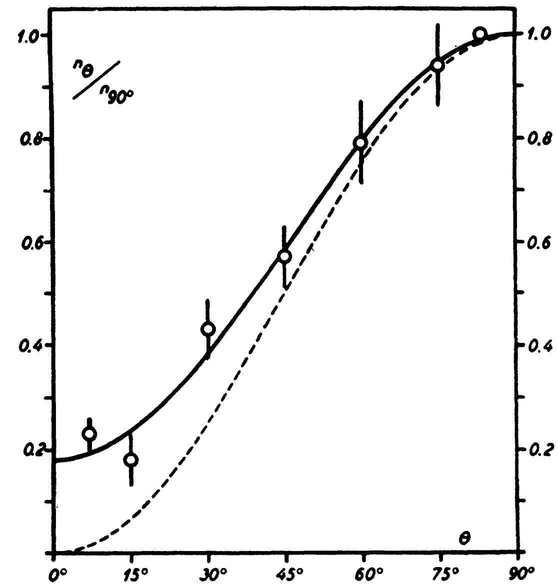

FIG. 2. Angular distribution of photo-protons.

to the different energies of forward and backward directed protons could have been expected, but the limited statistical material and the mentioned spread in pulse sizes did not allow their separation. In fact, for small values of θ the position of the peak was not very well defined, and therefore the number n_θ of pulses higher than 40 mm (see Fig. 1) was counted for the various values of θ. The result of the measurements, in which ^{24}Na was used as a γ-ray source, is given in Fig. 2, where the full-drawn curve corresponds to the formula

$$n_\theta/n_{90°}=a+b\sin^2\theta=0.18+0.82\sin^2\theta.$$

The ratio $a/(a+b)$ is found to be 0.18 ± 0.03, the ratio a/b to be 0.22 ± 0.04, in good agreement with the value 0.26 ± 0.08 given by Graham and Halban.[5] The present experiments give $\sigma_m/\sigma_e=0.33$ and $\sigma_m/\sigma_{\text{total}}=0.25$. The dotted curve in Fig. 2 corresponds to $\sigma_m=0$, and the figure clearly shows that there can be no doubt that both the photomagnetic and the photoelectric effect exist for the γ-energy concerned.

When determining the absolute cross section it is necessary to know the absolute strength of the γ-source and the range ρ of the protons. The author is indebted to Mr. Koefoed-Hansen for measuring the absolute strength of a weak sample of ^{24}Na by means of a special counting arrangement. Unfortunately, ρ is not known very accurately and, hence, the result may be given as follows, where a correction of +20 percent resulting from the absorption of the γ-rays and a correction of −20 percent resulting from the foot on the pulse-size distribution curve are included:

$$\sigma_{\text{total}}=0.8\cdot10^{-27}\cdot\frac{9\rho^2}{11-3\rho}\text{ cm}^2\pm20\text{ percent},$$

where ρ is the range in cm deuterium. Putting $\rho=1.5$ cm we get $\sigma_{\text{total}}=2.5\cdot10^{-27}$ cm^2 but, as we have seen, the result depends strongly on the value of ρ chosen.

A more detailed account of the work will soon be published in the Communications of the Danish Academy of Science.

[1] Chadwick, Feather, and Bretcher, Proc. Roy. Soc. **A163**, 366 (1937).
[2] Halban, Nature **141**, 644 (1938).
[3] R. J. Richardson and L. Emo, Phys. Rev. **53**, 234 (1938).
[4] F. E. Meyers and L. C. Van Atta, Phys. Rev. **61**, 19 (1942).
[5] G. A. R. Graham and H. Halban, Jr., Rev. Mod. Phys. **17**, 297 (1945).

Multiple Scattering with Energy Loss

LEONARD EYGES
University of Birmingham, Birmingham, England

September 23, 1948

FERMI[1] has found the distribution function for the lateral and angular displacements of charged particles which undergo multiple elastic scattering in passing through a layer of matter. In his treatment the energy loss which the particles suffer due to ionizing collisions is neglected. In this note we show that energy loss can be taken into account to a good approximation, and that this leads to a simple generalization of Fermi's distribution function.

The diffusion equation for the distribution function $F(t, y, \theta)$ is, in Rossi and Greisen's[1] notation and units,

$$\frac{\partial F}{\partial t}=-\theta\frac{\partial F}{\partial y}+\frac{1}{W^2}\frac{\partial^2 F}{\partial\theta^2}, \qquad (1)$$

where $W=2p\beta/E_s$. *We assume that p and β are functions of t*, i.e., we neglect the fact that a particle at t has traveled a somewhat greater distance than t due to the deviations caused by scattering. For the multiple scattering of high energy particles these deviations will be small and the approximation will be a good one. In (1) then we assume that W^2 is some known function of t, although not necessarily one for which there is an analytic expression. If we apply the Fourier transforms

$$F(t, y, \theta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}G(t, x, \zeta)\exp(i(xy+\zeta\theta))dxd\zeta, \qquad (2)$$

$$G(t, x, \zeta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(t, y, \theta)\exp[-i(xy+\zeta\theta)]dyd\theta, \qquad (3)$$

to (1) we are led to the equation for $G(t, x, \zeta)$,

$$\frac{\partial G}{\partial t}=x\frac{\partial G}{\partial\zeta}-\frac{\zeta^2}{W^2(t)}G. \qquad (4)$$

After introducing the two new variables

$$\xi=t+\zeta/x,\quad t'=t, \qquad (5)$$

Eq. (4) becomes:

$$\frac{\partial G}{\partial t'}=-\frac{x^2(\xi-t')^2}{W^2(t')}G. \qquad (6)$$

The solution of (6) is

$$G=H(\xi)\exp\left[-x^2\int_k^{t'}\frac{(\xi-\eta)^2}{W^2(\eta)}d\eta\right], \qquad (7)$$

where $H(\xi)$ is the "constant" of integration, and for convenience we have taken the lower limit in the integral to be some fixed number K. The boundary condition is that at $t=0$ there is one particle incident normally at $y=0$, i.e.,

$$F(0, y, \theta)=\delta(y)\delta(\theta). \tag{8}$$

The condition on G is then from (3)

$$G(0, x, \zeta)=1/2\pi. \tag{9}$$

If we write out (7) in terms of the original variables x, ζ, t, we get

$$G(t, x, \zeta)=H(t+\zeta/x)\exp\left[-x^2\int_k^t\frac{(t+\zeta/x-\eta)^2}{W^2(\eta)}d\eta\right]. \tag{10}$$

From (9) we see that

$$H(\zeta/x)=\frac{1}{2\pi}\exp\left[x^2\int_k^0\frac{(\zeta/x-\eta)^2}{W^2(\eta)}d\eta\right]. \tag{11}$$

Equation (11) gives the functional form of H, and therefore G is completely determined from (10) as

$$G(t, x, \zeta)=\frac{1}{2\pi}\exp\left[-x^2\int_0^t\frac{(\zeta/x+t-\eta)^2}{W^2(\eta)}d\eta\right]. \tag{12}$$

If we define three new functions of t by

$$A_0(t)=\int_0^t\frac{d\eta}{W^2(\eta)}, \tag{13a}$$

$$A_1(t)=\int_0^t\frac{(t-\eta)d\eta}{W^2(\eta)}, \tag{13b}$$

$$A_2(t)=\int_0^t\frac{(t-\eta)^2}{W^2(\eta)}d\eta, \tag{13c}$$

we can write G in the simple form

$$G(t, x, \zeta)=\frac{1}{2\pi}\exp[-(A_0\zeta^2+2A_1\zeta x+A_2x^2)].$$

This expression for G can be put into (3) and the integrations with respect to x and ζ carried out, with the result

$$F(t, y, \theta)=\frac{1}{2\pi(B(t))^{\frac{1}{2}}}\exp\left(-\frac{\theta^2A_2-2y\theta A_1+y^2A_0}{4A_0B}\right), \tag{14}$$

where $B(t)=A_0A_2-A_1^2$. If we assume that W^2 is constant, it is easy to see that (4) reduces to the Fermi solution as given by Rossi and Greisen. If we integrate over y, we get for the angular distribution irrespective of displacement

$$\int_{-\infty}^{\infty}F(t, y, \theta)dy=\frac{1}{2(\pi A_0)^{\frac{1}{2}}}\exp\left(-\frac{\theta^2}{4A_0}\right). \tag{15}$$

Similarly, for the lateral distribution independent of angle

$$\int_{-\infty}^{\infty}F(t, y, \theta)d\theta=\frac{1}{2(\pi A_2)^{\frac{1}{2}}}\exp\left(-\frac{y^2}{4A_2}\right). \tag{16}$$

We can gain a little insight into our formula by comparing (15) and (16). The angular distribution defined by (15) depends on t through A_0, i.e., through $\int_0^t(d\eta/W^2(\eta))$, whereas the radial distribution (16) depends on t through $\int_0^t[(t-\eta)^2/W^2(\eta)]d\eta$. Because of the factor $(t-\eta)^2$ large values of $t-\eta$ are weighted more heavily in the last integral than in the preceding one. This is caused by the fact that a given angular deflection produces a larger *radial* displacement at t the farther from t that it occurs, whereas all angular deflections at any intermediate thickness contribute equally to the total angular deflection at t.

In general the integrals A_1, A_1, A_2 which determine the distribution can be obtained by simple numerical integrations using the range *vs.* momentum curves given, e.g., in Rossi and Greisen's article. In some special cases the A's can be found by direct integration. If, for example, we consider particles with energy much greater than their rest energy, it is a good approximation to assume the rate of momentum loss is independent of the momentum and that $\beta=1$. Then the momentum p of a particle at depth t radiation lengths is $p_0-\epsilon t$, where ϵ is the constant momentum loss per radiation length. Then $W^2(t)=[4(p_0-\epsilon t)^2/E_s^2]$ and the A's become:

$$A_0(t)=\frac{E_s^2t}{4p_0(p_0-\epsilon t)}, \tag{17a}$$

$$A_1(t)=\frac{E_s^2}{4\epsilon^2}\left[\ln\frac{p_0}{p_0-\epsilon t}-\frac{\epsilon t}{p_0}\right], \tag{17b}$$

$$A_2(t)=\frac{E_s^2}{4\epsilon^2}\left[2t-\frac{t^2\epsilon}{p_0}-2\frac{(p_0-\epsilon t)}{\epsilon}\ln\frac{p_0}{p_0-\epsilon t}\right]. \tag{17c}$$

It is interesting to note that if one wished to take energy loss roughly into account using Fermi's original distribution function by taking some *mean* value for $W^2(t)$ that this mean value could not be simultaneously correct for both (15) and (16), i.e., for both the angular and radial distributions.

It is a pleasure to thank Dr. Max Krook for a discussion on the first part of this work and Professor R. E. Peierls for criticizing the manuscript.

[1] B. Rossi and K. Greisen, Rev. Mod. Phys. 13, 267 (1941).

Isotope Effect in the Spectrum of Ne II

K. MURAKAWA AND S. SUWA
Institute of Science and Technology, Komaba, Meguroku, Tokyo, Japan
October 4, 1948

THE spectrum of Ne II was excited in an alumnum hollow cathode discharge tube which was filled with neon of a few mm Hg pressure. It was found that, when the neon was quite free from impurities, the energy of the discharge was spent mainly in exciting the spectrum of the cathode material and only a very weak spectrum of Ne II was obtained, so the experimental arrangement which was finally adopted contained a purifying equipment that absorbed impurity gases only loosely. Under this condition the doublet spectrum[1] was strong, while the quartet spectrum was relatively weak. The fine structure was examined

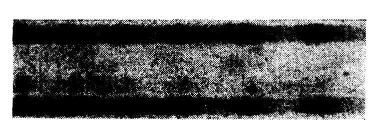

FIG. 1. Enlargement of interference patterns of Ne II λλ3378 and 3393 taken with a quartz Lummer plate of thickness 4.4 mm.