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Energy Straggling of Protons

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The energy straggling of protons of different energies penetrating foils of mica or beryllium of different thicknesses has been measured using the resonance radiation from aluminium and fluorine targets as energy indicators. The experimental results are in good agreement with the theory of straggling.

THE method of using the sharp resonances in the proton capture processes as energy indicator for protons, in the way previously described in an investigation of the stopping power of beryllium,[1] makes it also possible to measure the energy straggling caused by a thin foil. In fact, the resonance peaks show not only a shift, when a foil is inserted in the proton beam, but also a broadening, which is sufficiently great to justify a more quantitative comparison with the theory of energy straggling.

The energy of the protons in a homogeneous beam will, by the passage of a thin foil, be distributed over a finite range corresponding to a standard deviation Ω which, according to the theory,[2] should be approximately given by

$$\Omega^2 = (4\pi e^4/M)\cdot(Z/A)\cdot(t), \qquad (1)$$

where M is the mass and e the charge of the protons, Z the atomic number and A the mass number of the foil material, and where t is the thickness of the foil measured in weight per unit area.

In the previous investigation of the stopping power of beryllium[1] only an upper limit for the energy straggling in this substance was obtained, because it was not known, to what extent the foils, which had been prepared by evaporation, were homogeneous in thickness.

According to Eq. (1) the standard deviation should be proportional to the square root of the thickness t, and the factor of proportionality should be approximately independent of both the energy and of the foil material for not too small an energy of the protons, because the ratio of Z to A is close to the value $\frac{1}{2}$ for all elements (except hydrogen).

It is, therefore, also possible to compare the theory with experiments carried out with foils consisting of a mixture of different elements. The measurements were consequently mainly performed with foils of mica, which has the advantage that it can easily be split into very thin foils of homogeneous thicknesses, and that the homogeneity can be checked by optical means both before and after the experiments.

The technique and apparatus were the same as in the earlier work,[1] but it proved necessary to defocus the proton beam to 5 mm in diameter and to use currents less than 0.1 micro amp. in order to prevent destruction of the foils. Measurements at the lowest proton energies were carried out with the molecular beam H_2^+ of the van de Graaff generator, because the focusing was unstable at low voltages, but this should not influence the result as the molecules will split up as soon as they hit the foil.

The mica foils were prepared in thicknesses down to 0.3 mg per cm^2. They were used in the form of small disks with a diameter of 11 mm, but their thicknesses were determined before they were cut out, by measuring weight and area of pieces more than ten times bigger, which in mercury light had shown the same interference color over the whole of their area. A few foils were also silvered and investigated in an interferometer[3] by the method of Tolansky,[4] and the thicknesses were found to be constant at least within 0.1 percent, also for foils bombarded. Only two foils, which unfortunately had been heated too much by the proton current, showed 10 to 20 interference fringes at small spots, but were homogeneous over the whole area between the

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[1] C. B. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 648 (1948).

[2] Cf. N. Bohr, Kgl. Danske Vid. Selsk. mat.-fys. Medd. **XVIII**, 8 (1948), formula (3.4.5).

[3] We are very much indebted to cand. mag. Rahbek for carrying out this investigation.

[4] S. Tolansky, Proc. Roy. Soc. **A186**, 261 (1945) and **A191**, 182 (1947).

spots. The undisturbed area was about 90 percent of the whole area. These spots could also be seen directly as a metallic-like tint, probably caused by a decomposition of the mica, but the weight and the stopping power, as well as the straggling had the same values as for other foils of the same thickness.

The thicknesses of a few small foils, the areas of which could not be measured with sufficient accuracy, because of irregularities in the edges, were determined by comparing their stopping powers to that of a foil of known thickness.

Furthermore, the interferometer measurements did not only give information about the homogeneity of the foils, but also about their absolute thicknesses measured in angstrom units. The relative thicknesses of the different foils, determined by weighing, by the optical means and from the stopping power, agreed within the experimental accuracy.

TABLE I. The columns contain: 1. foil material, 2. foil thickness in mg per cm^2, 3. resonance used as indicator (figures in kev), 4. stopping power 's' of the foil, measured in kev per mg/cm^2, 5. standard deviation Ω_1 without foil, in kev, 6. standard deviation Ω_2 with foil, in kev, 7. straggling Ω in kev and 8. $\Omega/(t)^{\frac{1}{2}}$ in kev (mg/cm^2)$^{-\frac{1}{2}}$. The theoretical value of $\Omega/(t)^{\frac{1}{2}}$ is according to formula (1):

$$(2Z\cdot 2\pi e^4/AM)^{\frac{1}{2}}=(2Z/A)^{\frac{1}{2}}8.85 \text{ kev (mg/cm}^2)^{-\frac{1}{2}}.$$

Substance	t mg/cm^2	Resonance	s kev/(mg/cm^2)	Ω_1 kev	Ω_2 kev	Ω kev	$\Omega/(t)^{\frac{1}{2}}$ kev/(mg/cm^2)$^{\frac{1}{2}}$
1	2	3	4	5	6	7	8
Be	0.222	Al-503	327	4.4	6.1	4.2	8.9
—	0.222	Al-630	283	1.45	5.1	4.9	10.4
—	0.222	Al-630	283	4.2	5.3	3.3	6.9
—	0.222	Al-986	219	1.4	5.0	4.8	10.2
—	0.222	Al-986	219	1.7	4.2	3.9	8.2
—	0.222	Al-1112	195	2.9	5.5	4.7	9.9
—	0.222	Al-1112	195	2.9	5.3	4.5	9.4
—	0.222	Al-1255	188	1.7	4.9	4.6	9.7
—	0.245	Al-986	216	1.25	4.2	4.0	8.2
—	0.609	F-339	377	4.2*	9.5*	7.2	9.2
—	0.609	Al-986	207	2.5	6.6	6.1	7.9
		experimental mean value for Be					8.99
		value for Be according to Eq. (1)					8.3
Mica	0.336	F-339	300	3.6*	5.8*	4.5	7.8
—	0.336	Al-630	218	3.4	5.6	4.4	7.6
—	0.336	Al-986	176	1.7	5.2	4.9	8.5
—	0.336	Al-986	176	1.7	5.0	4.7	8.1
—	0.441	Al-986	172	1.6	6.4	6.2	9.3
—	0.441	Al-986	172	1.6	6.1	5.9	8.8
—	0.441	Al-986	172	1.6	5.9	5.7	8.5
—	0.441	Al-986	172	1.5	6.3	6.1	9.2
—	0.66	Al-986	170	1.7	8.0	7.8	9.6
—	0.66	Al-986	170	1.7	7.6	7.4	9.2
—	0.76	Al-986	170	3.5	8.0	7.2	8.3
—	1.02	Al-986	170	1.7	8.7	8.5	8.5
—	1.02	Al-986	170	1.9	9.0	8.8	8.7
—	1.02	Al-986	170	1.9	9.4	9.2	9.1
—	1.02	Al-986	170	2.1	9.5	9.3	9.2
—	1.02	Al-986	170	2.1	9.1	8.9	8.8
—	1.23	Al-986	174	3.0	9.5	9.0	8.3
—	1.71	Al-986	166	3.8	12.5	11.9	9.1
		experimental mean value for mica					8.70
		value for mica according to Eq. (1)					8.8

* 0.42 times the half-width. Standard deviation not determined (see text).

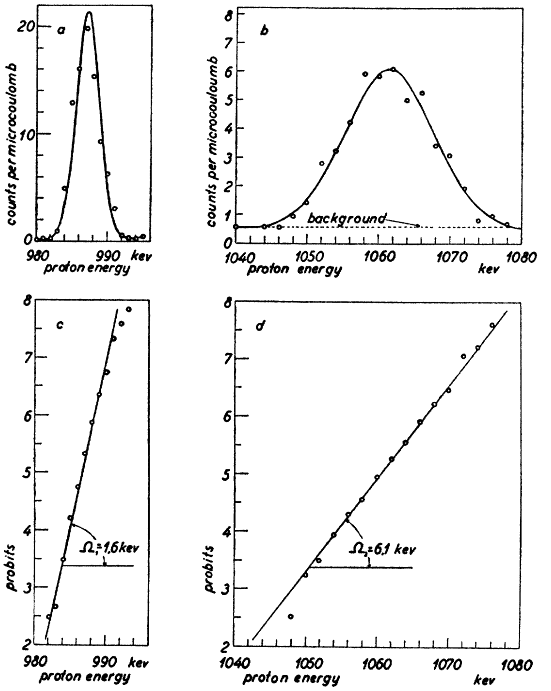

FIG. 1. The 986 kev Al resonance measured (a) without foil and (b) with a mica foil of thickness 0.441 mg per cm^2 inserted in the proton beam. In (c) and (d) the results from (a) and (b) are transformed into straight lines, the slopes of which give the standard deviations.

The sharp aluminum resonances, especially the 986 kev resonance, have been used as indicators in the high energy region, whereas the rather broad, but strong fluorine line at 339 kev has been used at the lowest energies, because the scattering and stopping powers of the foils are greater at low energies, and very low proton currents are therefore demanded. Targets with stopping powers ranging from 4 to 10 kev were used, depending on the foil thicknesses.

In Fig. 1 is shown an example from a measurement in the 1 Mev region. In Fig. 1a is plotted a measurement without foil and in Fig. 1b a measurement with a foil of thickness 0.441 mg per cm^2 inserted in the beam. In order to determine the standard deviation corresponding to the values plotted in these figures, the distribution is approached by the Gaussian, even though it should not be the case in Fig. 1a, where the

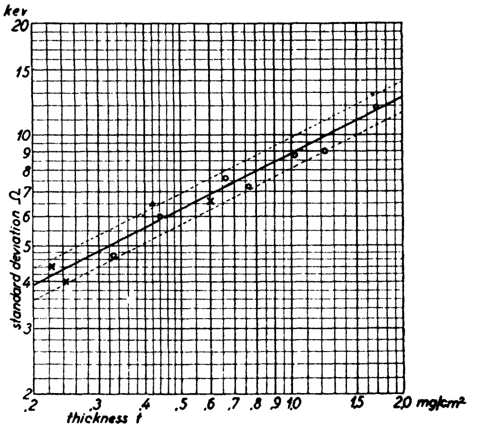

FIG. 2. The straggling Ω for beryllium foils (crosses) and mica foils (circles) as function of the thickness of the foils. The full drawn line is the theoretical Eq. 1 for substances with $2Z=A$. The dotted lines are the 10 percent lines.

width is mainly due to the thickness of the aluminum target used. The standard deviations for the two series of measurements can then be found by transforming the results into straight lines,[5] the slopes of which give the standard deviations Ω_1 and Ω_2 corresponding to the measurements without and with the foil respectively. By this method one takes into account all the points of the resonance peak except those which are only slightly above the background, and the determination of the standard deviations should, therefore, be rather unambiguous. The straight lines are drawn in Figs. 1c and 1d. The two curves on Figs. 1a and 1b are Gaussian curves with the standard deviations found in Figs. 1c and 1d respestively.

The true straggling Ω is calculated from Ω_1 and Ω_2, by the formula

$$\Omega=(\Omega_2^2-\Omega_1^2)^{\frac{1}{2}}, \qquad (2)$$

since the standard deviations add up geometrically.

In the measurements with fluorine as indicator the transformed curves deviated very much from straight lines, because of the long tails of the Breit-Wigner curve. In these cases, therefore, the straggling was found directly, by determining that Gaussian distribution, which combined with the curve obtained without foil, gave the best fit to the curve obtained with foil inserted.

The results of all the straggling measurements are given in Table I, and in Fig. 2 the mean of the values obtained for each foil is plotted in double logarithmic scale as a function of the thicknesses of the foils. The measurements with the sharp and strong line Al-986 are the most reliable.

Both from the curve and from the figures in the last column of the table it is seen that the results for beryllium and mica agree well with formula (1), whereas previous measurements by Briggs[6] of the straggling of fast α-rays in mica gave values, which were about 40 percent larger than those corresponding to the equivalence for α-rays of expression (1). Later measurements by Bennett[7] are however more in agreement with ours, giving values only about 15 percent in excess of the theoretical estimate.

The simple theoretical formula (1) is expected to represent a good approximation for fast particles in substances of comparatively small atomic number, in which case all the atomic electrons may be assumed to contribute effectively to the stopping and straggling process. This assumption should be well fulfilled in the case of beryllium for the proton energies in question, and also for mica it is approximately justified, at any rate for the experiments with proton energies of the order of 1 Mev. For smaller energies it may be necessary to take into account, that the strongest bound electrons no longer contribute to the phenomenon.[2] This circumstance might in fact explain the somewhat smaller straggling found at the lower proton energies, although too much significance cannot, of course, be given to the experimental evidence regarding this point.

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[5] R. A. Fisher and F. Yates, *Statistical Tables* (Oliver and Boyd Ltd., London, 1938).

[6] G. H. Briggs, Proc. Roy. Soc. **A114**, 313 (1927).

[7] W. E. Bennett, Proc. Roy. Soc. **A155**, 419 (1936).