The Absorption of High Energy Quanta. I.

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(Received March 1, 1948)

The absorption of quanta between 11 and 20 Mev has been measured with the x-rays from the 22-Mev betatron as a source. The high energy tip of the x-ray spectrum has been isolated for these measurements by using a threshold detector method. The detectors used were copper, iron, and carbon, with which absorption coefficient values were obtained at 11.04, 13.73, and 19.10 Mev, respectively. Aluminum, iron, copper, and lead were used as absorbers. The observed absorption coefficients are in good agreement with existing theory except in the case of lead. The absorption in lead is smaller than predicted. This is probably due to the failure of the Born approximation in the calculation of the pair-production cross section.

INTRODUCTION

BEFORE the advent of the 22-Mev betatron,[1] means did not exist to make detailed tests of the absorption of quanta of energies up to 20 Mev. The betatron is a source of x-rays produced from very nearly monokinetic electrons, the energy of which is continuously variable and accurately reproducible. The energy scale calibration is made both by electrical measurements and by comparing observed threshold energies[2] with known mass data.

Compton absorption has been tested[3] at length at low γ-ray energies with no apparent deviations from the prediction of Klein and Nishina.[4] There seems to be no good reason why this formula should not remain accurate at 20 Mev. Since the only other important contribution to the total absorption results from the production of electron-positron pairs,[5] absorption measurements involving quantum energies between 10 and 20 Mev will largely be tests of the predictions of this process. The atomic photo-effect contributes at most $\frac{1}{2}$ percent (see reference 5b, pp. 124, 160, and 216) of the total absorption in lead and is much smaller in the other absorbers used in the experiments described below. The cross section for the nuclear photo-effect is never large, the largest estimated by Bothe and Gentner[6] being 5×10^{-26} cm^2 for the activation of Cu^{62}. This is of the order of 1 percent of the total absorption cross section for copper. Thus, Compton effect and pair production are the important absorption processes in this energy range.

This paper will describe experiments in which absorption of quanta of energies between 10 and 20 Mev is measured. The observed values are compared with theoretical predictions.

EXPERIMENTAL METHOD

The experiments reported here were done partly in 1942–43,[7] and completed recently.[8] The same general techniques were employed in the two groups of experiments. Improvements in geometry and betatron yield make the recent experiments much more reliable.

The transmission of monochromatic quanta through an absorber of thickness x follows the law:

$$I/I_0=\exp(-\tau x), \qquad (1)$$

where τ is called the absorption coefficient. τ varies with the atomic number (Z) of the absorber and with the energy ($E=h\nu$) of the quanta. The x-rays from the betatron are not monochromatic but form a continuous spectrum. On the other hand, τ varies rather slowly with

[1] D. W. Kerst, Rev. Sci. Inst. **13**, 387 (1942).
[2] J. McElhinney, A. O. Hanson, and R. B. Duffield, Phys. Rev. **74**, 1257 (1948).
[3] For example, G. E. M. Jauncy and G. G. Harvey, Phys. Rev. **37**, 698 (1931); S. J. M. Allen, Phys. Rev. **27**, 266 (1926); J. Read and C. C. Lauritsen, Phys. Rev. **45**, 433 (1934); C. Y. Chao, Phys. Rev. **36**, 1519 (1930).
[4] O. Klein and Y. Nishina, Zeits. f. Physik **52**, 853 (1929); Y. Nishina, **52**, 869 (1929).
[5] (a) H. A. Bethe and W. Heitler, Proc. Roy. Soc. **146**, 83 (1934); (b) W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1936), Sections 20, 22.
[6] W. Bothe and W. Gentner, Zeits. f. Physik **106**, 236 (1937).
[7] G. D. Adams and R. K. Clark, Phys. Rev. **63**, 60A (1943); G. D. Adams, Thesis, University of Illinois, 1943 (unpublished). Submitted in partial fulfillment of the thesis requirement in the graduate school of the University of Illinois, September 17, 1943. Publication was withheld temporarily at the direction of OSRD.
[8] G. D. Adams and A. T. Nordsieck Phys. Rev. **74**, 1255 (1948).

quantum energy in the energy range being considered for all Z, so it is practical to determine absorption coefficients by measurements of the transmission of a narrow band of the spectrum. The lower bound of a band used in these experiments is the photo-nuclear threshold for activation of an element which is used for monitor and detector of the radiation. The upper bound is the maximum energy in the spectrum and is the energy to which electrons are accelerated in the betatron.

Figure 1 illustrates the experimental arrangement for all experiments. The width of the beam from the betatron at half-maximum intensity is about 9° at 20 Mev. The monitor and detector samples are also of about this width at the irradiation position; the absorbers are somewhat larger. The purpose of the monitor is to integrate small fluctuations in beam intensity and irradiation time. The detector-to-monitor ratio of the activities induced in those samples measures the relative transmission of an absorber placed between the monitor and detector. The absorption coefficient is determined from (1) by using transmission *vs.* thickness data.

In the earlier experiments, the induced activity was read in two ways: with Lauritsen electroscopes and with thin-walled Geiger counters. The electroscope work is less reliable largely due to uncertainties in the leakage rate. For the counter work, the detector was made a cylinder to improve the counting statistics. In each case, the thickness of both monitor and detector exceeds the range of the positrons forming the induced activity.

The reactions used, with half-life of the induced activity, threshold energy, and spectral band width, are given in Table I. The carbon reaction

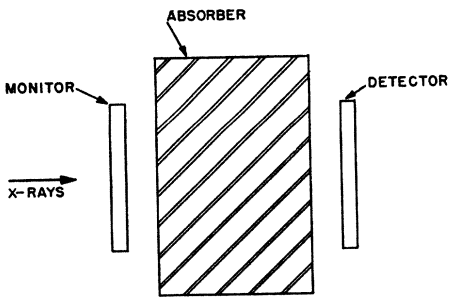

FIG. 1. Experimental arrangement.

was not used in the early experiments largely because of low activation yield. In the recent experiments, the beam intensity was about 100 roentgens per minute, measured one meter from the target in a thick-walled ionization chamber.

Transmission measurements have been made with carbon, aluminum, iron, copper, and lead as absorbers. In the early work, the absorption coefficient was in every case determined from a transmission curve involving 7 to 12 different absorber thicknesses. Within experimental error, the exponential transmission law of Eq. (1) was obeyed for the spectral band used. In the later experiments, transmission curves were taken as before to confirm the exponential behavior. The bulk of the data, however, was obtained using a single thickness of each absorber equivalent to two to three half-value layers. Calculations[9] indicated this range of thicknesses would permit the best statistics per unit operating time. The optimum thickness depends principally on the ratio of background to induced activity. The irradiation and counting times were each about one half-life for all experiments.

RESULTS

Before converting the raw data into absorption coefficients, corrections from three sources must be applied:

(a) General counter background of the order of 15 counts per minute.
(b) Counts arising from operation of the betatron during counting. This correction was usually much smaller than background, but detectable.
(c) Spurious activities induced in the detectors.

A spurious activity is present in the copper detectors which is attributed to $_{29}Cu^{65}(n,\gamma)_{29}Cu^{66}$ but is identified only by a half-life of 5.3 ± 0.2 min. This correction amounts to a few times the background correction, being largest when the absorber is lead. This amount is determined by measuring the spurious activity count at several energies below the 10.5-minute threshold and extrapolating to the operating energy. A check on the monitor value can be obtained by subtracting the total counts induced on successive runs with and without the absorber at constant

[9] Similar to those presented by M. E. Rose and M. M. Shapiro, Phys. Rev. **74**, 1213 (1948).

betatron output. The values obtained by these two methods are in good agreement.

The iron detectors used exhibited a spurious activity with a half-life of 10 ± 1 minutes and with the $_{29}Cu^{63}(\gamma,n)_{29}Cu^{62}$ threshold and thus appears to be a copper impurity. From the activity, the concentration of this impurity was estimated at 0.1 percent. It was not possible to separate this activity from that of Fe^{52} by half-life measurements, but the iron activity threshold is clearly marked by a change in slope when plotting induced activity *versus* activation energy. The amount of the spurious activity was determined by the extrapolation method only. The error arising from this impurity is small since it involves the small difference between the absorption coefficients for energies determined by the iron and copper reactions separately, and with the iron weighted heavily.

It was thought advisable to preserve the data as nearly as possible in the form taken. Consequently, the calculated absorption coefficients were corrected by an amount derived from the fractional contribution to the counts caused by this spurious activity. This correction amounts to about 1 percent.

A satisfactory detector consisting principally of carbon was hard to find. Graphite absorbs large quantities of atmospheric oxygen and nitrogen on its surfaces. The (γ,n) threshold is energetically higher for carbon than for either oxygen or nitrogen; hence the 2.1 and 10 minutes periods of these elements, respectively, are activated. The material used was polystyrene —$(CH)_n$—and was much better in this respect. Some activity of roughly similar periods was still detectable. The correction in this case is of the order of the uncertainty in the background count.

A weight equal to half the number of points involved was assigned to the absorption coefficients determined from the transmission curve runs for the purpose of combining these values with those obtained from the single thickness runs. Each point of the single thickness runs was used at unit weight. The errors indicated in Table II are the probable errors determined from the spread of values in the individual transmission determinations. In some cases this is somewhat larger than that estimated from the numbers of counts comprising the determination. This excess exhibits no apparent trend and is presumed to be due to variations in counter sensitivity. Sensitivity variations of the order of 2 percent were found by checking at convenient intervals. At the counting rate used (averaging about 2000 counts per minute), the chance of losing counts through lack of resolution is negligible.

TABLE I.

Reaction	Half-life (minutes)	Threshold (Mev)	Spectral band width (Mev)
$_{29}Cu^{63}(\gamma,n)_{29}Cu^{62}$	10.5	10.9 ± 0.1	0.5
$_{26}Fe^{54}(\gamma,n)_{26}Fe^{53}$	8.9	13.9 ± 0.3	1.6
$_{6}C^{12}(\gamma,n)_{6}C^{11}$	20.5	18.7 ± 0.1	1.7

The measured absorption coefficients are displayed in Table II. Theoretical comparison values and the difference (theoretical minus experimental) are given in adjacent columns. This subtraction is done in an unnatural way but permits easy comparison with the experimental error. The probable errors are mostly about 1 percent. Note that the only important discrepancy occurs when the absorber is lead. This will be discussed at some length below.

THEORETICAL VALUES

The calculation of the theoretical absorption coefficients proceeds from existing cross section formulae for the two important absorption processes. The exact calculation of Klein and Nishina[4] gives the form used for the Compton effect cross section:

$$\sigma_c = 2\pi r_0^2 Z\left\{\frac{1+\gamma}{\gamma}\left[\frac{2(1+\gamma)}{1+2\gamma}-\frac{\log(1+2\gamma)}{\gamma}\right]+\frac{\log(1+2\gamma)}{2\gamma}-\frac{1+3\gamma}{(1+2\gamma)^2}\right\}. \quad (2)$$

A calculation by Bethe and Heitler,[5] using the Born approximation, gives a cross section for the production of electron-positron pairs:

$$\sigma_P = (28/9)\alpha r_0^2 Z(Z+1)\{\log(2\gamma)-109/42\}. \quad (3)$$

In both (2) and (3), $\gamma=\bar{E}/mc^2$, $\bar{E}$ = mean quantum energy of the spectral band used for inducing

TABLE II. Absorption coefficients, in cm^{-1}, with theoretical comparison values calculated for the mean energy of the spectral band used.

Absorber	Cu detector $\bar{E}=11.04$ Mev theor.	exper.	th-exp.	Fe detector $\bar{E}=13.73$ Mev theor.	exper.	th-exp.	C detector $\bar{E}=19.10$ Mev theor.	exper.	th-exp.
Al	0.0613	0.0605±0.0013	+0.0008	0.0594	0.0596±0.0006	−0.0002	0.0595	0.0604±0.0016	−0.0009
Fe	0.233	0.231 ±0.003	+0.002	0.239	0.240 ±0.003	−0.001	0.257	0.262 ±0.002	−0.005
Cu	0.266	0.276 ±0.003	−0.010	0.277	0.285 ±0.003	−0.008	0.299	0.307 ±0.006	−0.008
Pb	0.612	0.569 ±0.007	+0.043	0.682	0.625 ±0.007	+0.057	0.791	0.695 ±0.004	+0.096

the activities observed, $\alpha=e^2/\hbar c$, $r_0=e^2/mc^2$, and $Z=$atomic number of the absorber.

The Z-dependence of the pair-production cross section was calculated to be Z^2 for the nuclear effect and has been increased to Z^2+Z to include additional absorption resulting from pair formation in the field of the orbital electrons.[10,11] As used, here, the contribution of the triplets so-formed to the whole absorption coefficient amounts to about 3 percent for aluminum and decreases slowly to about 1 percent for lead. This method of including the triplet contribution is not rigorous, but it is probably in error by less than a factor of 2.[11]

Some of the secondary Compton quanta are sufficiently energetic to activate the detectors. By the definition of the cross section given by (2) these should not be counted, but since they are, a correction has been estimated and applied to the theoretical comparison values. The amount of the correction depends on the width of the spectral band used and the relative size of Compton to total cross section. It ranges from 0.18 to 1.6 percent of the total cross section and can be written in such form as to reduce the theoretical absorption coefficient in that proportion. The validity of the estimate may be questioned on the basis of lack of knowledge of the bremsstrahlung spectrum shape. Two reasonable choices showed that the correction is almost independent of this shape.

The mean energy ($\bar{E}$) of the spectral band used is the energy of the average quantum. The average quantum is found by weighting the relative number of quanta at each energy of an assumed bremsstrahlung spectrum with an estimated cross section variation. Reasonable choices for the spectrum and cross section produced very slightly different values for $\bar{E}$. The form used for the spectrum in the *forward* direction is one due to Schiff[12] which, expressed in intensity, is

$$\Gamma=8\{2(1-E/E_0)(\log\epsilon-1)+(E/E_0)^2(\log\epsilon-\tfrac{1}{2})\}, \quad (4)$$

where

$E=$quantum energy in spectrum,
$E_0=$electron energy generating spectrum,
$1/\epsilon^2=1/\epsilon_1^2+1/\epsilon_2^2$,
$\epsilon_1=(2E_0/mc^2)[(E_0/E)-1]$,
and $\epsilon_2=191Z^{-\frac{1}{3}}$.

When the cross sections have been calculated for $\bar{E}$ with the corrections indicated, the absorption coefficient is

$$\tau=N(\sigma_{\text{Compton}}+\sigma_{\text{pair}}), \quad (5)$$

where N is the number of atoms per cc of the absorber. N was determined from measurements of the dimensions and mass of each absorber.

DEVIATIONS FROM THEORETICAL PREDICTIONS

The measured absorption coefficients of both copper and lead are appreciably different from those calculated as above, whereas the agreement is quite good for aluminum and iron. It is felt that no important change in either the Compton or pair-production cross sections could occur between iron and copper; hence the discrepancy at copper should be ascribed to some other absorption process. Copper was specifically chosen as an absorber in the attempt to demonstrate the absorption resulting from the (γ,n) process. If the indicated discrepancy, which involves a cross section of about 10^{-25} cm^2, is ascribed to

[10] F. Perrin, Comptes rendus **197**, 100 (1933); L. V. Groshev, J. Phys. U.S.S.R. **5**, 135 (1941); P. Nemirovsky, J. Phys. U.S.S.R. **11**, 94 (1947).
[11] K. M. Watson, Phys. Rev. **72**, 1060 (1947).
[12] Private communication from L. I. Schiff to D. W. Kerst.

this process, the agreement with other determinations[6,13] is good for the highest energy but otherwise poor. More data will be required to ascertain this behavior, especially relative to absorption in iron.

The discrepancy in lead, on the other hand, indicates the absorption in *less* than that predicted by theory. The correction for screening would appear in this direction, but theoretically (see reference 5(b), pp. 197–198, 201) this is not appreciable below 25 Mev in lead. Thus, it would appear that the accuracy of the formulas (2) and (3) should be questioned.

The Klein-Nishina formula is an exact calculation on which many detailed tests have been performed[3] at lower energies. One would feel that this formula should still be accurate in this energy range.

The calculation of the pair-production cross section, however, involves the Born approximation in which it is assumed that $Ze^2/\hbar v \ll 1$. In the case of lead, the least value of this quantity is $82/137=0.60$, which does not fulfill the stated assumption. Thus, it seems reasonable that this formula may be in error. If all of the discrepancy is attributed to the pair-production calculation, then the fractional discrepancy in the theoretical pair-production cross section may be written:

$$\delta = 1 - \frac{(\tau/N) - \sigma_{\text{Compton}}}{\sigma_{\text{pair}}},$$

where τ is the measured absorption coefficient and σ_{Compton}, σ_{pair} are both theoretical values. The energy dependence of the discrepancy is well fitted by

$$\delta = 0.0482 \exp(0.0277\bar{E}/mc^2),$$

as indicated in Table III. It is possible that the agreement in the case of iron absorbers is just chance agreement occasioned by equality in magnitude of the discrepancy δ at $Z=26$ and, typically, the atomic (γ,n) cross section. If this is not the case, and if the leading coefficient in δ contains a Z^n term, the agreement in iron rules out n smaller than 3. Experiments with other elements, preferably of $Z>60$, will help determine this dependence. It should be noted that at the highest energy the discrepancy in lead is larger than the whole Compton effect contribution by about 3 times the experimental probable error. Thus, one is at least forced to admit some inaccuracy in the pair-production calculations. Preliminary experiments using uranium absorbers show a larger discrepancy δ with a similar energy dependence. More work is required here to obtain a value for the absorption coefficient which has a reasonably small experimental error.

[13] J. McElhinney (to be published).

TABLE III. The discrepancy, δ, in lead at three mean energies.

$\bar{E}$ (Mev)	11.04	13.73	19.10
δ exptl.	0.088	0.100	0.136
δ fitted	0.0878	0.1022	0.1360

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Professor D. W. Kerst for stimulating criticisms during the course of the experiments, and to Professor A. T. Nordsieck for discussions mainly trying to understand the nature of the discrepancy found here. He wishes to thank Mr. R. K. Clark for assisting in part of the early experiments, and Mr. R. W. Dressel for his very capable workmanship and assistance in planning and making the recent measurements. These recent experiments have been supported by the Office of Naval Research.