Angular Dependence of Coincidences between Fission Neutrons*

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Coincidences between the neutrons from neutron-induced fission of U^{235} were measured using fast neutron counters, working on the principle of proton recoil. From measurements at angles of 30°, 60°, 90°, 135°, and 180° between the directions of the fission neutrons, it was found that the number of coincidences is fairly constant from 30° to 90°, and increases by about a factor 2 from 90° to 180°. This result can be explained if it is assumed that the fission neutrons most probably come from opposite fragments.

IT is known that the phenomenon of fission is accompanied by the emission of neutrons, and from the existence of self-sustaining chain reactions it can be inferred that more than one neutron is emitted, on the average, during each fission process. On the other hand, it has been shown[1] that there exists an angular correlation between the direction of the neutrons and the direction of the fragments, the neutrons being preferentially emitted in the same direction as the fragments. These facts lead one to believe that coincidences between neutrons from a fission source should be detectable, and that a measurement of the angular distribution between neutron-neutron coincidences could yield information pertinent to the mechanism of emission of fission neutrons. The experiments described in this paper were performed to measure this angular distribution of neutron-neutron coincidences from a source of U^{235} irradiated with neutrons.

INSTRUMENTATION

The fission source consisted of a U^{235} radiator bombarded by a beam of neutrons emerging from a hole in the concrete shield of the Oak Ridge pile. The fast neutrons from the source were detected by two proportional counters mounted in a plane, perpendicular to the beam in such a way that their angular separation could be conveniently changed by rotating one of the counters around the axis defined by the beam itself. After amplification the pulses from the two counters were fed to a coincidence circuit.

A view of the experimental arrangement is shown in Fig. 1. The beam from the pile, containing slow and fast neutrons and γ-rays, could be intercepted by a heavy shutter, sliding along the pile wall, for the protection of the experimenters working on the equipment. When allowed to emerge from the pile, the beam was collected in an appropriate catcher. The location, as well as the collimation of the beam, was determined by exposing photographic plates.

The U^{235} sample was situated in a thin brass ring (visible in Fig. 1 between the counters) at the center of the beam, which, at that position, has a diameter of $\sim$5 cm.

The counters are proportional counters filled with a mixture of A and CO_2 at about 2 atmospheres of pressure. They work on the principle of proton recoil, the hydrogeneous material being present in the form of 20 polythene films (5

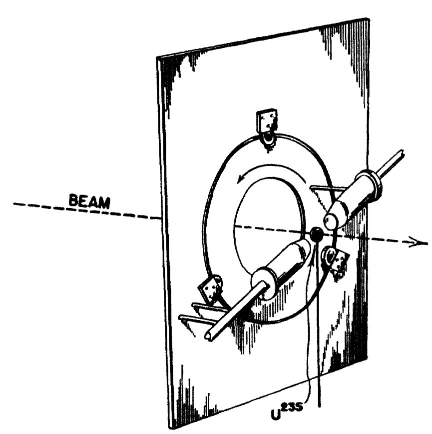

FIG. 1.

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[1] R. R. Wilson, Phys. Rev. 72, 189 (1947).

Table I.

Beam shutter closed	55 counts/min.
Beam shutter opened, no radiator	420 counts/min.
Beam shutter opened, Pb scatterer in place of radiator	529 counts/min.
Beam shutter opened, Cd covered Pb scatterer	558 counts/min.
Beam shutter opened, U^{235} radiator	26130 counts/min.

Table II. U^{235} radiator—observed angular distribution of $n-n$ coincidences.

Angular separation	30°	60°	90°	135°	180°
Counts per minute	2.39	1.33	1.22	1.83	2.40
Probable statistical error	0.12	0.11	0.08	0.11	0.09

mg/cm² thick) perpendicular to the wire and spaced at a distance of $\frac{5}{32}''$. Because of the high pressure, the recoil protons produce a large number of ions in the space between two successive films and, in general, give pulses larger than those from secondary electrons of γ-radiations. With proper setting of amplification and pulse height discrimination, the efficiency of these counters was found to be of the order of 1 percent for the fast neutrons from a PoBe source, while the efficiency for γ-rays is much smaller. The efficiency obviously depends on the energy of the neutrons; however, the thickness of the polythene films was chosen in such a way as to minimize the variation of efficiency for neutrons of energy above 1 Mev.

The amplifiers used are of the type described by Jordan and Bell.[2] The pulse height selectors of these amplifiers had been replaced by a circuit designed to minimize the variable delays resulting from the different size of the pulses accepted. After pulse height selection the pulses were fed to a coincidence circuit[3] whose resolving time could be varied from $\sim 10^{-7}$ to $\sim 10^{-6}$ sec.

DESCRIPTION OF MEASUREMENTS AND RESULTS

A typical set of single counting rates obtained with the counters located as shown in Fig. 1 is given in the following Table I.

These data show that the greatest part of the counts recorded with the U^{235} radiator were due to the fast neutrons which originated in this source, and that the background resulting from uncollimated or scattered fast neutrons from the beam, and to γ-rays, could be neglected. The fact that very few γ-rays from the U^{235} sample were counted was verified by the dependence of the counting rate on pulse height selector setting.

Because of the low efficiency of the counters and the small solid angle that they covered, the expected number of coincidences was about 2 per minute. It was of the utmost importance, therefore, to reduce the random coincidences to a minimum.

For this purpose a preliminary experiment was performed to establish the minimum value of resolving time which could be used without losing the true neutron-neutron coincidences.

In order to measure the resolving time two U^{235} samples were placed in the neutron beam, at a distance of about one meter; each one of the two counters was located close to one of these samples, so that they would record the fast neutrons from two independent fission sources. From a measurement of random coincidences in these conditions, the resolving time was obtained for different values of the circuit time-constants. For these same values, the true coincidences were measured (after subtracting the random rate) with the counters at an angular separation of 180°, as in Fig. 1. In Fig. 2 the number of true coincidences is plotted as a function of resolving time.

This preliminary experiment showed that one could not take full advantage of the speed of the electronic circuit since a considerable fraction of true coincidences were lost at resolving times smaller than 0.3 μsec. (probably as a consequence of variable lag times in the proportional counters). Since it was not practical to reduce the intensity of the source, nor to increase the solid angle covered by the counters, the measurement had to be performed with a random coincidence background considerably larger than the effect to be studied. This situation required long runs to collect statistically significant results, with numerous precautions to avoid errors caused by variations in the performance of the instrument or in the power level of the pile.

All the measurements were performed with the pile at constant nominal power level, and in the last part of the experiment the flux of the beam was monitored at least once every day by means

[2] W. H. Jordan and P. R. Bell, Rev. Sci. Inst. **18**, 703 (1947).
[3] P. R. Bell, S. DeBenedetti, and J. E. Francis, Jr., Phys. Rev. **72**, 160 (1947).

of In foils. It was found to be constant within a few percent.

After a first measurement of the ratio of coincidences at 90° and 180°, two series of measurements were performed, one for angular separations of 60°, 90°, 135°, and 180°, and the other for 30°, 90°, and 180°. Each series lasted a week or more, and consisted of a repetition of individual runs, each including the whole range of observations. A run consisted of counts of coincidences at different angular separation, alternated with counts of random coincidences (performed with different radiators for each counter as described above). A count of coincidences lasted 20 minutes and was preceded and followed by counts of single pulses in each counter. The coincidence counts were alternated in such a way as to minimize the effects of slow instrumental variations.

Slow variations in the single counting rate, caused by changes in counter efficiency, were observed; these were compensated by adjusting the counter voltage, and the single counting rate was kept close to 420×64 counts/min. The single counting rates before and after each coincidence measurement were averaged, and the coincidence reading was reduced to 420×64 single counts/min., assuming that both true and random coincidence varied in proportion to the product of the single counting rates. This correction, however, seldom exceeded 10 percent.

The results of the individual runs were averaged and analyzed statistically to make sure that the deviations did not exceed the statistical errors. From the random coincidences recorded it was found that the resolving time of the instrument was remarkably constant over periods of months; also the number of true coincidences for the same angular separation did not show variations larger than the expected statistical fluctuations. The consistency of the data, despite the high number of random coincidences and the long time involved in the measurement, can be regarded as a check on the reliability of the final results.

The possibility that scattering from surrounding materials (floor, walls, etc.) could falsify the results was considered. For this reason measurements for the same angular separation were repeated at different orientations; no appreciable difference was observed, indicating that this cause of errors could be disregarded.

Table II shows the experimental results obtained, after subtraction of the random coincidence rate of 7.2 counts/min.

The number of coincidences detected has a minimum at 90°, and apparently maxima at both 0° and 180°.

One can expect, however, that some of the coincidences at small angular separation are due to the same neutron which, after producing a recoil in the first counter, is scattered into the other. Thus the angular distribution of the coincidences does not necessarily correspond to the angular distribution of the direction of emission of distinct fission neutrons. In order to investigate this effect, and eventually correct the results for it, some measurements were performed with a Po−Be source of the same strength as the U^{235} radiator, replacing the U^{235}. Since neutrons are emitted individually by this source, all the coincidences observed above the random background are to be attributed to scattering. The results obtained for the same single counting rate as for the U^{235} radiator are:

	Po−Be source	
Angular separation	30°	180°
Coincidences per minute	8.24	7.02
Probable statistical error	0.10	0.09

It can be seen that no coincidences in excess of the random rate are observed at 180°, while a noticeable effect caused by scattering is present at 30°; at this angle the scattering contributes 1.22 ± 0.13 coincidences per minute.

Because of the difference in energy distribution, the effect of scattering cannot be expected to be exactly the same for fission neutrons and for PoBe neutrons; however, it seemed admissible to

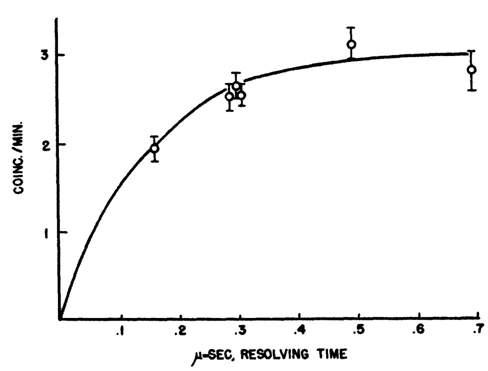

FIG. 2.

TABLE III. U^{235} radiator—angular distribution of $n-n$ coincidences after correction for scattering.

Angular separation	30°	60°	90°	135°	180°
Coincidences per minute	1.17	1.33	1.22	1.83	2.40
Probable statistical error	0.17	0.11	0.08	0.11	0.09

assume that the difference was not too large and to subtract the scattering effect, as measured from PoBe, from the angular distribution of the fission neutrons.

The angular distribution of the coincidences between two distinct fission neutrons corrected for scattering is plotted in Fig. 3 and reported in Table III.

The coincidence rate appears to be fairly constant from 0° to 90° and to increase by a factor 2 from 90° to 180°.

DISCUSSION OF RESULTS

The experimental results obtained can be expressed by stating that fission neutrons *are preferentially emitted in opposite directions.* If one assumes that the neutrons most likely go in the direction of the fragment from which they are emitted, it follows that fission neutrons *are preferentially emitted by opposite fragments.* A more quantitative reasoning (Appendix, formula 12) shows that, since the ratio of coincidences recorded for angular separations of 180° and 0° is roughly 2, *there are at least twice more neutron pairs emitted by opposite fragments then neutron pairs emitted by the same fragment.*

The angular distribution theoretically expected in the assumption of isotropic emission of neutrons by fragments in motion is discussed in some detail in the Appendix to this paper. In order to take full advantage of this discussion, however, one must make some assumptions on the spectrum of fission neutrons and, therefore, a more detailed analysis of the experimental results must be postponed until some data on the fission neutron spectrum is generally available.

It is appropriate at this point to mention that, as has been pointed out by Peierls,[4] the hypothesis of isotropic emission is inconsistent with some of the observed facts. Though it is reasonable to admit that the fission neutrons are emitted by the fragments in motion, it seems doubtful that the angular correlations can be explained entirely with the composition of neutron and fragment velocities, without assuming any anisotropy in the emission of neutrons in the fragment's system.

In the extreme case where the anisotropy in the fragment's system would be the most important cause of angular correlation it is obviously possible to make assumptions which will invalidate our conclusion about the preferential emission of neutrons by opposite fragments. The preferential emission in opposite directions could be explained, for instance, supposing that neutrons are emitted by the same fragment, but preferentially either

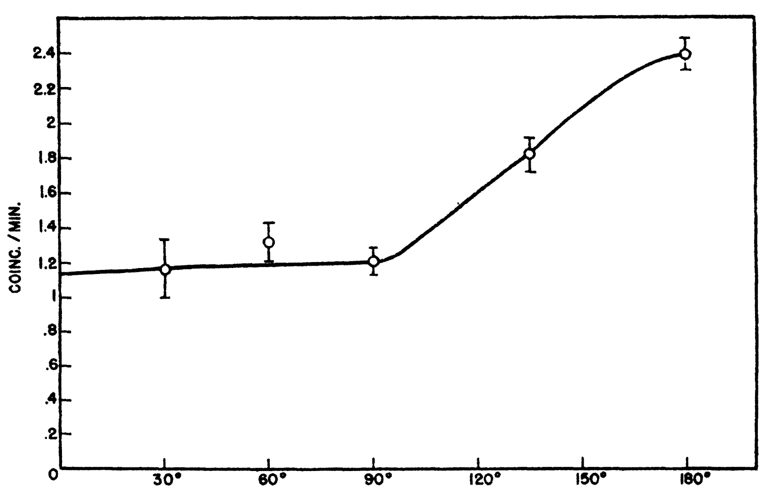

FIG. 3.

[4] R. Peierls, declassified British report, B-103.

from the region of the broken "fission neck" (where a large amount of deformation energy should be available) or from the diametrically opposite region (where the energy could be concentrated through ripple propagation). Any such hypothesis, however, seems so speculative, that the most reasonable interpretation of the experimental data still remains to suppose that, when more than one neutron is produced in fission, the different neutrons are most likely to be emitted by opposite fragments.

APPENDIX

Theory of the Experiment on the Assumption of Isotropic Emission in the Fragment's System

The outline of a theory which could eventually be used for the interpretation of the experimental data if the fission neutron spectrum was known is presented in this appendix. The following assumptions are made:

(a) The neutrons are emitted (evaporated) isotropically from the fragments in motion.
(b) The velocity of the fragments at the time of neutron emission is the same for all neutrons and is equal to the velocity which is obtained from the average fragment kinetic energy assuming fragments of equal mass.
(c) The energy spectrum of the neutrons in the fragment's system is the same for all neutrons (and does not depend on the number of neutrons emitted).

If η is the energy of a neutron in the fragment's system (*emission energy*), the energy E of the same neutron in the laboratory system is

$$E(\eta,\vartheta)=\eta[1-r^2+2r^2\cos^2\vartheta+2r\cos\vartheta(1-r^2+r^2\cos^2\vartheta)^{\frac{1}{2}}],\quad (1)$$

where ϑ is the angle (in the laboratory system) between the direction of the neutron and that of the fragment, and r is the ratio of the fragment's velocity to the neutron emission velocity.

If $\Phi(\eta)$ is the energy distribution of the neutrons in the fragment's system (emission neutron spectrum), the spectrum $N(E)$ of the neutrons in the laboratory system is given by[4]

$$N(E)=\text{const.}\times\int_{E(1-R)^2}^{E(1+R)^2}\Phi(\eta)\eta^{-\frac{1}{2}}d\eta,\quad (2)$$

where R is the ratio of the fragment's velocity to the neutron velocity in the laboratory system. We will assume in what follows that $\Phi(\eta)$ is determined from (2), by comparison with the experimental data on the laboratory fission neutron spectrum.

The probability per unit solid angle that neutrons of emission energy η emerge at an angle ϑ is

$$f(\eta,\vartheta)=\frac{1}{4\pi}(E/\eta)^{\frac{1}{2}}[1+r(1-(E/\eta)\sin^2\vartheta)^{\frac{1}{2}}]^{-1}.\quad (3)$$

If experiments on angular correlation are performed with neutron counters of known efficiency $\epsilon(E)$, in the ideal case

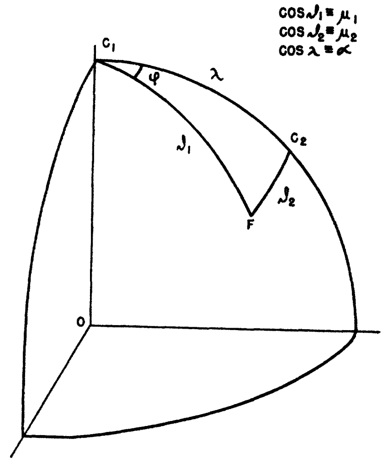

FIG. 4.

of *one* fragment emitting one neutron per unit time, the counting rate per unit solid angle as a function of ϑ is

$$\bar{\epsilon}F(\vartheta)=\int_0^\infty\Phi(\eta)f(\eta,\vartheta)\epsilon[E(\eta,\vartheta)]d\eta\quad (4)$$

where, with the normalization condition

$$\int F(\vartheta)d\omega=1,\quad (5)$$

$\bar{\epsilon}$ assumes the meaning of average counter efficiency, while (4) and (5) serve to define $F(\vartheta)$.

In this notation the angular distribution of neutron-fragment coincidences, as in Wilson's experiment, is given by const. $\times[F(\vartheta)+F(180°-\vartheta)]$, since there are two opposite fragments in each fission.

One can proceed now with the calculation of the angular distribution of neutron-neutron coincidences, studying first the case of emission of one neutron pair per fission. Let us consider (Fig. 4) a system of polar coordinates with its center in the neutron source and the polar axis in the direction of the first counter C_1. Let us measure the longitude φ from the plane C_1OC_2 (C_2 denoting the position of the second counter). A fragment emerging in the direction defined by the coordinates ϑ_1 and φ will be at an angle ϑ_2 with the second counter. Let ω be the solid angle covered by each of the two counters and let us define for brevity $\mu_1=\cos\vartheta_1$, $\mu_2=\cos\vartheta_2$, $\alpha=\cos\lambda$, where λ is the angular separation between the counters. If both neutrons of the pair are emitted by the same fragment the number of coincidences per fission is

$$c(\mu_1,\mu_2)=2\omega^2\bar{\epsilon}^2F(\mu_1)F(\mu_2)\quad (6)$$

while, if the two neutrons are emitted by opposite fragments, we have

$$c'(\mu_1,\mu_2)=\omega^2\bar{\epsilon}^2[F(\mu_1)F(-\mu_2)+F(-\mu_1)F(\mu_2)]\quad (7)$$

coincidences per fission.

However, we are interested in the number of coincidences per fission as a function of the angle λ between the two counters, for any arbitrary direction of the fragments. This will be expressed by averaging over the total solid angle so that

$$\begin{cases} c(\alpha)=\frac{1}{4\pi}\int c[\mu_1, \mu_2(\mu_1, \alpha, \varphi)]d\mu_1 d\varphi, \\ c'(\alpha)=\frac{1}{4\pi}\int c'[\mu_1, \mu_2(\mu_1, \alpha, \varphi)]d\mu_1 d\varphi. \end{cases} \quad (8)$$

If

$$F(\mu)=\Sigma\, a_l P_l(\mu) \quad (9)$$

represents the development of $F(\mu)$ in series of spherical harmonics, using the addition theorem, one obtains

$$c(\alpha)=2\omega^2\bar{\epsilon}^2\,\Sigma\,\frac{a_l^2}{2l+1}P_l(\alpha),$$
$$c'(\alpha)=2\omega^2\bar{\epsilon}^2\,\Sigma\,(-1)^l\frac{a_l^2}{2l+1}P_l(\alpha)=c(-\alpha). \quad (10)$$

Finally, if p_1 and p_2 are the average numbers of neutron pairs, respectively, originating from the same fragment and from opposite fragments, the number of coincidences per fission becomes

$$C(\alpha)=p_1 c(\alpha)+p_2 c(-\alpha). \quad (11)$$

This formula shows that, if $F(\vartheta)$ (and therefore $c(\alpha)$) is known, one can obtain the ratio p_1/p_2 from a measurement of coincidences at two different angles.

However, even without any assumptions on the fission neutron spectrum and on $F(\vartheta)$, one can obtain an upper or lower limit for the ratio p_1/p_2 simply considering that $c(\alpha)$ has a maximum for $\alpha=1$. In effect, from (11) one obtains

$$\frac{p_1}{p_2}=\frac{C(1)/C(-1)-c(-1)/c(1)}{1-c(-1)C(1)/c(1)C(-1)},$$

and, considering that both numerator and denominator are always positive, one has $p_1/p_2 \gtreqless 1$ for $C(1)/C(-1) \gtreqless 1$. Also from (11) one can write

$$C(1)/C(-1)=(p_1/p_2+c(-1)/c(1))/(p_1 c(-1)/p_2 c(1)+1),$$

and this leads to

$$\frac{p_1}{p_2}\gtrless C(1)/C(-1) \quad \text{for} \quad C(1)/C(-1)\gtrless 1. \quad (12)$$

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Microwave Breakdown of a Gas in a Cylindrical Cavity of Arbitrary Length*

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It is possible to consider breakdown in a cylindrical microwave cavity whose radius is large compared to its length as approaching the conditions of parallel plate breakdown. This assumption has been used to measure high frequency ionization coefficients. The present paper investigates the corrections to be made when the length is increased. Numerical results are given for cavities whose ratios of radius to length are as low as 0.5, and the method is applicable to any cylindrical cavity. The breakdown data in these longer cavities are used to extend the high frequency ionization coefficient curves for air by a factor of ten.

THE electrical breakdown of a gas at microwave frequencies has been discussed in two papers by the authors. The first[1] developed the principle of balancing the generation of electrons through ionization by collision against the loss of electrons through diffusion. The resulting breakdown criterion appeared as the solution of a characteristic value problem. A new ionization coefficient appropriate to the high frequency conditions was introduced. It is necessary to know this quantity as a function of the experimental conditions in order to compute the electric field for breakdown. Breakdown data were used to give experimental values of the ionization coefficient. The second paper[2] illustrated a computing technique for solving the boundary value problem for breakdown between coaxial cylinders. Comparison with experiment indicated that the breakdown theory is valid.

The present paper develops the breakdown criterion for the case of the TM_{010}-mode cylindrical cavity. The object of this computation is primarily to extend the range of the experimental data for the ionization coefficient beyond the region where the cavity height is small compared

* This work has been supported in part by the Signal Corps, the Air Materiel Command, and the O.N.R.

[1] M. A. Herlin and S. C. Brown, Phys. Rev. 74, 291 (1948).

[2] M. A. Herlin and S. C. Brown, Phys. Rev. 74, 910 (1948).