PHYSICAL REVIEW VOLUME 74, NUMBER 1 JULY 1, 1948

Theory of the Lithium Two-Alpha-Reactions. I. Angular Distribution of $Li^7(p,\alpha)\alpha$

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(Received March 16, 1948)

The reactions $Li^7(p,\alpha)\alpha$ and $Li^6(d,\alpha)\alpha$ have a special interest because the Bose statistics and consequent even parity of the final pair of alphas simplifies the analysis. It is assumed on reasonable grounds that Li^7 has odd parity and Li^6 even. This means that only $p, f\cdots$ protons entering Li^7 are relevant to this reaction and only entering $s, d\cdots$ deuterons of Li^6. An expression for the energy variation of the angular distribution in the Li^7 reaction is derived from the dispersion formula, with the specific assumption that only two levels of the compound nucleus, having angular momentum 0 and 2, are important in the range of energies employed in the experiments. (An alternative assumption is investigated but is more involved and it is considered less plausible.) The angular factor is $1+A(E)\cos^2\theta+B(E)\cos^4\theta$. A formula for $A(E)$ as far as it results from entering p waves has been derived previously by Critchfield and Teller, but it may not be adjusted to fit the data in the energy range up to 3 Mev even within the rather large experimental uncertainty. The effect of entering f waves is also included in the present analysis, and this contribution is not only essential to the existence of $B(E)$ but also makes it possible to obtain agreement with the recent measurements of $A(E)$. Certain matrix elements are treated as arbitrary parameters in obtaining this fit. In addition to the low-energy node apparent in the early data, a second node of $A(E)$ is required at higher energy. The formula derived for $B(E)$ contains the same resonance denominator as does $A(E)$ and is compatible with the present experimental results. The contrasting facts, first that the easy entrance of s waves makes the $Li^6(d,\alpha)\alpha$ reaction symmetric at very low energies and second that the p waves responsible for the Li^7 reaction at low energies may introduce asymmetry, are associated in a simple way with the qualitative experimental observation that both $A(E)$ and $B(E)$ rise more quickly in the Li^7 reaction than in the Li^6 reaction as the bombarding energy is increased from zero. Presentation of a detailed formulation of the Li^6 reaction is deferred until a later paper.

I. INTRODUCTION

THE nuclear reactions $Li^7(p,\alpha)\alpha$ and $Li^6(d,\alpha)\alpha$ are unique in having two alphas as their end products, and the Bose statistics obeyed by these alphas simplifies the theoretical treatment on the basis of the dispersion formula to a point where some measure of interpretation of the angular distributions is easily possible. The angular distribution of the $Li^7(p,\alpha)\alpha$ reaction was first investigated experimentally by Young, Ellett, and Plain,[1] and theoretically by Critchfield and Teller.[2] The angular distribution seemed to be of the form $1+A(E)\cos^2\theta$, with the value of $A(E)$ approaching a maximum at the top bombarding energy E of 400 kev. The form $1+A(E)\cos^2\theta$ follows most simply from the assumption that only p protons enter and are responsible for the reaction, f protons being repelled by the Coulomb barrier and s and d protons being ineffective because of the assumed odd parity of the target nucleus and the even parity of the final configuration. A form for $A(E)$ rising to a maximum and gradually tailing off to small values at high energies (through a node or not, depending on the existence of an unobserved node at very low energies) was shown to follow from the assumption of a broad s state and a narrow d state of the compound nucleus. Both the appearance of a maximum of $A(E)$ at 400 kev and the detection of a corresponding resonance maximum in the yield curve (relative to penetration factor) constructed from the yield data then available seem to have been spurious. The sensitivity of this inferred maximum in relative yield to the assumed nuclear radius has been discussed briefly by Eisner.[3]

Later experimental investigations[4–6] have shown that $A(E)$ increases to a maximum value of about 2 at about 1 Mev, and then decreases steadily but remains positive to the highest energies of the observations (3 Mev). The latest observations[6] show moreover that there is also a

[1] V. J. Young, A. Ellett, and G. J. Plain, Phys. Rev. **48**, 498 (1940).
[2] C. L. Critchfield and E. Teller, Phys. Rev. **60**, 10 (1941).
[3] E. Eisner, Phys. Rev. **65**, 85 (1944).
[4] C. D. Swartz, H. H. Rossi, B. Jennings, and D. R. Inglis, Phys. Rev. **65**, 80 (1944).
[5] S. Rubin, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. **71**, 212 (1947).
[6] N. P. Heydenburg, C. M. Hudson, D. R. Inglis, and W. D. Whitehead, Jr., Phys. Rev. **73**, 241 (1948).

term in $\cos^4\theta$: that the angular distribution is more accurately of the form $1+A(E)\cos^2\theta+B(E)\cos^4\theta$. The experiments are not yet sufficiently accurate to fix the dependence of the coefficient B on energy very well, but it is definitely negative and of the order of magnitude $-\frac{1}{2}$ throughout at least most of the range from 1 Mev to 3 Mev.

The reaction $Li^6(d,\alpha)\alpha$ involves the formation of the same compound nucleus Be^8 as is formed in $Li^7(p,\alpha)\alpha$ but the binding energy of a deuteron to Li^6 is 4.96 Mev higher than the binding energy of a proton to Li^7, so the compound nucleus is in a more highly excited state in the Li^6 reaction than in the Li^7 reaction, for comparable bombarding energies. The Li^6 spectrum thus makes it possible to explore a different part of the Be^8 spectrum than that explored as yet by the Li^7 reaction. Observations on the angular distribution of the alphas from the Li^6 reaction have been made recently by Heydenburg, Hudson, Inglis, and Whitehead.[7] The excitation curve displays a broad dip between a peak at 0.7 Mev and one above 3.5 Mev, the highest energy of the observations. Here again the angular distribution is limited to even powers of $\cos\theta$ by the Bose statistics of the product alphas, and the observations indicate that $A(E)$ rises, starting near 0.7 Mev, to a broad maximum of about unity at 2 Mev, while $B(E)$ is absent to almost 1.5 Mev and is small and positive at the higher energies.

Even though the validity of nuclear models, such as the alpha-model and the central-field model, for the calculation of finer details seems at present extremely questionable, the parities deduced on the basis of these models are almost beyond doubt, and it is assumed throughout this work that the parity of the ground state of Li^7 is odd while that of Li^6 is even. The results show that the data are compatible with these assumptions. Only in the case of flagrant disagreement would it seem to us worth while to investigate the contrary assumption.

In this paper we extend the theoretical interpretation of the Li^7 reaction to include the effects of entering f protons, and we shall present a similar interpretation of the Li^6 reaction in a subsequent paper. Some characteristic differences appear in the two reactions. The protons which enter the Li^7 reaction most easily, the p protons, are themselves responsible for the $\cos^2\theta$ term, so the necessity of penetrating a barrier does not make the asymmetry vanish at zero bombarding energy. The observed vanishing of $A(E)$ near $E=0$ is caused by a pure coincidence in the placing of the resonant state of the compound nucleus. In the Li^6 reaction, on the contrary, the deuterons which enter most easily are s deuterons which by themselves give spherical symmetry, and the penetration of d deuterons is necessary for the appearance of the asymmetry, so the coefficient $A(E)$ rises from zero for a more natural reason as E increases from zero, and this makes the rise more gradual than Li^7 as is observed. This qualitative agreement is perhaps the simplest experimental verification we have of the validity of the parities as given by the nuclear models.

A similar simple correlation is found between the theoretical expectation arising from the assumed parities of the two target nuclei and the qualitative features of the observed behavior of $B(E)$. The penetration of the f waves may be expected to be more meager than the penetration of the p waves in the Li^7 reaction by a factor of the same order of magnitude as the penetration of the d waves relative to the s waves in the Li^6 reaction. The product of a p wave and an f wave (which appears because the incoming matrix element is contained quadratically or bilinearly in the dispersion formula (1)) introduces a degree of complexity in the rotational properties sufficient to account for a $\cos^4\theta$ term, and this product term is rendered small by the penetration factor to the same degree as is the product term between s and d waves in the Li^6 reaction. The latter product only introduces a term as high as $\cos^2\theta$, and the square of the d wave involving the square of the penetration factor is required to make the $\cos^4\theta$ term, leaving the $\cos^4\theta$ term less prominent in the Li^6 reaction, as observed.

[7] N. P. Heydenburg, C. M. Hudson, D. R. Inglis, and W. D. Whitehead, Jr., Bull. Am. Phys. Soc. **23**, No. 3 (1948).

II. CALCULATION OF ANGULAR DISTRIBUTIONS FROM THE DISPERSION FORMULA

The calculation is based on the Breit-Wigner dispersion formula[8] extended to several compound states, which for a system prepared in a well defined initial state P may be written

$$\sigma(E,\theta)\sim\lambda\!\!\!^{-2}\,|\sum_r (P/H/r)(r/H/Q)/[E_p-E_r+(i/2)\Gamma_r]|^2. \quad (1)$$

The fact that the cross section is the square of a sum gives rise to interference between various states r of the compound nucleus. The "incoming" state P is characterized first by some quantization of the spin directions of the particle spin s and the target-nucleus "spin" I. A possible representation is that defined by the projections (along the beam direction) m_s and m_I. A more convenient and equivalent representation is that in which the sum of these two "spins" is given by a quantum number S, where

$$\mathbf{S}=\mathbf{s}+\mathbf{I}, \quad (2)$$

and by its projection m. In discussing the applications of (1), we first imagine that such a well defined initial state has been selected. The wave function of the prepared incoming state P is the product of a "spin" function $Z_m{}^{(S)}$ and a plane wave describing the orbital motion:

$$P_m{}^{(S)}=Z_m{}^{(S)}\sum_{l=0}^{\infty} i^l(2l+1)\psi_l(kr)P_l(\cos\theta_p) = \sum_{l=0}^{\infty}(lS0m/). \quad (3)$$

The functions $(lSm_lm/)=Z_m{}^{(S)}P_l(\cos\theta_p)e^{im_l\phi}\times i^l(2l+1)\psi_l(kr)$ are the normalized spin and angle factors of the incoming waves, each multiplied by a radial function depending on l. These functions do not describe proper states of the angular momentum j_r, since they diagonalize the projections of $\mathbf{l}$ and $\mathbf{S}$ separately rather than the vector sum of $\mathbf{l}$ and $\mathbf{S}$. Nowever, any one of the states $(lS0m/)$ appearing in (3) could be used, along with the other $(lSm_lm'/)$ having $m_l+m'=m$, to make up by linear combination a wave function of a proper state of the angular momentum j, for any j between $l-S$ and $l+S$. The transformation coefficient from $(lS0m/)$ to such a proper state of the angular momentum we call $(lS0m/lSjm)$. We may then write for the "incoming" matrix element,

$$(P/H/r)=\sum_l (lS0m/lSj_rm)(lSj_r/H/r). \quad (4)$$

The numerical factors $i^l(2l+1)$ are considered contained in $(lSj_r/H/r)$ and this matrix element also involves a radial integration over $\psi_l(kr)$ along with other factors. This $(lSj_r/H/r)$ might have been written $(lSj_rm/H/rm)$ but both wave functions diagonalize j so the matrix element is independent of m. $(lSj_r/H/r)$ then transforms from an incoming proper function of the angular momentum, having $j=j_r$ and a given l, to the resonant state r of the compound nucleus. No simple model which might allow the assignment of values of S and l to the nuclear states is to be expected to be valid, especially in the very highly excited states r of the compound nuclei encountered in nuclear reactions. The state r is rather to be thought of as a complicated mixture of states having various values of S and l, in such a way that neither of these possible quantum numbers would have any meaning. Even if the spin angular momentum S and the orbital angular momentum l were to have a meaning in a specific state r, the transformation from the incoming state S, l would in general involve transformation coefficients mixing spin and orbit because the "spin" I of the target nucleus generally contains orbital contributions. The vector-coupling situation for the transformation of the incoming state to the compound state r is thus a complicated one, usually involving a complex coefficient. If, for a given value of l, there are n values of S which combine with it to give a certain value of j_r, the n incoming proper wave functions $(Slj_r/)$ may be combined in n orthogonal linear combinations, $n-1$ of which have zero matrix elements of H with r and the other of which, X_{lr}, has a value of this matrix element $(X_{lr}/H/r)$. Then we may write

$$(lSj_r/H/r)=[lSj_r/X_{lr}](X_{lr}/H/r), \quad (5)$$

and this incoming matrix element may be

[8] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936); H. A. Bethe and G. Placzek, Phys. Rev. **51**, 450 (1937); H. A. Bethe, Rev. Mod. Phys. **9**, 71 (1937); G. Breit, Phys. Rev. **58**, 506 (1940). Cf. also H. Feshbach, D. C. Peasley, and V. F. Weiskopf, Phys. Rev. **71**, 145, 564 (1947).

further factored thus:[2]

$$(X_{lr}/H/r)=\phi_l(E)\alpha_{lr}', \tag{6}$$

where $\phi_l(E)$ is real and α_{lr}' is in general complex. The vector-coupling coefficients are expected to be complex but not to depend much on energy as one passes through a resonance, so they are contained almost entirely in α_{lr}', while the energy dependence of $\phi_l(E)$ arises almost entirely from the penetration of the Coulomb barrier. (In certain simple cases encountered in nuclear models α_{lr}' might be real, such as the case in which both incoming state and compound state are the same Russell-Saunders state, that is, in a transition with the magnitudes of spin and orbital momenta separately conserved and with no admixtures of other configurations of the compound nucleus.) Both of the reactions here considered are so highly exoergic that the "outgoing" matrix element $(r/H/Q)$ is assumed to be independent of energy during the passage of even a broad resonance. (A slow dependence of this factor on energy could also be included in $\phi_l(E)$.)

In Eq. (1), the angle θ which appears explicitly on the left side is implied in the final state Q on the right side. This means that the matrix element $(r/H/Q)$ describes not only the transition to the state of two alphas whose orbital motion is characterized by angular momentum matching that of the compound state, $L=j_r$ and $M_L=m$, but also describes the fact that observations are made at an angle θ, which brings in as a factor the angular wave function of this final state at θ, the associated Legendre polynomial $P_L{}^m(\cos\theta)$. Formally, the observed final state is a plane wave in the θ direction, and this angle factor is the angle part of the transformation coefficient from the outgoing radial wave (which includes an angle factor $P_L{}^m(\cos\alpha)e^{im\beta}$) to the plane wave Q, arising from the expansion

$$e^{ik\cdot R}=\sum_{m=1} i^n(2n+1)j_n(kR)\Big\{P_n(\cos\alpha)P_n(\cos\theta)$$

$$+2\sum_{n=0}^{n}\frac{(n-m)!}{(n+m)!}P_n{}^m(\cos\alpha)P_n{}^m(\cos\theta)\cos m(\phi-\beta)\Big\},$$

wherein α and β are the colatitude and azimuth angles of the position vector R, θ and ϕ those of the observation direction k.

Because products of incoming and outgoing matrix elements appear in (1), we combine the complex factors in one complex number α_{lr} defined as the product of α_{lr}' and all of $(r/H/Q)$ except the angle factor $P_j{}^m(\cos\theta)$. Then we have

$$\sigma_P(E,\theta)=4\pi\lambda\!\!\!^{-2}\Big|\sum_{lr}\{(lS0m/lSj_rm)$$

$$\times[lSj_r/X_{lr}]\phi_l(E)\alpha_{lr}P_{j_r}{}^m(\cos\theta)/$$

$$(E-E_r+(i/2)\Gamma_r)\}\Big|^2. \tag{7}$$

The preparation of the system into well defined states for the application of (1) would involve orienting the spins in just one quantized fashion before the impact, which is not done in the experiments. An unpolarized beam hitting an unpolarized target is equivalent to a repetition of the experiment equal numbers of times in the various possible prepared states P, and the cross section for finding an alpha in a small element of solid angle at θ is

$$\sigma(E,\theta)=\textstyle\sum_P \sigma_P(E,\theta). \tag{8}$$

Equations (7) and (8) together form the basis for the interpretation of the experiments for which rather extensive results have recently become available.

III. ANGULAR DISTRIBUTION OF THE Li^7+PROTON REACTION

In the reaction $Li^7(p,\alpha)\alpha$, the proton spin is $s=\frac{1}{2}$, the target nucleus has angular momentum $I=\frac{3}{2}$, and these combine to give "spin" states $S=1$ and $S=2$. The ground state of Li^7 is assumed to be odd, so incoming p, f, $h\cdots$ waves may give the even states of the compound nucleus required for the reaction, and the successively higher values of l find penetration more and more difficult, so it will suffice to consider only $l=1$ and $l=3$. The Bose statistics of the product alphas demands that the compound states of interest have not only even parity but also even angular momentum j_r. With $l=1$, the "spin" state $S=1$ leads to $j_r=0$ or 2, while $S=2$ leads only to $j_r=2$. With the incoming f wave, $S=1$ and $S=2$ each lead to $j_r=2$ or 4. We are then free to assume such disposition of virtual levels of the compound

TABLE I. Transformation coefficients $(lS0m/lSj_rm)$.

m	$(110m/112m)$	$(120m/122m)$	$(310m/312m)$	$(320m/322m)$	$(4/5)^{\frac{1}{2}}P_2^m(x)$
2	0	$-(2/3)^{\frac{1}{2}}$	0	$-14^{-\frac{1}{2}}$	$(3/2)^{\frac{1}{2}}(1-x^2)$
1	$2^{-\frac{1}{2}}$	$-6^{-\frac{1}{2}}$	$7^{-\frac{1}{2}}$	$2/14^{\frac{1}{2}}$	$6^{\frac{1}{2}}x(1-x^2)^{\frac{1}{2}}$
0	$(2/3)^{\frac{1}{2}}$	0	$-(3/7)^{\frac{1}{2}}$	0	$(1-3x^2)$
-1	$2^{-\frac{1}{2}}$	$6^{-\frac{1}{2}}$	$7^{-\frac{1}{2}}$	$-2/14^{\frac{1}{2}}$	
-2	0	$(2/3)^{\frac{1}{2}}$	0	$14^{-\frac{1}{2}}$	

nucleus, having $j_r=0$, 2, or 4, as may be necessary to account for the observations. The main trends of the observations may be attributed to an entering p wave, involving levels having not more than $j_r=0$ and 2, and it is rather gratifyingly found unnecessary to introduce any further states in order to explain the finer details arising from the f wave. That is, it may be assumed, in the interest of simplicity, that the f wave reacts only with the $j_r=2$ state(s), without invoking $j_r=4$.

The transformation coefficients $(lS0m/lSj_rm)$ appearing in (7) transform from a representation in which the projections of l and S are diagonal to that in which the total angular momentum is diagonal, and are the same as are used to describe Russell-Saunders states in the theory of atomic spectra. They have indeed been tabulated in a form convenient for the present work in the book of Condon and Shortley.[9] (Investigators preferring to make their own calculations complete from the beginning can do so without writing as many matrix elements as are listed in reference 2 by using a sum rule introduced by Breit and Darling[10] for the purpose of confining attention to those coefficients having $m_l=0$ in just such calculations as these.) The coefficients (9) for the states that concern us are evaluated in Table I. (Those with $l=1$ are also listed in reference 2.)

Our first set of assumptions about the compound nucleus is that it has just two states which contribute appreciably to the reaction in the energy range investigated, a state numbered $r=0$ having $j_0=0$ and having its half-width Γ_0 considerably greater than the range of energies covered by the experiments so that the energy variation of its "resonance denominator" may be neglected, and a state number 2 having $j_2=2$ and Γ_2 small enough to account for the rather rapid energy variation of the angular distribution of the product alphas. These are the assumptions about the compound states made in reference 2. With these states, the cross section given by (1) and (8) is

$$\begin{aligned}\sigma(E,\theta)&=4\pi\lambda^2\sum_{S,m}|H_0{}^PH_Q{}^0/(i\Gamma_0/2)\\&\qquad+H_2{}^PH_Q{}^2/(E-E_2+i\Gamma_2/2)|^2\\&=(16\pi\lambda^2/\Gamma_2{}^2)\sum_{S,m}|(\Gamma_2/i\Gamma_0)H_0{}^PH_Q{}^0\\&\qquad+H_2{}^PH_Q{}^2/(\epsilon+i)|^2\\&=[(16\pi\lambda^2/\Gamma_2{}^2)/(\epsilon^2+1)]\\&\times\sum_{S,m}|(\Gamma_2/i\Gamma_0)(\epsilon+i)H_0{}^PH_Q{}^0+H_2{}^PH_Q{}^2|^2.\end{aligned}\tag{9}$$

Here we have put

$$(E-E_2)/(\Gamma_2/2)=\epsilon,$$

which then measures the energy deviation from resonance with the state $r=2$ in units of its half-width. The summation indices S and m are implicit in the initial state P. Putting this in the more explicit notation of Eq. (7), we have

$$\begin{aligned}\sigma(E,\theta)&=[(16\pi\lambda^2/\Gamma_2{}^2)/(\epsilon^2+1)]\sum_{S,m}|(\Gamma_2/i\Gamma_0)(\epsilon+i)\phi_1\alpha_{10}\delta(S/1)\delta(m/0)\\&\qquad+\sum_{l=1,3}(lS0m/lS2m)[lS2/l2]\alpha_{l2}\phi_lP_2{}^m(\cos\theta)|^2\\&=[(16\pi\lambda^2/\Gamma_2{}^2)/(\epsilon^2+1)]\Big\{\sum_{m=-2}^{2}|\sum_{l=1,3}(l20m/l22m)[l22/l2]\alpha_{l2}\phi_l|^2(P_2{}^m)^2+\sum_{m=\pm1}|\sum_{l=1,3}(l10m/l12m)\\&\qquad\times[l12/l2]\alpha_{l2}\phi_l|^2(P_2{}^m)^2+|(\Gamma_2\alpha_{10}/i\Gamma_0)(\epsilon+i)\phi_1+\sum_{l=1,3}(l100/l120)[l12/l2]\alpha_{l2}\phi_lP_2{}^0|^2\Big\}.\end{aligned}\tag{10}$$

[9] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Oxford University Press, New York, 1935), pp. 76, 77.
[10] G. Breit and B. T. Darling, Phys. Rev. **71**, 402 (1947).

TABLE II. Sums used in the Li^7 reaction with $j_r=0$ and 2.

l_1	l_2	$(4/5)\sum_{m=-2}^{2}(l_1 20m/l_1 22m)\times(l_2 20m/l_2 22m)(P_2^m)^2$	$(4/5)\sum_{m=-1}^{1}(l_1 10m/l_1 12m)\times(l_2 10m/l_2 12m)(P_2^m)^2$
1	1	$2(1-x^2)$	$(2/3)(1+3x^2)$
1	3	$(3/7)^{\frac{1}{2}}(1-x^2)(1-5x^2)$	$-(2/7)^{\frac{1}{2}}(1-12x^2+15x^4)$
3	3	$(3/28)(1-x^2)(1+15x^2)$	$(3/7)(1-4x^2+7x^4)$

The sums over m appearing here (and more explicitly in Eq. (12)) are easily evaluated by use of Table I with the results shown in Table II (in which we put $\cos\theta=x$). The use of these sums is helpful in consolidating the result expressed in (10), but we are still left with a rather distressing number of arbitrary constants in addition to the energy-dependent factors ϕ_l which we consider to be just the penetration factors of the incoming waves penetrating the Coulomb barrier. At least we may first note that the first row of Table II contains no powers of x higher than x^2, and this leads immediately to the result that for $\phi_3=0$ the cross section varies with angle as $1+A(E)\cos^2\theta$, as may also be obtained from the transformation properties of the entering p wave. We further note in Table II that the $\cos^4\theta$ term comes in with the first power of ϕ_3, as anticipated in the introductory paragraphs above, and that there are no higher powers of $\cos\theta$ with ϕ_3^2.

The most conspicuous contributions to the angular distribution at fairly low energies are made by the terms in ϕ_1 alone, and with only these terms the analysis involves the arbitrary complex constants

$$[122/12]\alpha_{12},\quad [112/12]\alpha_{12},\quad \Gamma_2\alpha_{10}/i\Gamma_0,$$

and also the common factor $(16\pi\lambda^2/\Gamma_2^2)$ in which we need not be interested if we confine our attention to the angular distribution. The number of constants appearing in the equation and essential to the angular distribution may be reduced by dividing through by one of them, say by $[112/12]\alpha_{12}$ as was done in reference 2. This procedure, which simplifies the leading terms, neither simplifies nor complicates the terms in ϕ_3. In carrying it out, we define the remaining arbitrary constants in terms of their real parts R and imaginary parts I:

$$\begin{aligned}\Gamma_2\alpha_{10}/i\Gamma_0[112/12]\alpha_{12}&=R_0+iI_0,\\ [122/12]/[112/12]&=R_1+iI_1,\\ [312/32]\alpha_{32}/[112/12]\alpha_{12}&=R_2+iI_2,\\ [322/32]\alpha_{32}/[112/12]\alpha_{12}&=R_3+iI_3.\end{aligned}\tag{11}$$

With these definitions, the first summation in the last member of (10), for example, can be developed as follows:

$$\begin{aligned}&\sum_{m=-2}^{2}\Big|\sum_{l=1,3}(l20m/l22m)\phi_l(R_l+iI_l)\Big|^2(P_2^m)^2\\ &=\sum_m\{[\sum_l(l20m/l22m)\phi_lR_l]^2\\ &\qquad+[\sum_l(l20m/l22m)\phi_lI_l]^2\}(P_2^m)^2\\ &=(R_1^2+I_1^2)\phi_1^2\sum_m(120m/122m)^2(P_2^m)^2\\ &\quad+2(R_1R_3+I_1I_3)\phi_1\phi_3\sum_m(120m/122m)\\ &\quad\times(320m/322m)(P_2^m)^2+(R_3^2+I_3^2)\\ &\qquad\times\phi_3^2\sum_m(320m/322m)^2(P_2^m)^2.\end{aligned}\tag{12}$$

When the other terms are treated similarly and evaluations made by use of Tables I and II, Eq. (10) becomes

$$\begin{aligned}\sigma\sim(\epsilon^2+1)^{-1}\{&[(5/2)(R_1^2+I_1^2)(1-x^2)\\ &+(5/6)(1+3x^2)+(R_0^2+I_0^2)(1+\epsilon^2)\\ &+(10/3)^{\frac{1}{2}}(R_0\epsilon-I_0)(1-3x^2)]\phi_1^2\\ &+(3/7)^{\frac{1}{2}}[(5/2)(R_1R_3+I_1I_3)(1-x^2)(1-5x^2)\\ &-(5/6^{\frac{1}{2}})R_2(1-12x^2+15x^4)-5^{\frac{1}{2}}((R_0R_2+I_0I_2)\epsilon\\ &+R_0I_2-R_2I_0)(1-3x^2)]\phi_1\phi_3\\ &+(15/112)[(R_3^2+I_3^2)(1-x^2)(1+15x^2)\\ &\qquad+4(R_2^2+I_2^2)(1-4x^2+7x^4)]\phi_3^2\}.\end{aligned}\tag{13}$$

It is clear from Eqs. (8) and (10), among others, that there are no interference terms between the incoming states with $S=1$ and $S=2$, and consequently the phase of (R_1+iI_1) relative to (R_0+iI_0) cannot affect the result. If we consider these as entirely arbitrary parameters, we might without loss of generality of the result set $I_1=0$ (as suggested by footnote 9 of reference 2), and this is reflected in the form of (13). We prefer instead to keep our parameters uniquely defined in terms of the matrix elements even though these matrix elements are not in practice calculable, since expressing the parameters, which are handled as arbitrary, in this conceptually clearer

way does not interfere with the usefulness of the formulas. With the equation for the cross section in the form (13), it appears expedient to rename some of the combinations of arbitrary constants thus:

$$\begin{aligned} a&=(5/2)(R_1^2+I_1^2),\\ b&=(5/2)(3/7)^{\frac{1}{2}}(R_1R_3+I_1I_3),\\ c&=(5/14^{\frac{1}{2}})R_2,\\ d&=(15/7)^{\frac{1}{2}}(R_0R_2+I_0I_2),\\ e&=(15/112)(R_3^2+I_3^2). \end{aligned} \tag{14}$$

R_0 and I_0 together with these five are used as the seven independent real parameters. Among them, a and e are necessarily positive (or at least not negative). The formula for the cross section may then be written in the form

$$\sigma=C(\epsilon)[1+A(\epsilon)\cos^2\theta+B(\epsilon)\cos^4\theta], \tag{15}$$

with

$$\begin{aligned} A(\epsilon)&=[5/2-a-(30)^{\frac{1}{2}}(R_0\epsilon-I_0)\\ &\quad+3(2b+4c+d\epsilon+\alpha)\Phi\\ &\quad+(14e-4\gamma)\Phi^2]/D(\epsilon),\\ B(\epsilon)&=[5(b-3c)\Phi-(15e-7\gamma)\Phi^2]/D(\epsilon),\\ C(\epsilon)&=\pi(4\lambda[112/12]\alpha_{12}\phi_1/2^{\frac{1}{2}})^2D(\epsilon)/(\epsilon^2+1),\\ D(\epsilon)&=(R_0^2+I_0^2)(\epsilon^2+1)+5/6+a\\ &\quad+2(2/3)^{\frac{1}{2}}(R_0\epsilon-I_0)+(b-c-d\epsilon-\alpha)\Phi\\ &\quad+(e+\gamma)\Phi^2, \end{aligned} \tag{16}$$

where

$$\Phi=\phi_3/\phi_1$$

is the ratio of the penetration amplitudes of the incoming f and p waves, and the auxiliary constants α and γ (which were introduced as abbreviations for $(15/7)^{\frac{1}{2}}(R_0I_2-R_2I_0)$ and $(15/28)(R_2^2+I_2^2)$, respectively) may be expressed in terms of the independent parameters thus:

$$\begin{aligned} \alpha&=(R_0d-(6/5)^{\frac{1}{2}}(R_0^2+I_0^2)c)/I_0,\\ \gamma&=(3/10)\{c^2+((5/6)^{\frac{1}{2}}d-cR_0)^2/I_0^2\}. \end{aligned} \tag{17}$$

The common factor $C(\epsilon)$ contains a further matrix element not included in our listed parameters, but it need not necessarily be considered in a study of the angular distribution. It must be remembered that ϵ is related to the bombarding energy (in c.m. coordinates) and the resonant energy through the relation $\epsilon=2(E-E_2)/\Gamma_2$ so that the E_2 and Γ_2 must be considered as further arbitrary parameters. With no entering f wave, Φ reduces to zero and makes $B(E)=0$, as has been noted above, and the parameters involved in the determination of $A(E)$ are five: E_2, Γ_2, R_0, I_0, and a, of which Γ_2 and a are limited to positive values. When the terms in Φ but not those in Φ^2 are taken into account, we have the three additional arbitrary parameters, b, c, and d, and the terms in Φ^2 bring in the last parameter, e, which is positive.

The algebra is here presented more explicitly than is required to derive the form of the results used in the experimental comparisons below. This is done in order to display the extent to which the integrals originally appearing in the theory in principle determine, or from the empirical viewpoint are determined by, the final parameters which are adjusted to fit the data.

IV. COMPARISON WITH THE EXPERIMENTAL RESULTS

Since it depends on arbitrary assumptions about the compound nucleus, the theory here presented should be considered as merely the simplest of several alternative theories, and its adequacy must be judged by comparison with experiment. Unfortunately, the experimental determination of the coefficient $A(E)$ and particularly of $B(E)$ depend on taking rather small differences of observations subject to statistical and in some cases probably systematic errors, and the measurements at present available, which have only recently exhibited the term in $\cos^4\theta$ at all, must be regarded as preliminary. Under these circumstances the comparison between detailed theoretical and experimental results must be regarded more as a test whether the theoretical formula has sufficient flexibility to fit a typical experimental trend than as a final adjustment of the theory to the data. If the final data should require a more complicated theory, the understanding of it will have been aided by a study of the insufficient flexibility of the simpler theory. It is nevertheless gratifying that the theory here presented appears to have adequate flexibility to account for the general trend of the data over the fairly wide energy range in which they are now available.

The experimental measurements on $A(E)$ made by four different groups of investigators in partially overlapping energy regions are not in very close accord on the details of the curve. In reference 1 an S-shaped curve was drawn through the data up to 400 kev, and this does not seem

to join in a reasonable way with the curves of either reference 4 or 5. This S-shaped curve is reproduced as the short curve near the origin of Fig. 1. References 4 and 5 disagree by about 150 kev on the position of the principal steep ascent of the curve. However, these minor discrepancies leave little doubt that the main trend of the curve includes a sharp rise from practically zero at a small positive energy to a rather broad peak at about 1 Mev and a somewhat more gradual descent toward higher energies. The lower curve in Fig. 1 is drawn to fit all the data reasonably well with emphasis on avoiding sharp bends and inflections and with the knowledge that the theory does not require it to go to zero at zero bombarding energy. The only point which deviates from this compromise curve by more than 0.3 in A is the point whose originally plotted limits of uncertainty are reproduced by the vertical broken line in Fig. 1, the point from reference 4 at 875 kev, and it may be seen clearly in Fig. 3 of that paper that the reason this point was plotted too low was a prejudice against the possibility of a term in $\cos^4\theta$. (On the $\cos^2\theta$ plot, the 90° point was so far below a straight line through the others, probably the

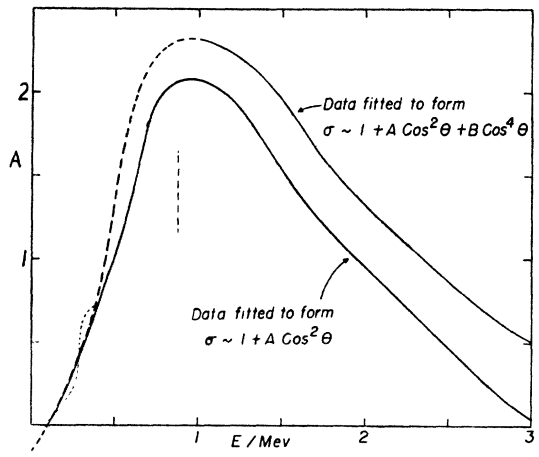

FIG. 1. The coefficient of the term in $\cos^2\theta$: curves summarizing the general trend of the observed angular distribution of $Li^7(p,\alpha)\alpha$ with varying energy. The lower curve is drawn in keeping with the interpretation of the earlier experimental papers, in which the term in $\cos^4\theta$ was not recognized. That these curves are uncertain in detail is indicated by discrepancies of overlapping data in the low energy part and by difficulties encountered in discriminating against scattered protons at the high energy end. The upper curve gives the general trend of the part of the angular distribution attributed to the $\cos^2\theta$ term after a part has been attributed to the term in $\cos^4\theta$.

combined effect of experimental fluctuations and of the negative value of $B(E)$, that it was ignored in determining the slope, since repetition of the observation had become impossible because of the advent of the war.) The curve avoids the inflections of the S-shaped curve at low energies and is consequently drawn with a node at about 100 kev. In reference 1, there seems to be a little evidence of a very natural prejudice against the possible appearance of negative values of A at low energies: the lowest point at 100 kev seems to be plotted in their Fig. 3 with an indication of the upper limit but with no indication of a lower limit. The scatter of points on their Fig. 2 would seem to permit a slightly negative A at 100 kev. The intensity is so low at these energies that the observations have unfortunately not been extended to energies sufficiently below 100 kev to have detected the negative value of $A(E)$ which would be expected there if there is a node near 100 kev. Experimental proof of the existence of a node rather than a gradual approach to zero would be desirable as a test of the theory.

The lower curve in Fig. 1 is based on fitting the data as well as possible to the simplified form of the angular distribution $1+A(E)\cos^2\theta$, with neglect of the observations at 10° and 20° when these were available in the more recent data in order to facilitate a comparison with the older data.

The upper curve in Fig. 1 represents $A(E)$ as determined by fitting the data to the more adequate form for the angular distribution $1+A(E)\cos^2\theta+B(E)\cos^4\theta$. This is, of course, the curve for $A(E)$ which should correspond to the theoretical expression (16). The part above 1 Mev is taken from reference 6, where this fit was carried out. The broken portion of the curve below 1 Mev is an arbitrary extrapolation guided by the lower curve, the data in this region being inadequate for any determination of $B(E)$. (Preparations are being made for further observation of this region.) The upper curve is expected to approach the lower curve closely at very low energies, where $B(E)$ is expected to vanish, so this curve also has a node at 100 kev.

Since $A(E)$ is observed to have its most rapid variation and pass through its maximum in the

range of energies up to not much over 1 Mev, in which Φ is expected to remain well below unity, it may be expected that the gross features of the curve for $A(E)$ may be reproduced by the approximation neglecting terms in Φ. On the face of it, it would seem that almost any general form of curve could be reproduced over a fairly limited range by a formula containing as many as five parameters. The equation for $A(\epsilon)$ when $\Phi=0$,

$$A(\epsilon)=\frac{5/2-a-2(6)^{\frac{1}{2}}(R_0\epsilon-I_0)}{(R_0^2+I_0^2)(\epsilon^2+1)+5/6+a+2(2/3)^{\frac{1}{2}}(R_0\epsilon-I_0)} \tag{18}$$

(which corresponds[11] to Eq. (12) of reference 2), does, however, seem to have a peculiar fitness for reproducing a curve which rises to a single maximum and falls to a node at one end of the observed range and to a small value but not a node at the other end. The numerator is linear in ϵ and thus has only one node, which can be made to match the experimental node[1,4] at about $E=100$ kev in the laboratory coordinate system.[12] For convenience in plotting a curve to compare with the experimental curve, Eq. (18) may be written with fewer terms:

$$A(E)=(C_1E+C_2)/(E^2+C_3E+C_4), \quad (18.1)$$

where the four parameters are expressed in the previous five parameters a, R_0, I_0, E_2, and Γ_2 as follows:

$$\begin{aligned}
C_1&=-6^{\frac{1}{2}}R_0\Gamma_2/\rho, \quad \rho=R_0^2+I_0^2,\\
C_2&=[(5/2-a+2(6)^{\frac{1}{2}}I_0)(\Gamma_2/4)\\
&\qquad +6^{\frac{1}{2}}E_2R_0]\Gamma_2/\rho, \quad (19)\\
C_3&=(2/3)^{\frac{1}{2}}R_0\Gamma_2/\rho-2E_2,\\
C_4&=(5/6+a-2(2/3)^{\frac{1}{2}}I_0)\Gamma_2^2/4\rho\\
&\qquad -(2/3)^{\frac{1}{2}}(R_0\Gamma_2/\rho)E_2+E_2^2+\Gamma_2^2/4.
\end{aligned}$$

The shape of the curve thus depends on only these four parameters and the previous five parameters are not uniquely determined by fitting the curve to the experimental data, even in this approximation which neglects terms in Φ.

This curve is plotted as curve *II* in Fig. 2, with the constants determined to match the experimental curve in the following respects: $A(0.1\text{ Mev})=0$, $A(3\text{ Mev})=\frac{1}{2}$, $E_{\max}=0.95$ Mev, $A(E_{\max})=2.32$. This curve has a considerably sharper maximum than the experimental curve *I* of Fig. 2, the latter being a copy of the upper curve in Fig. 1 which was drawn simply as a reasonable compromise between the slightly divergent and uncertain data. The constants C for curve *II* are $C_1=0.92$ Mev, $C_2=-0.092$ (Mev)^2, $C_3=-1.5$ Mev and $C_4=0.86$ (Mev)^2. A typical set of the previous five parameters compatible with this curve is

$$\begin{aligned}
&E_2=0.6\text{ Mev}; \quad \Gamma_2=0.1\text{ Mev};\\
&R_0=-0.04; \quad I_0=-0.1;\\
&a=(5/2)(R_1^2+I_1^2)=0.23.
\end{aligned}$$

The domain over which these sets of parameters may vary is limited by the requirement that a be

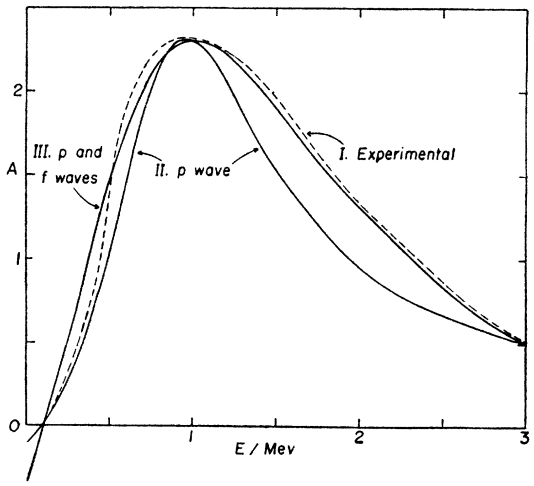

FIG. 2. The coefficient of the term in $\cos^2\theta$: comparison between theory and experiment. The curve taken to represent the rather uncertain experimental results is the same as the upper curve of Fig. 1. The theory resulting from only entering p waves gives too sharp a peak when fitted at the low energy node, at the peak, and at the highest energy. The theory including entering f waves is sufficiently adaptable to fit the experimental trend satisfactorily over this energy range.

[11] The correspondence between the coefficients becomes clear if one puts $\cos2\xi=1-2\sin^2\xi$ and notes that $a=(5/2)\tan^2\xi$ and that the terms in β^2 in numerator and denominator have the ratio 3 when $\sin\xi=a=0$ and the ratio -1 when $\sin\xi=1$ (or $a=\infty$).

[12] For convenience in comparison with experiment, at this point we consider the definition $\epsilon=2(E-E_2)/\Gamma_2$ to hold in the laboratory system, which is permitted since Γ_2 is now treated as an arbitrary parameter. This parameter Γ_2 is then (8/7) times the half-width appropriate to the theory, and for consistency a factor $(8/7)^2$ should henceforth be inserted in the expression in (16) defining $C(\epsilon)$.

TABLE III. Barrier penetrabilities for p and f waves.

$\frac{R}{e^2/mc^2}$	E/mc^2	1	1.5	2	3	4	6
1.5	C_1	1.73	1.23	0.90	0.51	0.28	0.04
	C_3	6.53	5.54	4.87	3.97	3.35	2.52
	Φ	0.0055	0.0087	0.0121	0.0193	0.0266	0.0381
	$5.06+2\ln\phi_1$	0.70	1.73	2.44	3.32	3.92	4.90
2.25	C_1	1.00	0.58	0.33	0.07		
	C_3	4.17	3.38	2.80	2.07	1.58	0.96
	Φ	0.028	0.038	0.050	0.065		
	$3.34+2\ln\phi_1$	0.83	1.81	2.44	2.56		
	$\ln Y(90°)$	1.61	2.10	2.44	3.20	4.26	

positive. E_2 remains fixed, and the rapidity of variation of the other parameters is indicated by the statement that, while a varies from 0 to 0.5, Γ_2 varies from 0.2 Mev to 0, R_0 from -0.07 to 0, and the ratio I_0/R_0 from 2.6 to 2.4. The smallness of $R_0^2+I_0^2$ relative to $R_1^2+I_1^2$ presumably arises primarily from the small ratio Γ_2/Γ_0, rather than from the other coefficients appearing in Eqs. (11), since the latter contain complicated integrals having no apparent reason to be very much larger for one state than another.

The next step in increasing complexity is to take into account the effect of the f wave of entering protons by including the terms containing Φ in the expression for $A(\epsilon)$ in Eq. (16). As we have noted above, Φ is essentially the ratio of the penetrabilities of the f and p wave functions (not probability amplitudes). These penetrabilities should properly be computed from the amplitudes, at the edge of the nucleus, of the regular and irregular solutions of the wave equation with a Coulomb field. Of these, only the regular solutions are at present available for f waves in the published literature.[13] We shall for the present content ourselves with a rough indication of their variation with energy obtained from the WKB approximation, which is admittedly unreliable for energies near the top of the barrier such as we are concerned with here. In Table III are shown data on the energy variation of $\Phi=\phi_3/\phi_1$, for two assumed nuclear radii, $R=1.5e^2/mc^2$ and $R=2.25e^2/mc^2$. These about span the range of uncertainty of R—they correspond to $\frac{1}{2}A^{\frac{1}{3}}e^2/mc^2$ plus half the range of the internuclear interaction and to about $0.7A^{\frac{1}{3}}e^2/mc^2$ plus the full range of the interaction, respectively. The data are computed from Eq. (631) of Bethe's *Nuclear Dynamics* article[14] by putting our ϕ_l equal to $(B_l-E)^{-\frac{1}{4}}e^{-C_l}$, where $B_l=U_l(R)$ is the height of the barrier. Here $U_l(r)=l(l+1)\hbar^2/(2Mr^2)+3e^2/r$, with M the reduced mass, and $C_l=2(M/\hbar^2)\int(U_l-E)^{\frac{1}{2}}dr$ integrated through the barrier. With $R=1.5e^2/mc^2$, B_1 is $7.19mc^2$ and B_3 is $33mc^2$. With $R=2.25e^2/mc^2$, B_1 is $3.6mc^2$ and B_3 is $15mc^2$, and with this large radius an energy of even $4mc^2$ is over the p wave barrier. In the table are also given the values of $\log\phi_1^2$ within a constant made to match the experimental data for $\log Y(90°)$ (taken from Fig. 4 of reference 6) at $E=2mc^2$. Equations (15) and (16) above show that the 90° yield can have a complicated dependence on energy through the explicit appearance of ϵ in $C(\epsilon)$ and $D(\epsilon)$, but because of the exponential nature of the penetration factor a large part of this energy variation is still expected to arise from the factor ϕ_1^2. For the sake of making this comparison, the experimental data for $\log Y(90°)$ are displayed as the last row of Table III. These data are taken from Fig. 4 of reference 6 (after converting energies from the center-of-mass system in mc^2 to the laboratory system in Mev). The traditional way of plotting $\log Y$ against $E^{-\frac{1}{2}}$ as followed in that figure arises from the expectation of a straight line in a much cruder approximation than we have used here, valid only even further below the top of the barrier, and the comparison of yields in Table III is more significant than the deviation from a straight line in that figure.

In the comparison between $\ln\phi_1^2$ and $\ln Y(90°)$ in Table III the two R's both lead to the expectation of a sharper variation with energy than is observed in the region below $2mc^2$, as though the existence of a resonance, presumably the state $j_r=2$ at 0.6 Mev, elevates the observed value there through the resonance denominator in $C(\epsilon)$. Aside from this, the value $R=1.5e^2/mc^2$ provides as satisfactory agreement as can be expected from the WKB approximation in a light nucleus. (The larger value or R might show up better, too, were it not that the approach to the top of the barrier invalidates the estimates completely at intermediate energies.)

[13] F. L. Yost, J. A. Wheeler and G. Breit, Terr. Mag. **40**, 443 (1935).

[14] H. A. Bethe, Rev. Mod. Phys. **9**, 178 (1937).

The main purpose of Table III is to exhibit the variation of Φ with energy. With the smaller radius, the relative penetrability Φ varies almost linearly with energy (or more nearly as the 1.1 power). With the larger radius the more questionable calculation gives a variation more nearly like the 0.85 power of the energy, which at least suggests that the variation is probably not very far from linear for a small range of radii R in the neighborhood of $1.5e^2/mc^2$.

In Eq. (16) we may then put

$$\Phi = KE/mc^2. \tag{20}$$

Since this is multiplied everywhere by arbitrary constants, the exact value of the numerical coefficient K is of no importance, though its order of magnitude is of considerable interest because one would presume offhand that several of the larger ratios of matrix elements in (14) would have the same order of magnitude, and that the coefficients of Φ in (16) would be of the same order of magnitude as the terms without Φ, except where fortuitous cancellations of the matrix elements cause some combinations of coefficients to vanish. The experimental fact is that $B(\epsilon)$, which contains Φ as a factor, is, in the energy range 2 to $6mc^2$, about one-fourth to half as large in magnitude as is $A(\epsilon)$, which has a leading term without Φ. The leading term in A cannot be fortuitously small at all these energies (it is known to be so at 100 kev), so Φ is expected to be of order of magnitude $\frac{1}{2}$ at $4mc^2$, for example, suggesting that one would prefer to find K of order of magnitude $\frac{1}{10}$, rather than 0.006 to 0.03 as suggested by the calculations for $R=1.5$ to $2.25e^2/mc^2$. While one might take this as indicative of a large nuclear radius, our ignorance of the matrix elements involved actually allows us to conclude only that the coefficient $5(b-3c)$ which determines the size of $B(\epsilon)$ in (16) is probably considerably larger in magnitude than, say, $2(6)^{\frac{1}{2}}R_0$ which is responsible for the size of $A(\epsilon)$.

The linear relation (20) in (15) and (16) yields

$$A(E) = (C_0E^2 + C_1E + C_2)/(E^2 + C_3E + C_4) \tag{21}$$

in place of (18.1), and for the coefficient of the $\cos^4$ term

$$B(E) = (C_5E^2 + C_6E)/(E^2 + C_3E + C_4). \tag{22}$$

These new coefficients C_n are no longer defined by (19) but instead involve the additional arbitrary parameters b, c, d, and e, as well as the constant K. Equation (21) which includes the effect of an entering f wave contains one more arbitrary coefficient C_n than does (18.1), which resulted from an entering p wave only. Whereas (18.1) gave somewhat too sharp a peak to fit the experimental data, the additional freedom provided by the C_0 term in (21) is sufficient to make a satisfactory fit with the data. This is shown by curve *III* in Fig. 2, which represents Eq. (21) with the constants[15] chosen to satisfy the following requirements: $A(0.1\ \text{Mev})=0$, $A(3\ \text{Mev})=\frac{1}{2}$, $A(2\ \text{Mev})=1.6$, $E_{\max}=1$ Mev, $A(E_{\max})=2.3$. While these requirements are stated in "round numbers" and may not give the best possible fit to the experimental curve *I*, the curve does show that (21) permits freedom to make the peak rather broad as required by the experimental results. Raising the curve as we have done on the high energy side of the peak raises it on the low energy side also and makes it steeper at very low energies than suggested by the low energy experiments. A slightly better fit with the various low energy experiments could perhaps be obtained by placing the node at about 200 kev and the peak at about 900 kev. In view of the tentative and discrepant nature of the experimental results below 1 Mev, which have not yet been analyzed for a term in $\cos^4\theta$, the agreement of curve *III* with curve *I* may be considered satisfactory, and a distinct improvement over curve *II*.

The f wave was included primarily in order to explain the $\cos^4\theta$ term, but we see now that the additional freedom that it introduces also helps to account for the energy dependence of the $\cos^2\theta$ term, insofar as the rather preliminary experimental data at present available may be trusted. Our assumptions about the states of the compound nucleus and about the barrier penetration must also be regarded as tentative, but within the limitations of these assumptions it should be noted that the inclusion of the f wave results not simply in additional freedom to fit the data within a limited energy range but also requires quite different behavior of $A(E)$ over

[15] They are $C_0=-0.99$, $C_1=4.34$ $C_2=,-0.43$, $C_3=-0.98$, $C_4=1.24$.

an extended energy range. In particular, the quadratic form of the numerator of (21) requires either no nodes or two, and the interpretation of the low-energy data in terms of a node near 100 kev requires another node at some higher energy. The available experimental data at the high energy end of the curve, near 3 Mev, are particularly uncertain because of the inconvenient range of scattered protons at those energies, but they seem at least to be compatible with a descent toward a node. Another important aspect of the greater freedom introduced by the f wave is that the correlation between the angular distribution and the expected position of a resonance in the excitation curve has been lost. Whereas with only a p wave curve *II* of Fig. 2 required a resonance (not observed) in the 90° yield at about 0.6 Mev, the number of parameters E_2, Γ_2, R_0, a, b, etc., has been increased by four in including the f wave, while the number of coefficients C_n uniquely determined by the data has been increased by only one, so it may unfortunately no longer be expected that the "parameters" E_2, Γ_2, etc., may be determined or even seriously limited.

As for the energy variation of $B(E)$, comparison of (22) with (16) and (20) shows that C_5 contains a factor K^2, so that C_5E^2 may be expected to be much smaller than C_6E, at least at energies below 1 or 2 Mev, since the coefficient $5(b-3c)$ may not be fortuitously small as we have seen above. Because the experimental determination of B depends on taking second differences of values subject to statistical and other fluctuations (and because the observation at the highest energy, 3 Mev, is made more doubtful than the rest by interference from scattered protons), the trend of B with energy is far from well established. The curves resulting from a couple of trial values of C_6 and C_5 (one with $C_5=0$ and the other with C_5 relatively much larger than would seem most likely) are shown in Fig. 3, along with the scattered experimental points. The resonance denominator here contains the same constants[15] as used for curve *III* of Fig. 2. The form of the curves gives hope of obtaining agreement with more definite experimental data if they should become available.

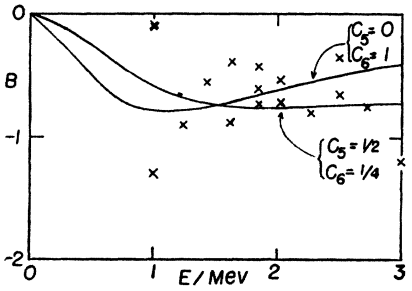

FIG. 3. The coefficient of the term in $\cos^4\theta$. Plausible variations of the two remaining arbitrary parameters would be expected to leave the theoretical curve between or near the two curves shown. The scattered preliminary experimental points are also shown.

V. ALTERNATIVE SET OF COMPOUND STATES

The assumption investigated above, that the angular distribution is to be attributed to a broad state of the compound nucleus with $j_r=0$ and a narrower state with $j_r=2$, seems to us the simplest and most natural basis for the interpretation of the experimental results, simplest because it involves fewest arbitrary parameters, and most natural because the gradual energy dependence of the angular distribution suggests a low density of levels of the compound nucleus Be^8 in this region of excitation, and one expects to encounter low angular momenta at those energies where the density of levels is very low. The great breadth of the state with $j_r=0$ is naturally associated with the ease with which such a state is expected to break up into two alphas.

It must however be recognized that the possibility of fitting the data with the above assumptions does not constitute a complete demonstration of the validity of those assumptions, and this investigation would be very incomplete without a display of some of the qualitative features of results obtainable from alternative assumptions. The simplest modification of the assumptions used above is to drop the requirement that Γ_0 be very large. This alteration complicates all the formulas considerably, but the development parallels so closely the calculation already carried out that we wish here only to note that its final formula for $A(E)$ is analogous to (21) with the numerator including powers of E up to the fifth and the denominator up to a term in E^4, the latter being small of order K^2.

We consider here in more detail the possibility

that there are two virtual states of the compound nucleus, each having $J_r=2$, but differing in E_r and Γ_r. We develop from Eqs. (7) and (8) as before, and the incoming states are the same as considered above. We find for the factor in the cross section which determines the angular distribution,

$$\begin{aligned}\sigma\sim&\{[\textstyle\sum C_{111r}/(\epsilon_r^2+1)+((\epsilon_1\epsilon_2+1)D_{111}\\&+(\epsilon_1-\epsilon_2)E_{111})/(\epsilon_1^2+1)(\epsilon_2^2+1)](1/3+x^2)\\&+[\textstyle\sum C_{112r}/(\epsilon_r^2+1)+((\epsilon_1\epsilon_2+1)D_{112}\\&+(\epsilon_1-\epsilon_2)E_{112})/(\epsilon_1^2+1)(\epsilon_2^2+1)](1-x^2)\}\\&-7^{-\frac{1}{2}}\{2^{\frac{1}{2}}[\textstyle\sum C_{131r}/(\epsilon_r^2+1)+((\epsilon_1\epsilon_2+1)D_{131}\\&+(\epsilon_1-\epsilon_2)E_{131})/(\epsilon_1^2+1)(\epsilon_2^2+1)](1-12x^2\\&+15x^4)+3^{\frac{1}{2}}[\textstyle\sum C_{133r}/(\epsilon_r^2+1)+((\epsilon_1\epsilon_2+1)D_{133}\\&+(\epsilon_1-\epsilon_2)E_{133})/(\epsilon_1^2+1)(\epsilon_2^2+1)](1-6x^2\\&+5_x^4)\}\Phi+(3/28)\{[\textstyle\sum C_{331r}/(\epsilon_r^2+1)\\&+((\epsilon_1\epsilon_2+1)D_{331}+(\epsilon_1-\epsilon_2)E_{331})/(\epsilon_1^2+1)\\&\times(\epsilon_2^2+1)](1+14x^2-15x^4)+4[\textstyle\sum C_{333r}/\\&(\epsilon_r^2+1)+((\epsilon_1\epsilon_2+1)D_{333}+(\epsilon_1-\epsilon_2)E_{333})/\\&(\epsilon_1^2+1)(\epsilon_2^2+1)](1-4x^2+7x^4)\}\Phi^2.\end{aligned}$$

The wealth of constants appearing here are defined as follows in terms of the integrals originally entering the theory:

$$\begin{aligned}&C_{abSr}=R_{aSr}R_{bSr}+I_{aSr}I_{bSr},\\&D_{abS}=R_{aS1}R_{bS2}+R_{aS2}R_{bS1}+I_{aS1}I_{bS2}+I_{aS2}I_{bS1},\\&E_{abS}=R_{aS1}I_{bS2}-I_{aS1}R_{bS2}+R_{bS1}I_{aS2}-I_{bS1}R_{aS2},\\&R_{lSr}+iI_{lSr}=(\Gamma_1/\Gamma_r)[lSj_r/lj_r]\alpha_{lr}/[11j_1/1j_1]\alpha_{11}.\end{aligned}$$

We then have the conditions

$$R_{111}=1,\quad I_{111}=0,$$

and the other R_{1Sr} and I_{1Sr} may be considered as arbitrary parameters, which amounts to the same thing as treating the C's, D's, and E's as arbitrary parameters with the conditions

$$C_{111}=1,\quad C_{1112}=E_{111}^2+D_{111}^2/4.$$

Neglecting terms containing Φ, we obtain $\sigma\sim 1+A\cos^2\theta$ as the result of an entering p wave, with

$$A=\frac{b(\epsilon_2^2+1)+d(\epsilon_1\epsilon_2+1)+f(\epsilon_1-\epsilon_2)+\{\frac{3}{4}[(c+d)^2+(e+f)^2]-g\}(\epsilon_1^2+1)}{a(\epsilon_2^2+1)+c(\epsilon_1\epsilon_2+1)+e(\epsilon_1-\epsilon_2)+g(\epsilon_1^2+1)}.$$

Here we have renamed the parameters to simplify the appearance of the equation, and of them, $a\geqslant\frac{1}{3}$ and $g\geqslant 0$. This contains seven parameters explicitly, and there are four more implicit in the definitions of the ϵ's, as compared with three explicit and two implicit parameters in Eq. (18) above. With so many parameters there is without doubt much more freedom than is required to fit the experimental results and any correlation achieved would seem less significant than with the fewer parameters of the simpler interpretation presented above. It is, of course, still possible that future more accurate data might require this greater freedom, or perhaps even a more complicated set of assumptions involving a state of the compound nucleus with $j_r=4$.