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The Magnetic Moment of the Electron†

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A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L=1$ and $g_S=2(1.00119\pm0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

1. INTRODUCTION

One of the important conclusions derived from the relativistic Dirac theory of the electron is that the electron possesses an angular momentum of $\frac{1}{2}$ measured in units of $h/2\pi$ and that with this angular momentum is associated a magnetic moment of one Bohr magneton. This conclusion substantiates earlier conclusions based on an analysis of the experimental data on the anomalous Zeeman effect. Indeed, all relevant experimental data have been in substantial agreement with this conclusion.

A direct measurement of the electron moment can most easily be made by a measurement of the g value of an atomic energy state. Direct determinations of the g values of atomic states from measurements of the frequencies of Zeeman lines in known magnetic fields, as, for example, in the work of Kinsler and Houston,[1] have yielded no significant differences between the measured atomic gyromagnetic ratios and the values consequent from the Dirac theory.* Millman and Kusch[2] have measured the magnetic moments of various nuclei, in particular that of the proton, in terms of the magnetic moment of the electron, assumed to be one Bohr magneton. The magnetic moments so found agree with those dependent on a measurement of a magnetic field in terms of classical standards to within about 0.14 percent ±0.5 percent. This again indicates that the g value of the electron is 2, to within the stated precision. It seems certain that any discrepancy with the theoretical value will be small.

The growth of various techniques of microwave and r-f spectroscopy makes available a series of new tools for the investigation of the detailed structure of atomic spectra. These techniques make it possible to resolve extremely minute details of structure and to determine the relative positions of energy levels to a very high degree of precision. The recent experiments of Lamb and Retherford[3] on the fine structure of hydrogen indicate that the Dirac theory does not adequately describe the hydrogen atom and that all the detailed conclusions of the Dirac theory are, therefore, presumably suspect to some degree.

In the recent measurements of Nafe, Nelson, and Rabi[4] of the hyperfine spectrum of the ground states of atomic hydrogen and deuterium, deviations of the zero field level splittings from the values predicted from theory were found. The theoretical values depend on a knowledge of the nuclear magnetic moment of the nucleus (known only in terms of an assumed value of the electron moment) as well as on the assumption that the magnetic moment of the electron is one Bohr magneton. Breit[5] has suggested that the discrepancy may be removed by the assumption that the electron possesses an "intrinsic" mag-

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[1] L. E. Kinsler and W. V. Houston, Phys. Rev. **45**, 104 (1934); **46**, 533 (1934).

* See, however, a consideration of these measurements in Section 6.

[2] S. Millman and P. Kusch, Phys. Rev. **60**, 91 (1941).

[3] W. E. Lamb, Jr. and R. C. Retherford, Phys. Rev. **72**, 241 (1947).

[4] J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947). J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).

[5] G. Breit, Phys. Rev. **72**, 984 (1947).

netic moment, i.e., a magnetic moment over and above that deduced from the Dirac theory.

The present experiments were undertaken to utilize the power of the atomic beam magnetic resonance method to provide an accurate determination of the electron moment. Preliminary results of the present experiments were given in two Letters to the Editor.[6] Subsequent to the publication of preliminary results of our experiments, Schwinger[7] has published results of theoretical investigation which indicate that the magnetic moment of the electron is, indeed, to be modified as the result of the interaction of the electron with the radiation field.

A deviation of the magnetic moment of the electron from the accepted value of one Bohr magneton could be detected by a precise measurement of the magnetic moment of an atom in a state in which the coupling of the electron spin with the orbital angular momentum is sufficiently well known. An absolute measurement of the magnetic moment requires a measurement of the Zeeman splitting of the zero field energy level in a known magnetic field. At the present time it is difficult to produce magnetic fields which are accurately known in terms of absolute standards and of sufficient magnitude to be useful in atomic beam determinations of the Zeeman splittings of energy levels. However, the frequencies of lines in the Zeeman spectrum of atoms (that is, the differences between atomic energy levels) may be determined by use of readily available techniques to within one part in ten or twenty thousand, and where precision is limited by statistical errors, as, for example, those arising from the least count of an instrument, a considerable improvement in precision may be obtained by a suitable repetition of observations. From measurements of the frequencies of Zeeman lines in two atomic states arising in either the same or different atoms but in the same constant magnetic fields, it is possible to deduce the ratio of the values of the atomic gyromagnetic ratios of these two states. If the spin and orbit vectors are coupled in the same way in the two states, the measured ratio yields no information about the fundamental g values of the electron. If the spin orbit coupling in the two states is different, however, the electron spin g value may be determined in terms of the orbital g value, provided only that suitable information is available, either on experimental or theoretical grounds, as to the validity of the assumed coupling. The principal limitations on such an experimental determination of the electron spin g value are the accuracy in the determination of the line frequencies (limitations imposed by a frequency meter and by the line widths) and the stability and homogeneity of the magnetic field.

In actual practice the single atomic level described above is split into two or more h.f.s. levels because of the presence of the nuclear angular momentum. In such a case the interpretation of data on line frequencies becomes considerably more complicated.

2. THEORY OF THE EXPERIMENT

In this and the following section is developed the elementary theory on the basis of which the experimental data have actually been reduced. Corrections to the results of these sections are considered in Section 6.

The Hamiltonian of the interaction of the electrons in an atom with a constant applied magnetic field H may be written as:

$$\mathcal{H} = g_L\mu_0 L_z H_z + g_S\mu_0 S_z H_z, \tag{1}$$

in which g_L and g_S are the orbital and electron spin gyromagnetic ratios, μ_0 is the Bohr magneton, and L_z and S_z are the operators of the z components of orbital and spin angular momentum. The diamagnetic energy is negligible at the fields employed in this experiment ($H<500$ gauss). For the calculation of matrix elements diagonal in the total atomic angular momentum J, i.e., neglecting atomic Paschen-Back effects, this Hamiltonian may be written:

$$\begin{aligned}\mathcal{H} &= g_L\mu_0 H_z(J|L_z|J)J_z + g_S\mu_0 H_z(J|S_z|J)J_z\\ &= g_L\mu_0 H_z\alpha_L J_z + g_S\mu_0 H_z\alpha_S J_z\\ &= g_J\mu_0 H_z J_z.\end{aligned} \tag{2}$$

The constants α_L and α_S are determined by the electronic wave functions. For Russell-Saunders coupling:

$$\alpha_S = [J(J+1)+S(S+1)-L(L+1)]/2J(J+1), \tag{3}$$

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[6] P. Kusch and H. M. Foley, Phys. Rev. **72**, 1256 (1947); H. M. Foley and P. Kusch, Phys. Rev. **73**, 412 (1948).

[7] J. Schwinger, Phys. Rev. **73**, 416 (1948).

TABLE I. The hyperfine structure separations and the nuclear g values of the atoms whose spectra have been observed.

	$(\Delta W/h)\times 10^{-6}$ sec.$^{-1}$	I	g_I
Na^{23}	1771.75	3/2	-0.0008039
Ga^{69}	2677.56	3/2	-0.0007239
Ga^{71}	3402.09	3/2	-0.0009218
In^{115}	11413	9/2	-0.000664

and

$$\alpha_L=[J(J+1)+L(L+1)-S(S+1)]/2J(J+1).$$

The ratio of the g_J values of two atomic states in the same or different atoms is:

$$g_{J1}/g_{J2}=[(g_L\alpha_{L1}+g_S\alpha_{S1})/(g_L\alpha_{L2}+g_S\alpha_{S2})], \quad (4)$$

in which it is assumed that the values of g_L and g_S are independent of the atomic state. If the fundamental gyromagnetic ratios differ from the conventional values by small amounts, then

$$g_S=2(1+\delta_S),$$
$$g_L=1+\delta_L,$$

and

$$g_{J1}/g_{J2}=[(2\alpha_{S1}+\alpha_{L1})/(2\alpha_{S2}+\alpha_{L2})]+2[(\alpha_{S1}\alpha_{L1}-\alpha_{L1}\alpha_{S2})/(2\alpha_{S2}+\alpha_{L2})^2]\times\{\delta_S-\delta_L\}. \quad (5)$$

Thus if the constants α_{S1}, α_{L1}, α_{S2}, α_{L2} are known from the state of coupling of the atomic levels, the quantity $\{\delta_S-\delta_L\}$ can be determined from the ratio of the atomic g_J values. Clearly, no experiment of this type can distinguish between an effect produced by a small change from the previously accepted values of the spin or the orbital gyromagnetic ratios.

The measurement of the g values of two different atomic energy states in an experiment in which the magnetic field is known in terms of classical standards would, in principle, yield the absolute values of g_S and of g_L separately. However, such a determination would depend on a precisely known value of e/m. Of the experimental methods which have been used to evaluate e/m, the determination from the Zeeman splitting of singlet states assumes that $g_L=1$, and the magnetic deflection results make use of the corresponding assumption of the Lorentz force law. Other determinations of e/m appear either to be of intrinsically low accuracy or to have a rather doubtful theoretical background. In the remainder of the present paper we shall adopt the convention $g_L=1$, and we shall express the experimentally determined quantity $(\delta_S-\delta_L)$ as δ_S.

If hyperfine interactions were absent or entirely negligible the ratio of the frequencies of the Zeeman lines of two atomic states would give directly the ratio of the atomic g_J values. By Zeeman lines we mean here lines resulting from transitions between the magnetic levels of a single atomic energy state. In the atomic states which were studied in the present experiments and at the values of the magnetic field at which lines were observed, the splitting of the energy levels into a hyperfine structure must be taken into account. The existence of this hyperfine structure complicates the analysis of the data from which the ratio of the g_J values is obtained; at the same time the possibility of observing a number of lines of different frequency resulting from hyperfine transitions within each atomic level gives a means of checking the self-consistency of data and improving the accuracy of the results.

3. DESCRIPTION OF THE ENERGY LEVELS

For the energy levels with $J=\frac{1}{2}$ the relevant part of the Hamiltonian is taken to be:

$$\mathcal{H}=a\mathbf{I}\cdot\mathbf{J}+g_I\mu_0 I_z H_z+g_J\mu_0 J_z H_z, \quad (6)$$

in which $\mathbf{I}$ is the nuclear spin operator and $\mathbf{J}$ is the operator of the total electronic angular momentum. The solutions of the secular equations for the energy eigenvalues are:[8]

$$W_{I\pm\frac{1}{2},m_F}=-[\Delta W/2(2I+1)]+g_I\mu_0 H_z m_F \pm(\Delta W/2)[1+(4/(2I+1))m_F x+x^2]^{\frac{1}{2}}. \quad (7)$$

The "weak field" quantum numbers $F=I\pm\frac{1}{2}$, m_F are used to designate the levels. The parameter, x, is defined by

$$x=(g_J-g_I)(\mu_0 H/\Delta W), \quad (8)$$

and $\Delta W=2a$ is the zero field hyperfine separation.

The constants in the energy expression which are given in Table I have been determined with

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[8] S. Millman, I. I. Rabi, and J. R. Zacharias, Phys. Rev. 53, 384 (1938).

precision, by Kusch, Millman, and Rabi[9,10] for sodium, by Becker and Kusch[11] for gallium, and by Hardy and Millman[12] for indium. The results of this paper demand some modification of the values of g_I as given in Table I. However, g_I enters into the expression for the energies of the states only as a correction term, and hence reasonable changes in g_I do not affect our results.[13]

For the ${}^2P_{3/2}$ level of gallium it is necessary to include in the energy expression a term corresponding to a nuclear electric quadrupole interaction. In this case the Hamiltonian becomes

$$\mathcal{H}=a\mathbf{I}\cdot\mathbf{J}+2b\mathbf{I}\cdot\mathbf{J}(2\mathbf{I}\cdot\mathbf{J}+1)+\mu_0H_z(g_JJ_z+g_II_z). \quad (9)$$

Terms due to moments of a higher order than the quadrupole moment are not included. The resulting secular determinant of sixteen rows and columns factors according to the total magnetic quantum number; thus there are two secular equations of the first, second, and third degrees, and one quartic equation. The expressions from which the energy levels may be determined are given below for the case in which $I=\frac{3}{2}$. The complexity of the expressions becomes much greater for $I>\frac{3}{2}$. To simplify the equations we have written:

$$x=(g_J-g_I)(\mu_0H/a), \quad (10a)$$

an expression reminiscent of that employed in describing the energy levels of the states for which $J=\frac{1}{2}$,

$$y=(g_J+g_I)(\mu_0H/a) \quad (10b)$$

and

$$r=b/a.$$

For

$$F=3,\ m_F=\pm 3,\ W/a=(9/4)(1+11r)\pm\tfrac{3}{2}y,$$
$$F=3,\ m_F=\pm 2,\ W/a=\tfrac{3}{4}(1+17r)\pm y+\tfrac{1}{2}[x^2+9(1+8r)^2]^{\frac{1}{2}},$$
$$F=2,\ m_F=\pm 2,\ W/a=\tfrac{3}{4}(1+17r)\pm y-\tfrac{1}{2}[x^2+9(1+8r)^2]^{\frac{1}{2}}. \quad (11)$$

The cubic equations may be simplified by use of the substitution

$$W/a=(W_0/a)+(\delta W/a), \quad (12)$$

where W_0/a is the weak field, first-order approximation to the energy of the level. For $F=3$, $m_F=\pm 1$

$$W_0/a=(9/4)(1+11r)\pm\tfrac{1}{2}y \quad (13a)$$

and

$$(\delta W/a)^3+8(1+3r)(\delta W/a)^2-(x^2-15-120r)(\delta W/a)-2x^2=0. \quad (13b)$$

The value of $\delta W/a$ is the same for $m_F=\pm 1$. For $F=2$, $m_F=\pm 1$,

$$W_0/a=-\tfrac{3}{4}(1-r)\pm\tfrac{1}{2}y \quad (14a)$$

and

$$(\delta W/a)^3-(1+48r)(\delta W/a)^2-(x^2-24r-576r^2+6)(\delta W/a)+(1+24r)x^2=0. \quad (14b)$$

For $F=1$, $m_F=\pm 1$,

$$W_0/a=-(11/4)(1-9r)\pm\tfrac{1}{2}y \quad (15a)$$

and

$$(\delta W/a)^3-(7-24r)(\delta W/a)^2-(x^2-10+120r)(\delta W/a)-3x^2=0. \quad (15b)$$

The quartic equation, whose roots determine the energies of the states for which $m_F=0$, is:

$$\begin{aligned}(W/a)^4&+(5-99r)(W/a)^3+[(9/8)(2755r^2-266r-1)-(5/2)x^2](W/a)^2\\&-[(9/16)(57123r^3-9487r^2-163r+47)+(9/4)(1-55r)x^2](W/a)\\&+(4455/256)(1287r^4-1412r^3+114r^2+12r-1)-(135/32)\\&\times(363r^2-26r-1)x^2+(9/16)x^4=0. \quad (16)\end{aligned}$$

A graph showing the energies of the levels as a function of field has been given by BK. The interaction constants, a and b, have been deter-

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[9] P. Kusch, S. Millman, and I. I. Rabi, Phys. Rev. **57**, 765 (1940).

[10] S. Millman and P. Kusch, Phys. Rev. **58**, 438 (1940).

[11] G. E. Becker and P. Kusch, Phys. Rev. **73**, 584 (1948). (This reference is hereinafter referred to as BK.)

[12] T. C. Hardy and S. Millman, Phys. Rev. **61**, 459 (1942).

[13] All the nuclear g values, with the exception of those of the gallium and indium isotopes, depend on the assumption that the g_J value for the ${}^2S_{\frac{1}{2}}$ state of Rb and Cs is 2. The g_I values given for the isotopes of gallium and indium depend on the assumption that the g_J value for the ${}^2P_{\frac{1}{2}}$ states of these atoms is $\frac{2}{3}$. Not only are these assumptions subject to correction, but they are inconsistent as well. To place the g_I values of Ga and In on the scale used for all other nuclei, a reduction of 0.25 percent is required. An arithmetical error made by Becker and Kusch just compensates for the required reduction of 0.25 percent in the case of Ga^{69}. The g_I values of Ga^{69}, Ga^{71}, In^{115} on the same scale as that used in all other cases are -0.0007239, -0.0009195, and -0.0006616. These changes have no effect at all on the results presented in this paper.

mined to be

for Ga^{69}:

$$a = 190.790\times 10^6 \text{ sec.}^{-1}, \quad b = 2.6049\times 10^6 \text{ sec.}^{-1};$$

and for Ga^{71}:

$$a = 242.424\times 10^6 \text{ sec.}^{-1}, \quad b = 1.6416\times 10^6 \text{ sec.}^{-1}.$$

It is to be noted that all expressions for the levels determine the energy of a level in the same units in which ΔW, a, and b are known. Experimentally, these quantities have been determined in sec.$^{-1}$ and observations of line frequencies, of course, lead to separations of energy levels expressed in terms of sec.$^{-1}$. The quantities x and y always contain the factor $\mu_0 H/a$ (or $\mu_0 H/\Delta W$) where a and ΔW are expressed in ergs. If we express all frequency measurements in terms of megacycles per second, then the quantity above becomes:

$$(\mu_0 H/ha)\times 10^{-6} = (eH/4\pi ma)\times 10^{-6} = H'/a \quad (17)$$

or

$$H' = 1.3998H.$$

In all subsequent discussion we use the quantity H' as a measure of the field.

It should be pointed out that in previous experiments with lithium and potassium in the $J=\frac{1}{2}$ states, observations made at very low fields and at high fields yielded self-consistent values of the interaction constants. This fact indicates that the Hamiltonian function given above is adequate to describe the hyperfine structure of alkali atoms in the $^2S_{\frac{1}{2}}$ state within experimental error. Thus it was unnecessary in the present experiment to carry out observations of all the possible transitions and over a range of values of the absolute magnetic field. For the $^2P_{3/2}$ state of gallium the self-consistency of the frequencies of four lines at a field strength of 380 gauss, under which conditions there is intermediate decoupling of the nuclear spin and the atomic angular momentum, was regarded as a confirmation of the adequacy of the Hamiltonian. Moreover, from a study of the h.f.s. pattern of gallium in the $^2P_{3/2}$ state at very low magnetic fields, BK conclude that no moments of a higher order than the quadrupole need to be included in the Hamiltonian to describe the energy levels.

For the accuracy required in these experiments, perturbation approximations to the energies of the levels were inadequate and, in all cases, accurate solutions to the secular equations were found.

In the present series of experiments the direction of the oscillating magnetic field was perpendicular to the constant magnetic field. Thus the allowed transitions are given, in weak field notation, by $\Delta F = \pm 1, 0$, and $\Delta m_F = \pm 1$. Under very strong magnetic field conditions the nuclear and atomic angular momenta are decoupled, and the transitions may be classified according to $\Delta m_J = \pm 1$, $\Delta m_I = 0$ or $\Delta m_J = 0$, $\Delta m_I = \pm 1$. Whether or not a particular field is "weak" or "strong" for a particular atomic state depends on the value of x (or y) which corresponds to that field. This, in turn, depends on the value of ΔW or a.

The decisions as to what magnetic field strength should be employed and which lines should be observed were made on the basis of the following considerations. Since the present experiments are directed toward the measurement of an atomic magnetic moment the lines to be observed should be selected to possess the greatest possible field sensitivity. Under very strong field conditions the field sensitivity of the transitions $\Delta m_I = \pm 1$ is so small that these lines are not useful for the purpose of this experiment. The transitions $\Delta m_J = \pm 1$ show adequate field sensitivity, but in most cases these lines occur at frequencies which are difficult to obtain experimentally. Under very weak field conditions the transitions $\Delta F = 0$ are the most suitable for this experiment, as the frequencies of these lines are very nearly proportional to the field strength and are sufficiently field sensitive. Very weak fields ($H < 100$ gauss) could not be employed in this experiment because of the residual inhomogeneity of the field.

The value of the magnetic field used in most of these experiments (~400 gauss) represents very weak field conditions for $^2P_{\frac{1}{2}}(In^{115})$, $^2P_{\frac{1}{2}}(Ga^{69})$; for $^2S_{\frac{1}{2}}(Na^{23})$ a considerable departure from weak field conditions appears, and for gallium in the $^2P_{3/2}$ state the field is very strong. Only a few lines in the spectrum of $^2P_{3/2}(Ga)$ of frequency less than 10^9 sec.$^{-1}$ permit a satisfactory determination of g_J.

In practice the large field sensitivity of some lines, particularly in the $^2S_{\frac{1}{2}}$(Na) spectrum, did not bring about a proportional increase in the precision of measurement of the field because of the lack of complete homogeneity in the constant field.

In principle it is possible, from the observed frequencies of lines in the spectra of atoms in two different states, to calculate directly the ratio g_{J1}/g_{J2}. However, such a procedure is extremely laborious. We have, instead, calculated the quantity H' for each observed line. If a discrepancy in the value of H' occurs for atoms in two different states and under conditions for which H' is known to be identical, the discrepancy may be removed by an adjustment of the g_J values. This is evident since in the expressions for the energies of the levels, H' always occurs in the product g_JH'.[14] Suppose the assumed values of g_J are g_{J1}^0 and g_{J2}^0 for two different states. The corresponding values of H' are H_1' and H_2'. $\Delta H' = H_1' - H_2'$. Then:

$$g_{J1}/g_{J2} = (g_{J1}^0/g_{J2}^0)[1+(\Delta H'/H')]. \quad (18)$$

4. APPARATUS AND PROCEDURE

The general procedures and instrumental requirements for the observation of lines in the radio frequency spectra of atoms have been discussed in a number of papers. The molecular beam apparatus used in the experiments described in this paper was originally designed as an apparatus for the study of the radio frequency spectra of molecules in very high magnetic fields. Accordingly, the deflecting fields are long and may be operated at high flux densities in order to deflect molecules with a total moment of the order of one nuclear magneton. The magnetic field in which transitions occur is 48 cm long and can operate at fields as high as twenty thousand gauss. Unfortunately, the field is not entirely homogeneous, and the use of a large fraction of the length of the transition field for the observation of extremely field sensitive atomic lines broadens the lines excessively. In all of the present experiments, the effective length of the transition field was reduced to 2 cm by the expedient of reducing the length of the r-f field through which the beam passes. The deflecting fields were operated at a very low level of flux density to permit refocussing of atoms whose moment is of the order of one Bohr magneton.

For the purpose of consecutive measurement of lines in the radio frequency spectra of two different atomic species, such as gallium and sodium, a special oven chamber was constructed in which an oven containing gallium and another containing sodium were mounted on a platform attached to a ground joint which could be rotated from the outside of the apparatus. This arrangement permitted rapid interchange and adjustment of the two ovens.

All frequency measurements were made on a General Radio heterodyne frequency meter, model 620A. The meter may be used to determine frequencies to about one part in twenty thousand, but the precision in reading may be somewhat greater or less than this, depending on the linearity of the scale at a particular reading, on the thermal stability of the meter and other factors. Because of the limit of accuracy in the location of the centers of the resonance curves imposed by their width, it was not worth while to use more accurate frequency measuring equipment. The uncertainties in frequency measurement imposed by the meter and by the line widths are statistical in character and the precision is improved by judicious repetition of observations. Readings on the wave meter are determined in terms of a crystal within the wave meter. The frequency of the crystal in the wave meter which we used differed from its nominal value by 0.0045 percent, as determined by comparison with signals from WWV. The correction is of no importance in the measurements of the frequencies of the lines of gallium, since BK used the identical meter in determining the constants of gallium. The correction has been applied to all cases, however.

All experimental data considered in the experiments discussed in this paper are measurements of the frequencies of spectral lines. No knowledge

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[14] It is true that a term g_IH' also occurs. However, in the cases of Li^7, Na^{23}, Ga^{69}, Ga^{71}, and In^{115} the value of g_I has been determined in each case in terms of g_J of the ground state of the atom in which the nucleus occurs. The term g_IH' may then be written as $(g_I/g_J)g_JH'$ where the ratio g_I/g_J is a quantity independent of any assumption as to g_J. The only case in which g_IH' is an independent term is that of the $^2P_{3/2}$ state of gallium, where g_I is known in terms of $g_J(^2P_{\frac{1}{2}})$ and not in terms of $g_J(^2P_{3/2})$. However the term is so small that uncertainties in g_I have no appreciable effect on our results.

TABLE II. Observations of lines in the spectrum of the $^2P_{\frac{1}{2}}$ state of gallium.

Isotope	Line	Obs. $f\times10^{-6}$ sec.$^{-1}$	Calc. $f\times10^{-6}$ sec.$^{-1}$
71	$(2, 0)\leftrightarrow(2, -1)$	89.434	89.422
69	$(2, 0)\leftrightarrow(2, -1)$	89.867	89.870
71	$(1, 0)\leftrightarrow(1, -1)$	90.389	90.395
69	$(1, 0)\leftrightarrow(1, -1)$	90.637	90.634
71	$(2, -1)\leftrightarrow(2, -2)$	94.716	94.714
69	$(2, -1)\leftrightarrow(2, -2)$	96.848	96.848

of the magnetic field in which transitions occur is required, though it is necessary that transitions resulting in lines whose frequencies are to be compared, occur in the same magnetic field. In practice, a magnetic field is not entirely constant, and the rate of drift of the field depends upon a number of factors, of which the principal two are the condition of the storage cells which supply the exciting current to the magnet and the temperature stability of the d.c. circuit and the magnet. A drift is observed by noting a change in the frequency of a line with time and a very good correction for drift of field may be made by alternately measuring the frequencies of each of two lines to be compared and then, by suitable graphical or other methods, determining the frequencies of each of the lines at any arbitrary instant of time.

To reduce the time interval between observations on successive lines, and hence minimize the effects of a drifting field, all observations were made on the basis of a single frequency reading for each line. That is, no detailed plot was made of the reduction of beam intensity as a function of frequency; rather, the observer noted the frequency at which the maximum reduction of beam intensity occurred, the mean of the frequencies at which one-half the maximum reduction occurred or the mean frequency of any other pair of symmetrical points. On the basis of several of these observations of a single line, the observer recorded a single reading on a wave meter.

TABLE III. The comparison of the apparent values of H' in the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states of gallium.

$I(^2P_{\frac{1}{2}})$ $f\times10^{-6}$ sec.$^{-1}$	$II(^2P_{\frac{1}{2}})$ H'	$III(^2P_{3/2})$ isotope	$IV(^2P_{3/2})$ transition	$V(^2P_{3/2})$ $f\times10^{-6}$ sec.$^{-1}$	$VI(^2P_{3/2})$ H'	VII $\Delta H'$
96.832	527.24	69	$(3, 0)\leftrightarrow(2, 1)$	492.33	528.27	1.03
96.751	526.84	71	$(3, -1)\leftrightarrow(3, -2)$	459.77	527.63	0.79
96.620	526.19	69	$(3, 0)\leftrightarrow(3, -1)$	443.92	527.08	0.89
					Mean	0.90

TABLE IV. The comparison of the apparent values of H' in the $^2P_{\frac{1}{2}}$ and $^2P_{3/2}$ states of gallium.

$I(^2P_{\frac{1}{2}})$ $f\times10^{-6}$ sec.$^{-1}$	$II(^2P_{\frac{1}{2}})$ H'	III isotope	$IV(^2P_{3/2})$ transition	$V(^2P_{3/2})$ $f\times10^{-6}$ sec.$^{-1}$	$VI(^2P_{3/2})$ H'	VII $\Delta H'$	$VIII$
90.909	528.92	69	$(3, 0)\leftrightarrow(3, -1)$	447.06	529.79	0.87	3
90.910	528.93	69	$(3, -1)\leftrightarrow(3, -2)$	486.90	529.85	0.92	3
90.913	528.94	71	$(3, -3)\leftrightarrow(3, -2)$	491.14	529.90	0.96	2
90.921	528.99	71	$(3, -1)\leftrightarrow(3, -2)$	462.23	529.91	0.92	1
					Mean	0.91	

5. EXPERIMENTAL RESULTS

a. The Determination of the Ratio of the g_J Value of Gallium in the $^2P_{3/2}$ State and in the $^2P_{\frac{1}{2}}$ State

To estimate the precision with which the frequency of a line may be determined, six lines in the spectrum of gallium in the $^2P_{\frac{1}{2}}$ state were measured in rapid succession. Preliminary observation had indicated that the field did not drift at a rapid rate just prior to the observations. Each line frequency is, therefore, the result of a single observation. By use of constants given by BK the frequencies of all lines were calculated, where the $(2, -1)\leftrightarrow(2, -2)$ line of Ga^{69} was used to determine the value of the magnetic field. The results are shown in Table II. The agreement between calculated and observed frequencies is very good. The results indicate, in general, that line frequencies are, indeed, measurable to within one part in twenty thousand. The constants given by BK and used in the calculations are consistent for the two isotopes of gallium and the relationships which determine the frequencies of the lines are valid to within the error in measurement of the line frequencies.

It should be pointed out, however, that all the lines recorded in Table II lie in the same frequency range and have very nearly the same frequency dependence on field. This means that inhomogeneities in the magnetic field, will affect all lines in a similar way and that similar agreement may not occur when two lines in a spectrum have a markedly different frequency dependence on field. In fact, for the case of Na^{23}, to be discussed later, a small systematic discrepancy occurs between the magnetic field as calculated from two lines with markedly different field dependence.

Three lines in the $^2P_{3/2}$ state of gallium were observed alternately with the $(2, -1)\leftrightarrow(2, -2)$ line of Ga^{69} in the $^2P_{\frac{1}{2}}$ state. The results are shown in Table III. Column I gives the frequency of the $(2, -1)\leftrightarrow(2, -2)$ line of Ga^{69} in the $^2P_{\frac{1}{2}}$ state in megacycles per second. Column II gives the value of H' as previously defined and calculated on the basis of the assumption that $g_J(^2P_{\frac{1}{2}})=\frac{2}{3}$. Column III indicates the isotope of gallium in the spectrum of which the line tabulated in column IV has been measured. The lines in column IV are in the spectrum of the $^2P_{3/2}$ state. Column V lists the observed frequency in megacycles per second, and column VI the corresponding value of H' calculated on the basis of the assumption that $g_J(^2P_{3/2})=4/3$. Column VII lists the difference between the values of H' for the $^2P_{3/2}$ and the $^2P_{\frac{1}{2}}$ states. The data and results in Table III are to be considered as the result of an exploratory experiment. The rather large variation between the values of $\Delta H'$ can be ascribed to an insufficient number of successive observations of each line in the face of a drifting magnetic field and to insufficient care in locating the resonance minima. The rapid drift of field is apparent from an observation of the successive values of H' in columns II and VI. The deviation of the values of $\Delta H'$ from the mean can be explained by uncertainties, for the first line, of one part in ten thousand in frequency measurement, and for the second line of only slightly more than this.

In a repetition of the previous experiment, the frequencies of four lines in the spectrum of Ga in the $^2P_{3/2}$ state were measured alternately with the frequency of the $(1, 0)\leftrightarrow(1, -1)$ line of Ga^{69} in the $^2P_{\frac{1}{2}}$ state. The results are shown in Table IV. Column I gives the frequency of the $(1, 0)\leftrightarrow(1, -1)$ line of Ga^{69} in the $^2P_{\frac{1}{2}}$ state. The other columns are exactly as in Table III. Column VIII gives a weight assigned to each observation, based on an evaluation of experimental factors. It should be noted that the rate of drift of field for these observations is very much less than in the case of the data recorded in Table III. The particular lines in the spectrum of gallium in the $^2P_{3/2}$ state which have been measured are rather arbitrary; the lines themselves were not identified until rather elaborate reductions of data had been made. That is to say, observations were made at a field and within a frequency range known to contain lines with suitable frequency dependence on field. The assignment was subsequently made by use of the criterion that all lines in the spectrum of gallium in the $^2P_{3/2}$ state should determine substantially the same field. The assignment is unique. All the lines in the spectrum of gallium in the $^2P_{3/2}$ state which are recorded in the two tables have about the same frequency dependence on field with the exception of the first line in Table III, where the frequency dependence on field is about half as great as for the other lines. In all cases measurements have been made against a single line in the spectrum of gallium in the $^2P_{\frac{1}{2}}$ state, but auxiliary data were taken to make the assignment of the line definite.

TABLE V. Notation used in describing lines in the spectra of sodium, and of gallium in the $^2P_{\frac{1}{2}}$ state.

Line		Symbol
$(2, -2)\leftrightarrow(2, -1)$		Na*I*
$(1, 0)\leftrightarrow(1, -1)$		Na*II*
$(2, 0)\leftrightarrow(2, -1)$		Na*III*
$(1, 1)\leftrightarrow(1, 0)$		Na*IV*
$(2, 1)\leftrightarrow(2, 0)$		Na*V*
$(2, 2)\leftrightarrow(2, 1)$		Na*VI*
$(1, 0)\leftrightarrow(1, -1)$	69	Ga*I*
$(2, 0)\leftrightarrow(2, -1)$	69	Ga*II*

TABLE VI. The observed frequencies and apparent values of H' for several lines in the spectrum of sodium.

Time	Na*I*	Na*II*	Na*III*	Na*IV*	Na*V*	Na*VI*
3:10	418.10 (531.84)					
3:25						176.85 (531.77)
3:35	418.06 (531.80)					
3:45	418.00 (531.75)					
3:50	417.99 (531.74)					
3:55						176.83 (531.67)
4:15			260.51 (531.63)			
4:15			260.50 (531.60)			
4:30		261.34 (531.56)				
4:30		261.34 (531.56)				
4:35						176.81 (531.59)
4:40					206.77 (531.59)	
4:45				207.61 (531.53)		
4:50						176.78 (531.47)
5:00	417.72 (531.50)					

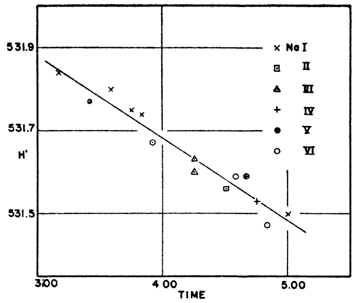

FIG. 1. Values of H' calculated from the observed frequencies of several lines in the spectrum of Na.

An uncertainty in the measurement of the frequency of the $(1,0)\leftrightarrow(1,-1)$ line of Ga^{69} in the $^2P_{\frac{1}{2}}$ state of one part in twenty thousand gives rise to an uncertainty in the corresponding H' of 0.03. The same fractional uncertainty in the measurement of the frequencies of the lines in the spectrum of the $^2P_{3/2}$ state gives rise to an uncertainty in the corresponding value of H' of 0.02. Thus the maximum variation among the values of $\Delta H'$ can be ascribed entirely to the limitation of frequency determination. In our experimental arrangement, a better frequency meter could not be expected to increase the precision of the result to a significant extent, since the width of the resonance curves rendered a significant measurement of frequency to a higher precision impossible.

The variation among the values of $\Delta H'$ shown in Table IV are statistical in character. We conclude that for these observations $\Delta H'=0.91\pm0.03$. The agreement of this value with the mean obtained from Table III indicates that the large scatter of values obtained in the first performance of the experiment is statistical.

On the basis of the assumption that $g_J(^2P_{3/2})=4/3$ and that $g_J(^2P_{\frac{1}{2}})=\frac{2}{3}$, the ratio of the apparent field as calculated from lines in the spectrum of the $^2P_{3/2}$ state to that calculated from lines in the spectrum of the $^2P_{\frac{1}{2}}$ state is 1.00172 ± 0.00006. Evidently the field is independent of the mechanism of measurement. To satisfy this requirement we must make

$$g_J(^2P_{3/2}\mathrm{Ga})/g_J(^2P_{\frac{1}{2}}\mathrm{Ga})=2(1.00172\pm0.00006)$$

where it is assumed that the deviation of the g_J values from their nominal values is small.

b. The Determination of the Ratio of the g_J Value of Sodium in the $^2S_{\frac{1}{2}}$ State and That of Gallium in the $^2P_{\frac{1}{2}}$ State

Six lines of the spectrum of sodium and two lines of the spectrum of gallium were used in the determination of the ratio of the g_J values. To assist in the discussion of the observations, the notation given in Table V is used. All lines of sodium are those of the $^2S_{\frac{1}{2}}$ state and the lines of gallium are of the $^2P_{\frac{1}{2}}$ state.

As in the previous experiment, a test was made to determine the consistency of frequency measurements. In Table VI are given the observed frequencies of the six lines of sodium and the time at which each frequency was measured. Directly beneath each frequency in megacycles per second is given, in parentheses. the value of H' corresponding to the observed frequency and calculated on the basis of the assumption that $g_J(^2S_{\frac{1}{2}})=2$. The data of Table VI are plotted in Fig. 1, where the calculated values of H' are plotted against the time. A straight line is drawn through the points and the average deviation of the points from the mean line is less than 0.03 units of H'. The maximum deviation is 0.04.

TABLE VII. The observed frequencies and values of H' of several lines in the spectrum of sodium and gallium.

Time	Na*I*	Na*II*	Na*III*	Ga*I*	Ga*II*
6:30				91.093 (529.97)+0.00	
6:40					90.319 (529.93)−0.01
6:50				91.087 (529.93)+0.01	
7:05			260.32 (531.16)+0.00		
7:05		261.17 (531.14)−0.02			
7:15	417.33 (531.15)+0.02				
7:40				91.060 (529.78)−0.01	
7:40					90.295 (529.79)+0.00
7:58			260.27 (531.02)−0.01		
8:02		261.12 (531.01)−0.01			
8:14	417.17 (531.00)+0.01				
8:28				91.038 (539.65)−0.02	
8:35					90.275 (529.68)+0.02
8:45			260.21 (530.89)+0.00		
8:49		261.05 (530.85)−0.05			
9:10	417.04 (530.88)+0.03				
9:10	417.05 (530.89)+0.04				
9:26				91.014 (529.52)−0.01	
9:26					90.248 (529.52)−0.01

That is, the maximum deviation of the magnetic field as calculated from the separate lines is one part in thirteen thousand from the average as determined from all observations.

From the frequencies of the Na*I* and Na*VI* lines, at the same instant of time, we have found for $\Delta\nu$ of Na the value 1771.60×10^6 sec.$^{-1}$. A change in the assumed frequency of either line of one part in twenty thousand will alter the $\Delta\nu$ by about 0.25×10^6 sec.$^{-1}$. The agreement of the result with the value of 1771.75×10^6 sec.$^{-1}$ given by Millman and Kusch is therefore very good. It is to be noted that the value of the magnetic field employed in these experiments is poorly chosen for the determination of $\Delta\nu$ as compared to the fields used by Millman and Kusch. The widths of the sodium lines was roughly proportional to the rate of variation of line frequency with magnetic field and are presumably due to inhomogeneities in the magnetic field. The observed width of the Na*I* line was about 0.280×10^6 sec.$^{-1}$, that of the Na*II* and Na*III* lines about 0.150×10^6 sec.$^{-1}$ and that of the Na*VI* line about 0.070×10^6 sec.$^{-1}$. The very good internal consistency noted above indicates that the measured center of the resonance curve is, indeed, the center of the line and that all line frequencies determine the same mean magnetic field.

Three lines in the sodium spectrum which have the greatest sensitivity to field were measured alternately with two lines in the spectrum of gallium. The data are given in Table VII. The time of observation is indicated in the first column, and in the other columns are given the observed line frequencies in megacycles per second. Directly below the observed frequency is recorded, in parentheses, the value of H' calculated on the basis of the assumption that $g_J(^2S_{\frac{1}{2}})=2$ and $g_J(^2P_{\frac{1}{2}})=\frac{2}{3}$. The quantity which appears directly after the value of H' is the difference between the observed H' and that calculated from a least squares solution of the data.

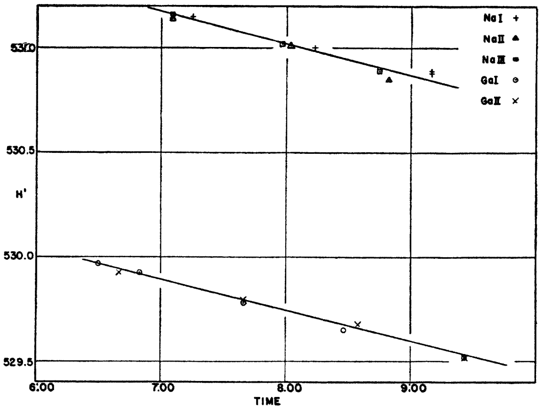

FIG. 2. Values of H' calculated from the observed frequencies of several lines in the spectra of Na and of Ga in the $^2P_{\frac{1}{2}}$ state.

The data are graphically presented in Fig. 2. An independent least squares solution of the variation of H' with time for sodium and for gallium gives very nearly the same rate of drift of field in both cases, as expected. A combined least squares solution for both sets of lines, where the rate of drift of field is the same in both cases, yields the result

$$\begin{aligned}&\text{for Na: } H'=531.319-0.002475t,\\&\text{for Ga: } H'=530.036-0.002475t,\end{aligned}\qquad(19)$$

where the time, t, is measured in minutes from 6:00. The difference between the observed and calculated values is indicated in the table. It is to be noted that the differences are all positive for the Na*I* line and negative or zero for the Na*II* and Na*III* lines. The effect probably represents a slightly unsymmetrical broadening of the lines by the inhomogeneities of the field. The difference in the value of H' for Na and that for Ga is $531.319-530.036=1.283$. To this value we assign the arbitrary precision value of ±0.030, which is the sum of the mean deviation of the values of H' yielded by Eq. (19) from those calculated for the individually observed Na lines and of the corresponding quantity for Ga. This uncertainty is obviously somewhat greater than the probable error calculated from the internal consistency of the data and is given in view of the fact that a systematic error may occur.

On the basis of the assumption that $g_J(^2S_{\frac{1}{2}})=2$ and that $g_J(^2P_{\frac{1}{2}})=\frac{2}{3}$, the ratio of the apparent field as calculated from Na to that as calculated from Ga is 1.00242 ± 0.00006. Other data has given the value for the same quantity of 1.00247 ± 0.00020. To satisfy the condition that the magnetic field is the same for a measurement of the frequencies of the gallium and sodium lines, we must set:

$$g_J(^2S_{\frac{1}{2}}\,\text{Na})/g_J(^2P_{\frac{1}{2}}\,\text{Ga})=3(1.00242\pm0.00006),$$

TABLE VIII. Notation used in identifying lines in the spectrum of indium in the $^2P_{\frac{1}{2}}$ state.

Line	Symbol
$(4, -4)\leftrightarrow(4, -3)$	In*I*
$(4, -3)\leftrightarrow(4, -2)$	In*II*
$(5, 5)\leftrightarrow(5, 4)$	In*III*
$(5, 4)\leftrightarrow(5, 3)$	In*IV*
$(5, 3)\leftrightarrow(5, 2)$	In*V*

where it is assumed that the deviation of the g_J values from their nominal values is small.

c. The Determination of the Ratio of the g_J Value of Na in the $^2S_{\frac{1}{2}}$ State and That of In in the $^2P_{\frac{1}{2}}$ State

The $\Delta\nu$ of In^{115} and its g_I have been determined by Hardy and Millman.[12] The low abundance of In^{113} together with the fact that its $\Delta\nu$ is almost identical to that of In^{115} makes it impossible to observe lines in the spectrum of In^{113} in our experiments. In fact, the frequencies of the In lines which we observe are predominantly due to the term $g_F\mu_0H/h$, and higher order terms, involving $\Delta\nu$, contribute at most about 3 percent to the frequencies of the lines. Under these conditions the small variation of the $\Delta\nu$ between the two isotopes of indium would not permit the independent observation of lines in the spectrum of both isotopes in our apparatus.

For the purposes of the comparison of the g_J values, we have used three lines in the spectrum of sodium, previously identified as Na*I*, Na*II* and Na*III*. Five lines in the spectrum of indium have been used, identified as indicated in Table VIII. The data were observed in exactly the same way as in the case of the comparison of the g_J values of gallium and sodium and the arrangement of Table IX, wherein are recorded the observations on sodium and indium, is identical to the arrangement of Table VII.

It can be seen from the table that the rate of drift of magnetic field during the experiment was very small. In making a least squares reduction of the data, the observations on the Na*II* line

TABLE IX. The observed frequencies and values of H' for several lines in the spectrum of indium and sodium.

Time	In*I*	In*II*	In*III*	In*IV*	In*V*	Na*II*	Na*III*	Na*IV*
3:45	35.4104 (514.56) −0.03							
3:47		35.1936 (514.58) −0.02						
3:53			33.0891 (514.57) −0.03					
3:54				33.2855 (514.64) +0.04				
3:55					33.4769 (514.57) −0.04			
4:08						254.210 (515.90) +0.03		
4:35							254.989 (515.88) −0.01	
4:40						254.166 (515.92) +0.03		
5:00							254.980 (515.84) −0.07	
5:10	35.4194 (514.69) +0.02							
5:12		35.1982 (514.64) −0.03						
5:20			33.0973 (514.70) +0.02					
5:22				33.2909 (514.72) +0.04				
5:24					33.4855 (514.70) +0.02			
5:47						254.181 (515.96) +0.01		
5:52							255.00 (515.91) −0.05	
5:54						254.186 (515.97) +0.01		
6:02								203.323 (515.944) −0.02
6:16	35.4228 (514.74) +0.02							
6:30		35.2032 (514.72) −0.02						
6:40				33.2929 (514.76) −0.01				
6:45					33.4879 (514.74) −0.04			

have been weighted by two because of experimental factors which made it very much easier to locate this line than the other two lines in the spectrum of sodium. The data may be adequately represented by assuming a linear drift of field. If t is the time in minutes from 3:45, we can express the result

$$\begin{aligned}&\text{for In: } H'=514.594+0.000861t;\\ &\text{for Na: } H'=515.846+0.000861t.\end{aligned}\qquad(20)$$

The difference in the value of H' for Na and In is 1.252. If all the sodium lines are weighted equally, this difference becomes 1.243. The first of these two values is believed to be the better one. The data are clearly inferior to those obtained in the case of gallium. To the value of 1.252 we assign the arbitrary precision value of ±0.050, obtained as in the previous experiment. This uncertainty is obviously somewhat greater than the probable error calculated from the internal consistency of the data.

On the basis of the assumption that $g_J(^2S_{\frac{1}{2}})=2$ and that $g_J(^2P_{\frac{1}{2}})=\frac{2}{3}$, the ratio of the apparent field as calculated from Na to that as calculated from In is 1.00243 ± 0.00010. To satisfy the condition that the magnetic field is the same for a measurement of the lines in each of the two spectra, we must set:

$$g_J(^2S_{\frac{1}{2}}\,\text{Na})/g_J(^2P_{\frac{1}{2}}\,\text{In})=3(1.00243\pm0.00010).$$

6. DISCUSSION OF RESULTS

The observed deviations of the ratios of the atomic g_J values from those calculated under the assumptions that pure LS coupling obtains and that the fundamental gyromagnetic ratios of spin and orbit have the values 2 and 1, respectively, may be attributed, in any particular case, to a deviation of the spin or orbit g values from their accepted values. It was shown in Section 2 that δ_S, defined by $g_S=2(1+\delta_S)$, can be computed from the observed ratio of the g values of two atomic states of different spectral term classifications. This quantity has been evaluated from the results of the three relevant experiments. The results appear in Table X. The error assigned to δ_S in each case follows directly from the error assigned to the corresponding ratio of the g values. Thus from the experimental results, the difference between the value of δ_S which results from the $^2P_{3/2}(\text{Ga})-{}^2P_{\frac{1}{2}}(\text{Ga})$ experiment and that which results from the $^2P_{\frac{1}{2}}(\text{Ga})-{}^2S_{\frac{1}{2}}(\text{Na})$ experiment is probably real. It is, however, quite possible that this discrepancy may be accounted for by small deviations of the properties of the atomic systems from the simple description implicit in the theory underlying the calculations by means of which the experimental data has been reduced. It is to be emphasized that the numerical results are independent of the precise values of any of the fundamental atomic constants.

The agreement of the values of δ_S in Table X is very strong evidence in support of the hypothesis that the fundamental spin gyromagnetic ratio does, in fact, differ from the accepted integral value by an amount of very nearly the magnitude of the quantities given in Table X. An alternative explanation of these results in terms of perturbations (e.g., configuration interactions) of the individual atomic states would require that the perturbations on the g values of the four states should have just such magnitudes as to give the agreement noted above. Aside from the question of the possible magnitude of such perturbations (discussed below) it should be pointed out that three different atomic species and three states of different spectral classification are involved in these experiments. It is very unlikely that under these conditions there should occur the systematic effect shown in these results. Schwinger[7] has reported the results of a theoretical study of the interaction of an electron with the quantized electromagnetic field. By a canonical transformation of the Hamiltonian the infinite interaction terms are separated from the rest of the Hamiltonian and are shown to be properly included in the experimental mass of the electron. The evaluation of the remaining finite terms in the case of an applied magnetic field leads to an interaction energy term corresponding to a spin moment of the electron increased from the accepted value of one Bohr magneton by a factor $(1+\alpha/2\pi)=1.00116$. This value is in very good agreement with our experimental results. The possible reasons for the small remaining discrepancies are discussed below.

The result of this experiment, that the electron spin g value differs from 2 by about 12 parts in ten thousand, may be compared with previous

TABLE X. Observed ratios of atomic g values and the corresponding values of δ_S.

Experimental ratio	δ_S
$g_J(^2P_{3/2}\mathrm{Ga})/g_J(^2P_{\frac{1}{2}}\mathrm{Ga})=2(1.00172\pm0.00006)$	0.00114 ± 0.00004
$g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_J(^2P_{\frac{1}{2}}\mathrm{Ga})=3(1.00242\pm0.00006)$	0.00121 ± 0.00003
$g_J(^2S_{\frac{1}{2}}\mathrm{Na})/g_J(^2P_{\frac{1}{2}}\mathrm{In})=3(1.00243\pm0.00010)$	0.00121 ± 0.00005

measurements of atomic g values in the Zeeman effect. In the classic work of Back[15] the uncertainty in the measured atomic g values is not less than one part in a thousand, and thus the effect would escape detection. In the somewhat more accurate work of Kinsler and Houston[1] on the determination of e/m from the Zeeman pattern in standard magnetic fields, singlet states were studied in almost all cases, so that no discrepancy corresponding to the effect described in the present paper could be found. However, in one experiment of this series, Kinsler[16] measured the g values of the two states $2p^53s\,^1P_1(\mathrm{Ne})$ and $2p^53s\,^3P_1(\mathrm{Ne})$. These two states arise from the same configuration and are the only states of this configuration with $J=1$. Application of the g sum rule to the experimental values yields 2.5017 ± 0.0016 for the sum of the atomic g values. In view of the assigned experimental uncertainty this value was considered by Kinsler to be in sufficiently good agreement with the value 2.5000 predicted on the basis of the accepted integral g values of the electron. Introduction of the spin moment g value reported in the present work gives a predicted value of the g sum of 2.5012.

There are several perturbations of the electronic states involved in the present experiments which in principle could bring about deviations of the atomic g values from the values given by the Russell-Saunders coupling formula. Estimates of the magnitude of these effects will be given in the following paragraphs.

In all of the atomic states employed in this experiment the configuration corresponds to a single valence electron outside of closed shells. Thus, the term classification is unique, and the question of the type of coupling does not arise for these states.

By far the largest configuration perturbation is the electrostatic interaction. If the excited configurations of the atomic system be described in first approximation by Russell-Saunders coupling it is well known that the electrostatic coupling will mix states only of the same values of total L and S. Thus the electrostatic coupling among pure LS states does not change the atomic g value in any approximation.

The effect on the g_J values of gallium and indium of the magnetic interactions with the excited configurations of the outer group of three electrons is small. The ordinary spin-orbit interaction with these configurations vanishes. A magnetic interaction with a ^4P state of magnitude 100 cm^{-1} would increase the $g_J(^2P_{\frac{1}{2}}\mathrm{Ga})$ by about 1×10^{-5}. The effects in indium might be somewhat larger than in gallium. The $g_J(^2P_{3/2}\mathrm{Ga})$ is affected by a still smaller amount than is the $g_J(^2P_{\frac{1}{2}}\mathrm{Ga})$, as the g_J of all the $J=\frac{3}{2}$ terms of the interacting configurations differ very little from $g_J(^2P_{3/2})$. Spin-orbit interactions with excited states of the core will produce effects no larger than those described above, because of the high excitation of these levels.

It appears that an important perturbation of the g_J values of gallium or of indium would occur only if in the lower excited configurations, i.e., those of the outer three electrons in each case, there is a very considerable departure from Russell-Saunders coupling. In this case the electrostatic configuration mixing could bring about an appreciable alteration from the Russell-Saunders values of the g_J values of the states employed in the present experiment. The evidence as to the character of the coupling in the excited states of gallium and of indium is meager. In the $4s4p^2\,^4P$ term of gallium the interval rule appears to be obeyed accurately, while in the corresponding term $5s5p^2\,^4P$ of indium the fine structure interval ratio differs by 17 percent from the Landé rule. The relative positions of the terms arising from the $5s5p^2$ configuration of Sb*III* seem to be accounted for by LS coupling with the addition of electrostatic interaction with other configurations.[17]

The effect of configuration interactions on the g_J values of the ground states of the alkali metal

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[15] E. Back and A. Landé, *Zeemaneffekt* (Verlag Julius Springer, Berlin, 1925). See also, R. Bacher and S. Goudsmit, *Atomic Energy States* (McGraw-Hill Book Company, Inc., New York, 1932).

[16] L. E. Kinsler, Phys. Rev. **46**, 533 (1934).

[17] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (University Press, Cambridge, 1935).

atoms were investigated by Phillips,[18] who indicated that these effects are negligible.

Relativistic effects on the g_J values of a Dirac electron have been considered by Margenau[19] who showed that the atomic g_J value of a single electron is decreased by a factor $1-[(2J+1)/2(J+1)][\bar{T}/mc^2]$ for $J=L+\frac{1}{2}$ and $1-[(2J+1)/2J][\bar{T}/mc^2]$ for $J=L-\frac{1}{2}$. $\bar{T}$ is the mean kinetic energy. For the alkali metals the reduction in the g_J value is about 1 part in 10^5. For gallium and indium the effect is somewhat larger because of penetration of the core and because of the larger numerical factor in the relativistic expression for the ${}^2P_{\frac{1}{2}}$ state. When approximate account is taken of the penetration the g_J values of the ${}^2P_{\frac{1}{2}}$ states of gallium and indium are reduced by four parts in 10^5, and the ${}^2P_{3/2}(\mathrm{Ga})g_J$ value is reduced by about two parts in 10^5.

The magnitude of the orientation dependence of the diamagnetic susceptibility, which in principle could affect the spectrum of the ${}^2P_{3/2}(\mathrm{Ga})$ state, is negligible (less than one part in 10^5) at the fields employed in this experiment.

Incipient electronic Paschen-Back effect produces a perturbation in the line frequencies of the ${}^2P_{3/2}(\mathrm{Ga})$ spectrum of about one part in 10^5.

The interaction of the valence electron spin with the diamagnetically induced moment of the core electrons reduces the g_J value of the atom. The calculation of this effect requires fairly accurate core and valence electron charge distributions. Such a calculation has been carried out for ${}^2S_{\frac{1}{2}}(\mathrm{Na})$ with Hartree fields and a corresponding optical electron wave function. The decrease in the g_J value is less than one part in 10^5. It is believed that this effect is also negligible for the other atomic states.

The application of the corrections estimated in this section to the experimental values of the g_J ratios of Table X affects the calculated values of δ_S by about one percent. This magnitude is well within the experimental uncertainty and these corrections have not been applied in Table X.

We write $g_S=2(1.00119\pm0.00005)$ as the best value of the spin gyromagnetic ratio obtainable from the present experiments. The discrepancy between individual values of δ_S indicates the existence of small residual systematic effects and further analysis might indicate that g_S differs somewhat from the value given here.

The original motivation for these experiments came from the suggestion of G. Breit with regard to the intrinsic magnetic moment of the electron. The actual method of testing Breit's hypothesis by comparing the g values of atomic states was suggested to us by I. I. Rabi. We wish to express our thanks to both Dr. Breit and Dr. Rabi for many helpful discussions and to Miss Zelda Marblestone who performed a considerable fraction of the rather extensive calculations involved in this work.

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