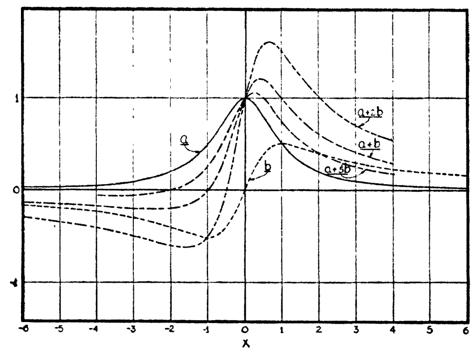

FIG. 2. Approximate energy dependence of the quantities A' and B' of Section III (plotted as ordinates in Fig. 1) to be expected in the case of a one-level resonance with background. Here $a=1/(1+x^2)$ and $b=x/(1+x^2)$; for comparison of these curves with those of Fig. 1, x is to be taken as $(E-E_0)/\Gamma$, where E_0 is the resonance energy and 2Γ is the half-value breadth.

the point at which a has a value equal to one-half of its maximum value. The maximum of B' falls at about 2.28 Mev. Taking the value of A' at 2.60 Mev as an approximate base line for A', the half-value point of A' falls at ~ 2.32 Mev, which may be looked at as agreeing reasonably well with 2.28 Mev, since there are so many uncertain factors in the adjustment of the curves to each other.

The tentative interpretation just discussed is defective in reproducing the exact shape of the right side of curve b of Fig. 2. Thus, e.g., the value of b for $x=2$ is 0.8 of its value at the maximum which falls at $x=1$. On the other hand, the value of B' at $2.19+2(2.88-2.19) = 2.37$ Mev is $165/250=0.66$ of the value at the maximum. The quantity B' is thus seen to decrease relatively somewhat faster beyond the maximum than b. The fit could be improved in a number of ways. Thus, for example, one can leave the zero line of B' undisplaced but fit the curve for B' to the right of its node by a curve of type $a+qb$, with a finite positive q, to the right of the node of B'. For $q=1$ the distance from the node to the maximum is 1.4, the point distant from the node by twice this amount is at $x=1.8$, and the ratio of $a+b$ at $x=1.8$ to its maximum value is $0.66/1.21=0.55$, which is less than the corresponding number for the curve b. For $q=2$ this number is 0.63, which is close to the experimental value. The fit can also be improved by displacing the zero line of B' and various combinations of the two ways can be used.

It is clear that the presence of several adjustable parameters in Eqs. (5.1), (5.2) would make a fit by one or another type of curve very questionable. This situation is further aggravated by the presence of unknown variations in the backgrounds. In addition, the cross product terms which matter for B are affected by both the phase and the amplitude of the nonresonant s wave.

(e) It is believed in view of the arguments just presented that the experimental material contains evidence of the presence of a quasi-stationary level at an energy corresponding to ~ 2.2-Mev incident proton energy and that the half-value breadth $2\Gamma \sim 200$ kev. It is also believed that there is an indication of another level which is stable towards dissociation into Be^7+n but unstable towards disintegration into Li^7+p. The behavior of the angular distribution coefficients A, B suggests that in both cases there is resonance to s protons and it is probable, therefore, that the compound nucleus is in an odd state for both of them.

(f) It was assumed throughout that Be^7 is left in its ground state after neutron emission. A reason for this assumption is the apparent absence of a second group of neutrons which would be expected to appear if Be^7 were left in an excited state analogous to the 450-kev level of Li^7. It should be pointed out, however, that this assumption is based on incomplete evidence. It is difficult to exclude the possibility of a weak neutron component of smaller energy which might be setting in $\sim 8\times 450/7=514$ kev above threshold and would confuse the interpretation. On the other hand, there is no special evidence in the curves of Fig. 1 for supposing that such a neutron group begins to appear at $\sim 1.86+0.51 = 2.37$ Mev. Both A' and B' decrease smoothly in this region.

It is more difficult to deal with the possibility of an excited level of Be^7 at a quite low energy taking part in the process. If it did, the parity of the level might even be different. On the other hand, it is somewhat improbable that there should be a level of a different parity so close to the ground level of Be^7.