where K is as given in the text, Eqs. (24) and (25).

Case 1: $2\pi\nu\tau_2 \gg 1$

Here

$$V_n^2 = V_{n0}^2 + K + iK/2\pi\nu\tau_2, \quad \text{(C32)}$$

so that at infinite frequency

$$V_{n\infty}^2 = V_{n0}^2 + K. \quad \text{(C33)}$$

The attenuation coefficient arising from (C32), and entering the solution of the wave equation in the form

$$\bar{Y} = \bar{Y}_0 e^{-\alpha x} \exp 2\pi i\nu(t - r/V_n),$$

where r is position, is

$$\alpha = K/2\tau_2 V_{n0}^3. \quad \text{(C34)}$$

Case 2: $2\pi\nu\tau_2 \ll 1$

Here

$$V_n^2 = V_{n0}^2 + K\cdot 2\pi i\nu\tau_2, \quad (35)$$

and at zero frequency

$$V_n^2 = V_{n0}^2, \quad \text{(C36)}$$

while the attenuation coefficient for this frequency range is

$$\alpha = 2\pi^2\nu^2\tau_2 K/V_{n0}^3. \quad \text{(C37)}$$

The peak attenuation occurs at the frequency given by

$$2\pi\nu\tau_2 = (V_{n0}^2/(V_{n0}^2 + K))^{\frac{1}{2}},$$

which is practically unity. The peak attenuation coefficient is then

$$\alpha_{\max} = \tfrac{1}{2}\pi\nu K/V_{n0}^3, \quad \text{(C38)}$$

as stated in the text.

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Relaxation Theory of Thermal Conduction in Helium II

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The equation for thermal diffusion derived in the foregoing paper agrees with the relaxation equation for second sound used previously (see reference 3), if the second sound velocity is corrected to zero frequency. The relaxation time τ is calculated to a higher accuracy. The peculiar property of the heat current in being proportional to the cube root of the temperature gradient can be phenomenologically reduced to the relation

$$u_s^2\tau_s = J,$$

where u_s is the velocity of the superfluid balancing the mass flow of normal particles transporting the energy. τ_s is a relaxation time derived from τ, the relaxation time responsible for the damping of second sound, by the conditions for frequency balance. J appears to be a simple function of temperature only, approaching $h/m\pi$ at temperatures below 1°K. The equations lead to a qualitative understanding of the reduction of thermal conductivity by mass flow found by Kapitza.

IN the previous paper[1] it has been shown that the wave equation for second sound degenerates into a diffusion equation approaching zero frequency and becomes

$$\langle\dot{Z}\rangle/\tau_1 = V_{s0}^2\nabla\nabla\cdot\bar{Z}; \quad V_{s0}^2 = x'(1-x)SQ^2/x^2C^2, \quad (1)$$

where x is the fraction of the liquid in the normal (excited) states, $\bar{Z}$ is the relative displacement vector of the normal fluid, τ_1 is the relaxation time for momentum exchanges between the two component streams. Take the divergence of this equation: assume very small amplitude and absence of ordinary sound, then we shall have, by (C6) of the previous paper,

$$\nabla\cdot\bar{Z} \propto \Delta x/x.$$

Thus:

$$(1/\tau_1)(\partial/\partial t)(\Delta x) = V_{s0}^2\nabla\nabla\cdot(\Delta x). \quad (2)$$

Here there is full relaxation so we have $\Delta x = x'\Delta T$,

[1] W. Band and L. Meyer, Phys. Rev. **74**, 386 (1948).

where $\Delta T = T - T_0$, T_0 being constant. Thus (2) can be written

$$(1/\tau_1)(\partial T/\partial t) = V_{s0}^2\nabla^2 T. \tag{3}$$

If this is compared with the equation for conservation of heat flow,

$$\rho C(\partial T/\partial t) = -\nabla \cdot W.$$

W being the heat current density, Eq. (3) yields*

$$\nabla \cdot W = -\rho\tau_1 C V_{s0}^2 \nabla \cdot \nabla T, \tag{4}$$

or

$$W = -\rho\tau_1 C V_{s0}^2 \nabla T. \tag{5}$$

From the experimental fact[2] that a heat current W in helium II is proportional to the cube root of the temperature gradient

$$W \propto (\nabla T)^{\frac{1}{3}}, \tag{6}$$

Eq. (5) means that

$$\tau_1 \propto 1/W^2, \tag{7}$$

because τ_1 is the only quantity in (5) which can depend on W in order to fulfill (6). As the heat flow is due to the diffusion of the normal particles (velocity u_x) with a density ρ_x and energy content ϵ, we can also write[3]

$$W = \rho_x \epsilon u_x. \tag{8}$$

Under the assumption that the energy content of the normal particles is the heat content of helium II, we get

$$\rho_x \epsilon = \rho Q = \rho \int_0^T C dT.$$

This permits us to calculate u_x from measurements on W and C.

Using (8) in (7) we find

$$\tau_1 \propto 1/W^2 \propto 1/u_x^2,$$

or

$$\tau_1 u_x^2 = \text{constant for constant temperature.} \tag{9}$$

The relaxation time τ_1 in itself is due to two processes: transitions from the superfluid state into the normal state with a time τ_s and the inverse process with a time τ_x. As both processes contribute to the disturbance of the opposing streams, the total disturbance is obtained by adding the two frequencies:

$$1/\tau_1 = 1/\tau_s + 1/\tau_x. \tag{10}$$

In the steady state the number of particles passing from one state to the other must be

TABLE I.

T °K	W cal./cm² sec. at 10^{-3} deg./cm	u_x cm/sec.	$u_x\tau_x = u_s\tau_s$ 10^{-4} cm	V_{s0} m/sec.	J
1.0	0.0134	28.1			
1.1	0.0217	23.2	9.8	17.00	0.0099
1.2	0.0347	20.1	8.1	17.06	0.0125
1.3	0.0533	17.4	6.5	17.20	0.0148
1.4	0.0791	15.3	5.5	17.41	0.0183
1.5	0.112	13.5	4.8	17.52	0.0225
1.6	0.155	11.9	4.2	17.66	0.0270
1.7	0.210	10.6	4.0	17.72	0.0363
1.8	0.258	9.0	3.7	17.23	0.0447
1.9	0.285	7.0	3.3	16.00	0.0510
2.0	0.275	5.0	2.9	14.14	0.0546
2.05	—	—	—	12.65	—
2.10	0.196	2.6	2.3	10.95	0.0552
2.15	—	—	—	7.75	—
2.186	—	—	—	—	—

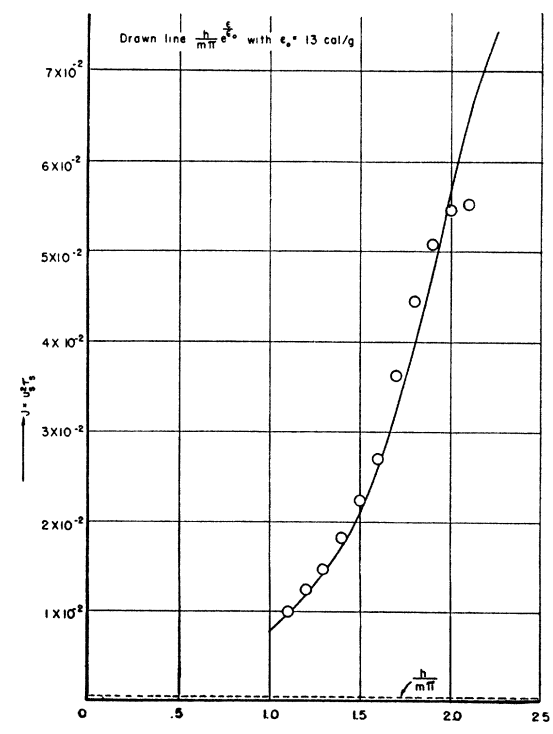

FIG. 1.

* If, as is later found to be the case, $1/\tau_1 \propto \dot{Z}^2$ or W^2, a factor 3 must be inserted on the left side of (2) on taking the divergence of (1). Similarly, a factor 3 must be inserted on the right side of (5) on integrating (4). These two factors cancel each other, so the result (5) is correct in any case.

[2] W. H. Keesom, B. F. Saris, and L. Meyer, Physica **7**, 870 (1940).

[3] W. Band and L. Meyer, Phys. Rev. **73**, 229 (1948).

equal in both directions:**

$$\rho_x/\tau_x = \rho_s/\tau_s. \tag{11}$$

The condition that the center of mass stays at rest is

$$\rho_x u_x + \rho_s u_s = 0. \tag{12}$$

Equations (10)–(12) enable us to replace τ_1 and u_x in (8) by τ_s and u_s leading to

$$\tau_s u_s^2 = J(T). \tag{13}$$

The evaluation of this function is given in Table I and Fig. 1.

The somewhat surprising result is that the values of $J(T)$ tend to approach $h/m\pi$ at temperatures below 1°K. However, as the velocities u_x and u_s in (12) are only drift velocities, this result cannot be related to the uncertainty principle, but must be considered as a transition probability relation. The whole $J(T)$ curve can be represented, purely empirically, by the equation

$$\tau_s u_s^2 = (h/m\pi)\ \exp\epsilon/\epsilon_0, \tag{13a}$$

where ϵ is the energy per excited particle (see reference 1) and ϵ_0 is a constant having the value 0.13 cal./g; m is the mass of the helium atom.

Equation (13) leads also to a simple understanding of the peculiar behavior of the heat current observed by Kapitza[4] in the presence of mass flow. In this case Eq. (11) remains valid, but (12) must be replaced by

$$\rho_s u_s + \rho_x u_x = \rho u, \tag{12a}$$

where ρu is the net mass flow. Express u_s in terms of u_x and u by means of (12a) and use the result in (13), then because of (8) and the equation derived from (10) and (11), namely,

$$1/\tau_1 = \rho/\rho_x\tau_s,$$

it is found that (13) is equivalent to

$$J/\tau_1 = (\rho/\rho_x)\{(\rho u/\rho_s)^2 + (\rho_x W/\rho_s\rho Q)^2 + 2(\rho_x\rho/\rho_s^2)(uW/\rho Q)\}. \tag{13b}$$

Use this in (12a) in the form*

$$\tfrac{1}{3}\rho C V_{s0}^2 d\nabla T = (1/\tau_1)dW \tag{14}$$

and integrate the result; we obtain

$$K^3\nabla T = (\rho/\rho_x)^2(\rho Q u)^2 W + (\rho/\rho_x)(\rho Q u)W^2 + W^3, \tag{15}$$

where K is the normal coefficient under zero mass flow.

It is clear from (15) that if $\rho Q u$ is the same order of magnitude as W—as it was in the measurements reported by Kapitza—and if the temperature is low enough to make ρ_x/ρ considerably less than unity, there should be a great increase in thermal resistance due to the mass flow, qualitatively independent of the direction of u. At temperatures sufficiently near T_λ the effect should change sign with the direction of mass flow. For very large heat currents ($W \gg \rho Q u$) the effect should become very small. The conclusions are all in agreement with Kapitza's observations.

We hope to be able to discuss the meaning of Eq. (13) in a later paper. We want to thank Professor E. A. Long for valuable discussions.

** This equation is strictly true only at zero temperature gradient. If account is taken of the fact that the normal particles are moving towards lower temperature where their number is less, it is easy to calculate from (13) the departure from the cube root law at higher heat currents. It is found that significant increases in thermal resistance are to be expected if the temperature gradients are higher than 0.01°/cm.

[4] P. L. Kapitza, J. Phys. U.S.S.R. **5**, 58 (1941).