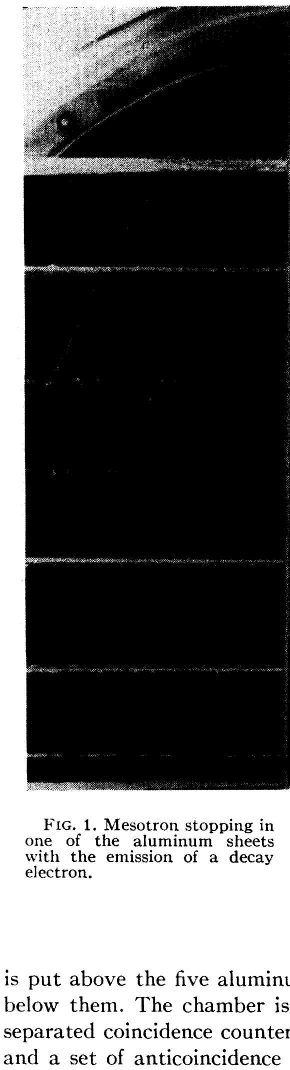

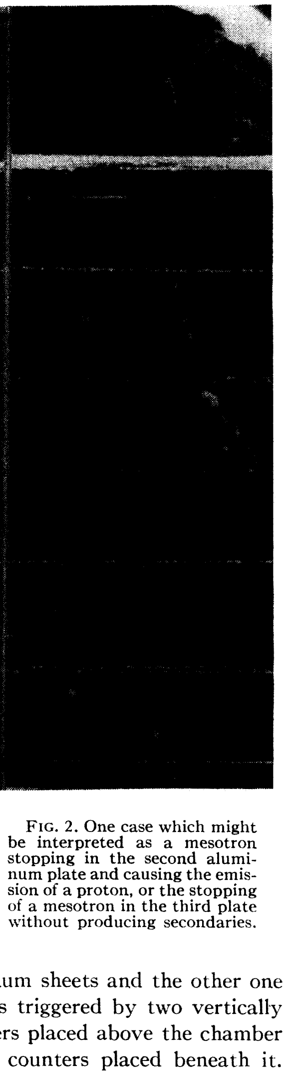

FIG. 1. Mesotron stopping in one of the aluminum sheets with the emission of a decay electron.

FIG. 2. One case which might be interpreted as a mesotron stopping in the second aluminum plate and causing the emission of a proton, or the stopping of a mesotron in the third plate without producing secondaries.

is put above the five aluminum sheets and the other one below them. The chamber is triggered by two vertically separated coincidence counters placed above the chamber and a set of anticoincidence counters placed beneath it. This method of triggering favors the detection of those charged particles which stop in the chamber. A lead block 11 inches thick is inserted between the two coincidence counters to filter out the electron component. The photographing is done with a stereoscopic type camera.

Up to the present, we have observed 40 mesotrons stopping within the cloud chamber. Of the 20 which stopped in the aluminum sheets, 12 showed decay particles of electronic character. Of the 20 mesotrons stopping in the lower lead plate, 7 showed decay particles of electronic character.

Figure 1 shows a mesotron stopping in one of the aluminum sheets with the emission of a decay electron. This decay electron penetrates through three aluminum sheets suffering scattering, which yields projected angles of 1.6°, 1.3°, and 1.5°, respectively. From the formula connecting the mean square of the projected scattering angles, the energy of the electron, and the thickness of the material through which the electron passes, we get from the above data an approximate value of the energy of the electron. The calculated energy is found to be 52 ± 10 Mev. In another case the decay electron passes through the first aluminum sheet and the upper lead plate and then goes out of the chamber. We calculate the lower limit of its energy to be about 10 Mev. These values are to be compared with those recently found by different authors.[1–6] For the rest of the decay electrons observed in our experiment, the visible tracks are shorter and are more unfavorable for energy estimation.

Figure 2 shows the one case which might be considered as the emission of a proton from the end of the mesotron track which stops in the second aluminum sheet. However, it might also be regarded as a mesotron stopping in the third aluminum sheet without producing any secondaries. From this and the 7 cases in which the mesotron is stopped in an aluminum sheet with no visible emission of a secondary, we conclude that either the probability of a mesotron producing a star after it stops in an aluminum sheet is rather small, or the energies of the prongs of the star are so small that the prongs have very little chance of getting out of the 0.081 cm of aluminum. This is in agreement with the result of Chang.[7] It is also in agreement with the recent results of Occhialini and Powell,[8] who have found no definite evidence for the production of stars by μ-mesons.

It is clear that our result does not support the hypothesis of the accelerated decay of the negative mesotrons but rather favors the hypothesis that some of the negative mesotrons are captured by the aluminum nuclei,[9] since of the 20 mesotrons stopping in the aluminum sheets, we have observed 8 giving no decay electrons.

The writers wish to express their thanks to Professor R. B. Brode for his encouragement and continuous interest in this work. They are also indebted to Professor W. B. Fretter and Dr. J. G. Retallack for their suggestions and help.

* This work was supported in part by the Atomic Energy Commission and the Office of Naval Research.

[1] Anderson, Adams, Lloyd, and Rau, Phys. Rev. **72**, 724 (1947).
[2] Adams, Anderson, Lloyd, Rau, and Saxena, Rev. Mod. Phys. **20**, 334 (1947).
[3] E. C. Fowler, R. L. Cool, and J. C. Street, Phys. Rev. **74**, 101 (1948).
[4] J. L. Zar, J. Hershkowitz, and E. Berezin, Phys. Rev. **74**, 111 (1948).
[5] Robert W. Thompson, Phys. Rev. **74**, 490 (1948).
[6] J. Steinberger, Phys. Rev. **74**, 500 (1948).
[7] W. Y. Chang, Phys. Rev. **74**, 1236 (1948).
[8] G. P. S. Occhialini and C. F. Powell, Nature **162**, 168 (1948).
[9] John A. Wheeler, Phys. Rev. **71**, 320 (1947); G. E. Valley and B. Rossi, Phys. Rev. **73**, 177 (1948); J. G. Retallack, Phys. Rev. **73**, 921 (1948); Harold K. Ticho, Phys. Rev. **74**, 492 (1948).

On Feenberg's Perturbation Formula

HERMAN FESHBACH
Brookhaven National Laboratory, Upton, Long Island, New York*
September 20, 1948

IN two recent papers,[1] Feenberg has derived perturbation formulas in which there are no repetitive matrix elements in the construction of a term of given order. Feenberg's calculation proceeded by effecting summations and regroupings of various infinite processes occurring in the Brillouin[2]-Wigner[3] formulation of perturbation theory. The purpose of the present note is the derivation of Feenberg's formula directly from the eigenvalue problem by a method of successive approximations. The derivation also has the advantage of simplicity.

The equations to be solved for the coefficients are:

$$(E-H_{mm})a_m=\sum_{m\neq n} H_{mn}a_n, \quad (1)$$

where the original eigenvalue problem is $H\psi=E\psi$ expanded in an orthonormal set of functions, a_m are the amplitudes in the expansion, and H_{mn} are the matrix elements of H.

Suppose now that the principal state in question is the k^{th} so that it is convenient to take

$$a_k=1, \quad (2)$$

then

$$(E-H_{mm})a_m=H_{mk}+\sum_{n\neq mk} H_{mn}a_n. \quad (3)$$

To set up the method of successive approximations, we must now write the equation determining $a_n(n\neq m, k)$. To avoid repetitive matrix elements, we separate the m and k terms in the expression for a_n

$$(E-H_{nn})a_n=H_{nk}+H_{nm}a_m+\sum_{p\neq nmk} H_{nk}a_p. \quad (4)$$

To determine a_p

$$(E-H_{pp})a_p=H_{pk}+H_{pm}a_m+H_{pn}a_n+\sum_{q\neq pnmk} H_{pq}a_q. \quad (5)$$

For a_q

$$(E-H_{qq})a_q=H_{qk}+H_{qm}a_m+H_{qn}a_n+H_{qp}a_p+\sum_{r\neq qpnmk} H_{qr}a_r, \quad (6)$$

etc. To obtain a_m to first order we need only drop the summation in (3). To obtain to second order we need only drop the summation in (4) and solve for a_n, etc. We shall give the second- and third-order formulas obtained in this way. It is convenient to define the symbols

$$(\mathcal{E}_{km}{}^*)_1=H_{mm}(\mathcal{E}_{km}{}^*)_2=H_{mm}+\sum_{n\neq km}\frac{H_{mn}H_{nm}}{E-(\mathcal{E}_{kmn}{}^*)_1},$$

$$(\mathcal{E}_{km}{}^*)_3=H_{mm}+\sum_{n\neq km}\frac{H_{mn}H_{nm}}{E-(\mathcal{E}_{kmn}{}^*)_2}+\sum_{\substack{n\neq km\\ p\neq kmn}}\frac{H_{mn}H_{np}H_{pm}}{[E-(\mathcal{E}_{kmn}{}^*)_2][E-(\mathcal{E}_{kmnp}{}^*)_1]}. \quad (7)$$

Then to *second order*

$$a_m[E-(\mathcal{E}_{km}{}^*)_2]=H_{mk}+\sum_{n\neq mk}\frac{H_{mn}H_{nk}}{E-(\epsilon\mathcal{E}_{mn}{}^*)_1}. \quad (8)$$

To *third order*

$$a_m[E-(\mathcal{E}_{km}{}^*)_3]=H_{mk}+\sum_{n\neq mk}\frac{H_{mn}H_{nk}}{E-(\mathcal{E}_{kmn}{}^*)_2}+\sum_{\substack{n\neq mk\\ p\neq nmk}}\frac{H_{mn}H_{np}H_{pk}}{[E-(\mathcal{E}_{kmn}{}^*)_2][E-(\mathcal{E}_{kmnp}{}^*)_1]}. \quad (9)$$

These formulas are precisely those of Feenberg's to the order indicated. The general formula obtained by him is given by replacing $(\epsilon_k\ldots{}^*)_i$ by $(\epsilon_k\ldots{}^*)_\infty$. Feenberg uses $\epsilon_k\ldots{}^*$ for this symbol. The general expression may be derived here but the details are lengthy and uninformative. It may be noted that the procedure rather obviously yields an exact expansion for any finite secular equation.

The writer is greatly indebted to Dr. Eugene Feenberg for pointing out the relevance of this particular derivation. This work was done while serving as a consultant at the Brookhaven National Laboratory, whose hospitality is hereby gratefully acknowledged.

* Research carried out at Brookhaven National Laboratory under the auspices of the Atomic Energy Commission while author was on summer leave from the Massachusetts Institute of Technology.

[1] E. Feenberg, Phys. Rev. **74**, 206 (1948); E. Feenberg, Phys. Rev., in press.
[2] L. Brillouin, J. de phys. et rad. **III**, 373 (1932).
[3] E. P. Wigner, Mathu-Naturwirs. Anzerg. d. Ungar Akad Wiss **LIII**, 475 (1935).

Penetrating Power of Extensive Shower Particles*

G. T. Reynolds and W. D. Hardin
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
October 4, 1948

An experiment has been performed to measure the penetrating power in lead of extensive shower particles. This experiment differed from others of a similar purpose in that an extensive shower was first selected by two Geiger counter trays which had nothing to do with the measurement of penetration, and then the behavior of the particles in the shower was analyzed by means of six other Geiger counter trays under various thicknesses of lead. This procedure guaranteed that a decrease in counting rate observed with increasing thickness of lead was a true measure of particle penetrating power, and not an effect due to the exclusion of certain classes of showers. Thus, the only requirement imposed on a particle was that it be associated with an extensive shower.

This method of observation had the experimental advantage that it allowed simultaneous sampling under several different thicknesses of lead, an advantage because of the low counting rate encountered in extensive shower detections. The counter array is shown in Fig. 1. Trays M_1

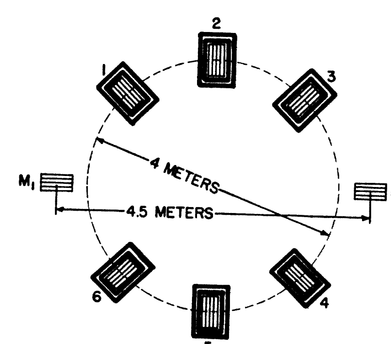

FIG. 1. Disposition of counter trays.

and M_2 are the master coincidence trays which selected the shower. They are separated by about 4.5 meters. Trays 1 through 6 were in lead boxes, spaced evenly along the