PHYSICAL REVIEW VOLUME 74, NUMBER 10 NOVEMBER 15, 1948

The Geiger Discharge

D. H. WILKINSON
Cavendish Laboratory, Cambridge, England
(Received March 17, 1948)

It is pointed out that those phenomena concerned with Geiger counter operation which are well understood are those associated with the motion of the positive ion sheath while those awaiting explanation are associated with its development. A theory of this development is presented and used to calculate the following phenomena:

(i) The relation between starting potential and counter variables.
(ii) The amount of charge generated in the discharge.
(iii) The shape of the plateau curve.
(iv) The velocity of propagation of the discharge down the wire.

Good agreement is obtained at all points.

The curve connecting the charge generated and the overvolts is shown to consist of two parts, each a straight line, their slopes bearing a ratio of 2:1. The plateau curve is shown to climb more steeply to its constant value, the greater the number of electrons liberated in the counter by the ionizing particle, and the effect is confirmed experimentally. The velocity of propagation is shown to vary roughly linearly with overvolts but to tend to a finite value at zero overvolts. All forms of counter behavior are shown to depend strongly on the ratio of charge generated in the counter to that originally on the wire.

INTRODUCTION

BROADLY speaking, those aspects of Geiger counter behavior which are well understood are those which are concerned with the radial motion across the counter of the positive ion sheath formed in intimate contact with the wire by the discharge. Of these, the most important are:

(i) The form of the voltage pulse as a function of time.
(ii) The dead-time phenomena.

The first has been elucidated by the work of Ramsey[1] and Montgomery and Montgomery,[2] and in more detail by van Gemert, den Hartog, and Muller,[3] and by Alder, Baldinger, Huber, and Metzger.[4] The second has been treated by van Gemert, den Hartog, and Muller,[5] and by Stever.[6]

These phenomena, however, occur after the end of the discharge proper, which has been quenched by the action of the positive ion sheath—all avalanche activity has ceased and the starting point of the discussion is the "burnt out" positive ion space charge sheath which remains. There are several important phenomena, however, which depend on the growth of the discharge itself, and to explain which, one has to enquire more closely into the actual mechanism of formation of the space charge sheath. The most important are:

(i) The relation between the counter starting potential (V_s) and the counter variables.
(ii) The quantity of charge generated in the discharge (q) and its dependence on counter overvolts ($V-V_s$), filling gas pressure (p), and counter geometry.
(iii) The shape of the plateau curve.
(iv) The velocity of propagation of the discharge down the counter wire (v) and its dependence on counter variables.

It is the object of this paper to discuss these four phenomena, and to attempt to explain the facts concerned with them.

THE DEVELOPMENT OF THE DISCHARGE

The characteristic feature of the Geiger discharge is that it spreads the whole length of the counter wire, and so there must exist some mechanism which enables the first electron avalanche to breed others and so on. This mechanism, in the case of the "self-quenching" counters at any rate, is the emission of photoelectrons from molecules of the gas under the action of photons proceeding from excited mole-

[1] W. E. Ramsey, Phys. Rev. **57**, 1022 (1940).
[2] C. G. Montgomery and D. D. Montgomery, Phys. Rev. **57**, 1030 (1940).
[3] A. G. M. Van Gemert, H. Den Hartog, and F. A. Muller, Physica **9**, 556 (1942).
[4] Alder, Baldinger, Huber, and Metzger, Hel. Phys. Acta **20**, 73 (1947).
[5] A. G. M. Van Gemert, H. Den Hartog, and F. A. Muller, Physica **9**, 658 (1942).
[6] H. G. Stever, Phys. Rev. **61**, 38 (1942).

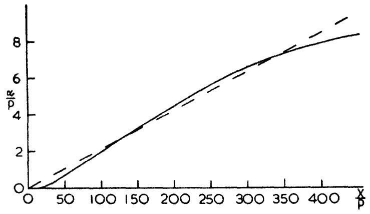

FIG. 1. α/p *versus* X/p for pure argon (after Kruithof and Penning). Solid line: experimental values; dotted line: linear approximation.

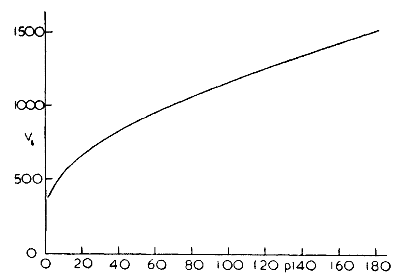

FIG. 2. Relation between V_s and p for the typical counter.

cules or ions formed in the first avalanche. This is shown by the beaded wire experiments of Stever[6,7] and others. It is the self-quenching counters which are considered here. We may assume a constant probability ϵ per ion of an avalanche that a further avalanche should arise by this mechanism. Then, if the first avalanche contains $N(1)$ ions, the condition for the Geiger discharge is

$$N(1)\epsilon>1$$

the threshold V_s being given by

$$N(1)\epsilon=1. \tag{1}$$

(Straggling effects in $N(1)$ are neglected, and will not influence much the greater part of the calculation.) $N(1)\epsilon$ has a Poisson distribution since $\epsilon \ll 1$. The value of ϵ is uncertain, but nearly all the results to follow depend only logarithmically on it. $N(1)$ is the gas amplification which would have obtained at V had not the divergent chain Geiger action set in. This is usually about 10^5 at V_s, so we will set $\epsilon=10^{-5}$.

Thus, for the calculation of problem (i) all that is required is $N(1)$ as a function of the counter variables.

THE FORM OF THE FIRST AVALANCHE

An electron, born inside the counter, will drift towards the wire under the influence of a field

$$X=\frac{V}{r\log b/a} \tag{2}$$

at a distance r from the axis. V is the applied counter voltage, b and a the cathode and anode radii. At some distance r_c ionization by collision begins, r_c being determined by some critical value of X/p, p being the pressure in the counter (in the following, X will be measured in volts/cm and p in mm Hg). Thus

$$\frac{r_c}{a}=\frac{V_s}{V_p}$$

where V_p is the starting potential for proportional counter action. So

$$N(1)=\exp\left(\int_a^{r_c}\alpha\cdot dr\right), \tag{3}$$

α being the first Townsend coefficient effective under the counter conditions. We now make the assumption on which the whole treatment is based, namely:

$$\alpha=\text{constant}\times X. \tag{4}$$

Generally α/p will be some function of X/p for electrons remaining in equilibrium with the field, and over the range of X/p of interest (about 60–300 for the average counter) this approximation is reasonable, as may be seen from Fig. 1 representing the results of Kruithof and Penning[8] for pure argon. Other gases behave similarly. Relations (23), (3) and (4) give together

$$N(1)=\left(\frac{r_c}{a}\right)^{Q_0c}, \tag{5}$$

where Q_0 is the charge per unit length of the counter wire, given by

$$\frac{Q_0}{V}=\frac{1}{2\log b/a},$$

and c is a constant related to that in (4).

If now by $N(1, r)$ we mean the number of ions

[7] H. G. Stever, Phys. Rev. **59**, 765 (1941).

[8] A. A. Kruithof and F. M. Penning, Physica **3**, 515 (1936).

produced by the avalanche between r_c and r

$$N(1, r) = \left(\frac{r_c}{r}\right)^{Q_{0c}}.* \tag{6}$$

DEPENDENCE OF V_s ON THE COUNTER VARIABLES

We may now at once give an answer to problem (i) using relations (1) and (5) and find, for the dependence of V_s on p

$$V_s \log\frac{k_1 V_s}{p} = \text{constant}, \tag{7}$$

where k_1 is a constant given by

$$\frac{k_1 V_s}{p} = \frac{r_c}{a}.$$

If we now consider a typical counter which we shall discuss throughout this paper where a numerical value is to be derived, having $b=1.0$ cm, $a=0.01$ cm, $V_s=1000$ volts, $V_p=200$ volts, $p=70$ mm Hg, we have $k_1=0.35$ mm Hg/volt and a relation between V_s and p given in Fig. 2.

The dependence of V_s on b and a may similarly be found to be given by, at a fixed p,

$$\frac{V_s}{\log b/a} \log\left(\frac{V_s}{k_2 a \log b/a}\right) = \text{constant},$$

k_2 being another constant of value 4.35×10^3 volts/cm for our typical counter considered above. V_s is shown in Fig. 3 as a function of a for several b values.

These relations are subject to two limitations. The first concerns r_c which should be considerably smaller than b, or the whole avalanche may not be formed. Thus the results may not be used at very low counter pressures or small b/a values. The second concerns the constancy of composition of the gas. This has been implicitly assumed as ϵ has been held constant. The fact that, for example, in argon-alcohol counters, V_s depends more rapidly on the partial pressure of the alcohol than on that of the argon, is interpreted simply as a decrease of ϵ with increasing alcohol pressure, which may be understood from many points of view.

These relations are in good accord with experience, and, rather than being a product of the theory, should be regarded as confirmation of the basic assumptions.

THE BUILDING-UP OF THE SPACE-CHARGE SHEATH

For the attacking of the main problems we must consider the way in which the space charge of positive ions is developed, and its manner of quenching the discharge. Before this can be done, we must establish three important characteristics of the sheath.

(i) It is uniform in structure and does not preserve any of the granular nature which might be expected of it, having been built up from discrete avalanches.
(ii) It is effectively stationary until the discharge has terminated in its immediate neighborhood.
(iii) It holds on to the wire by induction all the electrons formed in the avalanches.

These three characteristics will be discussed in turn:

(i) Although the avalanches building up the sheath are quite discrete, each one undergoes

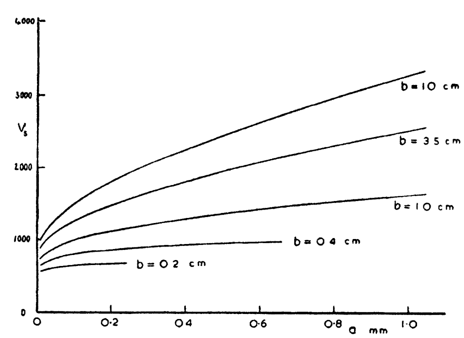

FIG. 3. Relation between V_s, a, and b for $p=7$ cm Hg.

* It is not suggested that this is a complete account of the first avalanche. It is merely desired to have some simple analytic expression for its form which is in adequate accord with the facts and on which the formal treatment of the space-charge build-up may be based. A detailed account of the first avalanche has been given by S. C. Brown [Phys. Rev. 62, 244 (1942)] for a helium-filled counter. Such an account should be given in a complete study, but would complicate enormously its development. Brown considers the space-charge effect of a single avalanche on itself. It is well known that this effect is all-important in cases of small b/a ratio—as Brown's results reveal—but in the specifically Geiger-counter geometry considered here it is relatively small. The diffusion results of the next section but one may be used to estimate the effect, which amounts to a few percent at the wire.

considerable diffusion in the course of its formation, and the positive ions from neighboring ones completely overlap. This diffusion effect may be roughly estimated by calculating that obtaining in a parallel plate chamber of electrode spacing $\delta=r_c-a$ across which is applied the potential difference V' between r_c and the wire.

$$V'=V\frac{\log r_c/a}{\log b/a}$$

$\sim$300 volts for our typical counter. Remembering that

$$\frac{v_e'}{D}=\frac{LeX}{\eta P},$$

where v_e' is the electron drift velocity, D the coefficient of diffusion, L Loschmidt's number, P 760 mm Hg expressed in dynes/cm^2, and η the Townsend coefficient giving the ratio between the mean electron agitation energy and $\frac{3}{2}kT$, we have: the probability of finding our electron between x and $x+dx$ from the plane of the center of gravity of the distribution at time t is

$$\frac{\exp\left(-\dfrac{x^2}{4Dt}\right)}{(4\pi Dt)^{\frac{1}{2}}}\cdot dx,$$

and the mean electron displacement, which may be taken as a measure of the spread of the avalanche, is

$$d=\left(\frac{4Dt}{\pi}\right)^{\frac{1}{2}}.$$

This gives a fractional displacement $f=d/\delta$ of

$$f=0.178\left(\frac{\eta}{V'}\right)^{\frac{1}{2}}$$
$$\sim 0.01(\eta)^{\frac{1}{2}}.$$

Now η for pure argon is already 310 for $X/p=5$ and, though considerably reduced by the quenching agent, is probably quite big in the $X/p\sim 100$ region in which we are interested. Letting then $\eta=100$ gives $f=0.1$ so that the lateral extension down the wire of an individual avalanche is about $2f(r_c-a)\sim 0.1$ mm. This, coupled with the fact that the number of avalanches per cm of counter wire is usually of the thousands order, as will be seen, shows that the positive ions are in fact quite mixed up, and we may assume the desired uniform space-charge sheath.

(ii) The assumption of the locally stationary positive ion sheath is based on the enormously greater mobility of the electrons than positive ions (the ratio is about 10^3:1). As we shall see, the local discharge lasts a few times 10^{-8} sec. in which time the positive ions formed in the beginning at the wire surface will have moved a few times 0.001 cm—a few tenths of a wire radius. Those at r_c will have moved about a fifth as much, and, as most of the avalanches are formed quite late on in the discharge, because of its exponential nature, these movements may be neglected.

(iii) The assumption that all the electrons formed are held on to the wire by induction of the positive ions may be justified by calculation. Applying Green's Theorem in electrostatics, we find that the fraction q_- of electrons which can escape from the wire is given by, using (6)

$$q_-=\frac{Q_0c\displaystyle\int_a^{r_c}\frac{1}{r}\cdot\left(\frac{r_c}{r}\right)^{Q_0c}\log\frac{r}{a}dr}{\log b/a\left(\dfrac{r_c}{a}\right)^{Q_0c}}$$
$$\sim\frac{1}{Q_0c}\cdot\frac{1}{\log b/a}.$$

Now, as seen, $r_c/a\sim 5$ and $\epsilon\sim 10^{-5}$ so $Q_0c\sim 7$ and $q_-\sim 0.03$. Thus all but about 3 percent are retained by induction.

THE CHARGE DEVELOPED IN THE COUNTER

To find the charge developed in the discharge we must examine the action of the space charge sheath. We assume that all the photoelectrons are born outside r_c. (This will be discussed later.) Thus, until a new photo-electron reaches r_c it will experience the same field (2) as if no space charge were present, since the positive ions retain an equal number of electrons on the wire. If we now imagine all the avalanches to fall in a length x of counter wire, and if we number them successively, $N(n, r)$ being the ionization produced by the nth avalanche between r_c and r, we have a field at r for the $(n+1)$th avalanche of

$$\frac{2}{r}\left\{Q_0-\frac{e}{x}\sum_1^n N(n,r)\right\}, \tag{8}$$

(implicitly assuming $x \gg r_c$ which we shall see is true). Thus, to calculate $N(n+1)$ the total number of electrons in the $(n+1)$th avalanche, we have

$$-\frac{1}{c}\cdot\frac{dN(n+1,r)}{N(n+1,r)}=\frac{dr}{r}\left\{Q_0-\frac{e}{x}\sum_1^n N(n,r)\right\}, \tag{9}$$

and so $N(n+1)<N(n)$.

The direct solution for the form of $N(n, r)$ is very difficult, but two approximations suggest themselves:

$$\text{(i)} \qquad N(n,r)=\left(\frac{r_c}{r}\right)^{Q(n)c}, \tag{10}$$

$$\text{(ii)} \qquad N(n,r)=A(n)\left(\frac{r_c}{r}\right)^{Q_0c}. \tag{11}$$

The first allows the form of successive avalanches to change, as it must, and so is more realistic. As both give the same results, the calculation will be described in terms of the first. The second will be used later. The calculation is easy and proceeds as follows. The sum in (8) is replaced by an integral since dN/dn is small and the final value of n is large. (10) is substituted in (9) and the integration over r performed, giving, since $N(n+1)\sim N(n)$,

$$\frac{1}{c}\log N(n)=Q_0\log\frac{r_c}{a}-\frac{e}{x}\int_a^{r_c}\left(\int_1^n\left(\frac{r_c}{r}\right)^{Q(n)c}dn\right)\frac{1}{r}dr. \tag{12}$$

Differentiating (12) with respect to n, and performing the integration over r gives an expression for $Q(n)$, which, combined with (10) and solved, gives

$$\frac{e}{x}(n-1)=\left(\frac{r_c}{a}\right)^{-Q(n)c}\left\{Q(n)+\frac{1}{c\log r_c/a}\right\}-\left(\frac{r_c}{a}\right)^{-Q_0c}\left\{Q_0+\frac{1}{c\log r_c/a}\right\}.$$

Neglecting small terms and retaining only the more powerful $Q(n)$ in the power term, setting the other equal to Q_0, gives

$$N(n)=\frac{1}{\dfrac{en}{Q_0x}+\theta}, \tag{13}$$

$$\left(\theta=\left(\frac{r_c}{a}\right)^{-Q_0c}=N(1)^{-1}\right),$$

showing an inverse linear dependence of $N(n)$ on n.

At some avalanche n_f we shall have

$$N(n_f)\epsilon=1,$$

and the discharge will converge. Thus the total charge qx generated in the length x of counter is given by

$$qx=e\int_1^{n_f}N(n)dn,$$

giving the simple result

$$q=Q_0\log\frac{\epsilon}{\theta}.$$

This may be expressed in terms of the very important ratio

$$m=\frac{q}{Q_0}$$

which is found to govern almost all aspects of counter behavior

$$m=\log\frac{\epsilon}{\theta}. \tag{14}$$

Now the chief experimental facts concerning the dependence of q on the counter variables are the following:

(i) q depends almost linearly on $V-V_s$ to begin with, most authors agreeing that the increase is a little more rapid than linear.[5, 6, 9–11]

---

[9] A. Nawijn, *Het Gasontladings Mechanisme van den Geiger-Müller Teller* (Drukkerij Hoogland-Delft, 1943).
[10] S. H. Liebson, Phys. Rev. **72**, 602 (1947).
[11] J. D. Craggs and A. A. Jaffe, Phys. Rev. **72**, 784 (1947).

(ii) At a well-defined overvoltage $(V_B - V_s)$ corresponding approximately to $m=1$, a sharp break occurs in the curve, the slope $dq/d(V-V_s)$ falling to about half its previous value, the dependence now being quite linear.[5, 9, 11]

(iii) For a given overvoltage $V-V_s$, q is practically independent of p and depends only slowly on b/a.[6, 9]

(iv) If the results for the pulse size q be expressed in terms of m, we have the relation, for the first part of the curve

$$m=\frac{V-V_s}{100}\times C, \tag{15}$$

C being a constant depending on the counter, its value ranging from about 0.4 to about 2 (see Stever,[6] and Nawijn[9] for extreme values).

We must now see whether relation (14) can explain all these facts. Using relation (5) and remembering that $r_c \propto V$ we find

$$m=\text{constant}\times\left\{(V-V_s)\log\frac{V_s}{V_p}+V\log\frac{V}{V_s}\right\} \tag{16}$$

for a given counter and pressure.

Now generally $(V-V_s)/V_s$ is small, and we write

$$\log\frac{V}{V_s}=\frac{V-V_s}{V_s},$$

giving

$$m=\text{constant}\times(V-V_s)\times\left\{1+\log\frac{V_s}{V_p}+\frac{V-V_s}{V_s}\right\}, \tag{17}$$

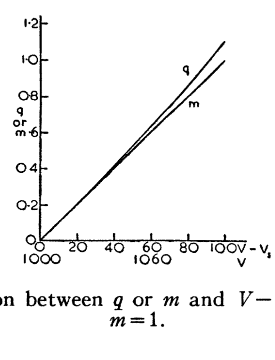

FIG. 4. Relation between q or m and $V-V_s$ or V up to $m=1$.

or

$$q=\text{constant}\times V(V-V_s)\left\{1+\log\frac{V_s}{V_p}+\frac{V-V_s}{V_s}\right\},$$

showing the desired sensibly linear dependence of m on $V-V_s$, with the almost linear dependence of q, the extra V term serving to give the noted slight departure from linearity. m and q are shown in Fig. 4 for our typical counter detailed above (which we now assume, in addition, has $C=1$) up to $m=1$, q being simply plotted as $V/V_s\times m$.

The dependence of q on p may be derived by eliminating the pressure from the constant in (16) and when this is done we find, neglecting the $(V-V_s)/V_s$ term in the curly bracket of (17)

$$q=\text{constant}\times(V-V_s)\frac{V}{V_s}\left(1+\left(\log\frac{V_s}{V_p}\right)^{-1}\right),$$

showing the almost complete lack of dependence of q on p, V_s as we have seen in relation (7), changing only slowly with p.

The dependence on b/a is similarly found, giving, with a "geometry-free" constant

$$q=\text{constant}\times\frac{V}{V_s}(V-V_s)\frac{1}{\log b/a},$$

giving the slow change of q with counter geometry.

To find the predicted value of the constant C we must have an equation free from undetermined constants. This may be found, with the help of ϵ to be

$$m=\frac{V-V_s}{V_s}\left\{1+\left(\log\frac{V_s}{V_p}\right)^{-1}\right\}\log\frac{1}{\epsilon} \tag{18}$$

giving, for the counter considering above, $C=1.86$. This C value lies in the range 0.4–2 noted in (iv) above, though it is a little on the high side. It is quite satisfactory in view of the nature of the calculation.

The theory has thus accounted quantitatively for facts (i), (iii), and (iv) above, and only (ii), the sharp break at $m=1$ awaits explanation. Expression (14) is clearly incapable of accounting for the phenomenon, giving a smooth indefinite increase of m with $V-V_s$. However, something is going to happen at $m=1$ since at this point the field at the wire has been reduced to zero, and further avalanches cannot multiply all the way to the wire. (Actually they must stop a little way from the wire even at $m=1$, and V_B will not occur quite at $m=1$ but at $(1-a/r_c)\sim 0.8$ as indeed is found by some experimenters, but we neglect this.)

For the calculation of this effect it is more convenient to use the simpler expression (11) for $N(n, r)$ which, we have noted above, gives the same answer for m in the $0<m<1$ range. We then have, treating unit length of counter wire, x having no effect on m as we have seen above,

$$A(n)=\frac{1}{(en/Q_0\theta)+1},$$

so if $m=1$ occurs when $n=n_m$ we have

$$\log\left(\frac{en_m}{Q_0\theta}+1\right)=1.$$

So for $n>n_m$ multiplication will proceed only as far as $a(n)$ and the total number of electrons generated so far in the discharge is

$$\Phi(n)=\frac{Q_0}{e}+\int_{n_m}^{n} A(n)\left(\frac{r_c}{a(n)}\right)^{Q_0c} dn, \quad (19)$$

$a(n)$ being the r value, for the nth avalanche, for which the positive ion space charge outside $a(n)$ is Q_0. So for $r>a(n)$ we always have

$$\Phi(n, r)=\frac{Q_0}{e}\log\left(\frac{en}{Q_0\theta}+1\right)\left(\frac{r_c}{r}\right)^{Q_0c}\theta.$$

Thus, if $a(n)$ increases by $da(n)$ from the nth to the $(n+1)$th avalanche $(n>n_m)$, the charge brought down by the $(n+1)$th avalanche must equal the positive ion charge between $a(n)$ and $a(n)+da(n)$. Thus, writing $a(n)-a=\Delta(n)$,

$$\frac{1}{(en/Q_0\theta)+1}\left(\frac{r_c}{a(n)}\right)^{Q_0c}=-\frac{d\Phi}{dr}\cdot\frac{d\Delta(n)}{dn},$$

$d\Phi/dr$ being evaluated at $r=a(n)$. Solution yields, noting that $\Delta/a\ll 1$

$$Q_0c\frac{\Delta(n)}{a}=\log\log\left(\frac{en}{Q_0\theta}+1\right). \quad (20)$$

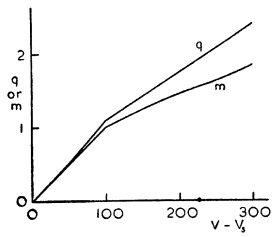

FIG. 5. Complete relation between q or m and $V-V_s$.

We now have an expression for $a(n)$ which may be inserted in (19). The solution is, under the fair approximation

$$\left(\frac{a(n)}{a}\right)^{Q_0c}=1+\frac{Q_0c\Delta(n)}{a},$$

$$\Phi=\frac{Q_0}{e}\left\{1+\frac{\bar{E}_i[1+\log\log((en_f/Q_0\theta)+1)]-\bar{E}_i(1)}{e}\right\}.$$

The e in the denominator of the expression in brackets is the base of the Napierian logarithms, and

$$\bar{E}_i(x)=\int_{-\infty}^{x}\frac{e^x}{x}\cdot dx.$$

Equation (20) may also be used to find n_f, giving

$$\left(\frac{en_f}{Q_0\theta}+1\right)\log\left(\frac{en_f}{Q_0\theta}+1\right)=\frac{\epsilon}{\theta}.$$

If now $m=1$ occurs at V_B we may rewrite (18) for $V<V_B$ as

$$m=\frac{V-V_s}{V_B-V_s},$$

or, for any V, using (14)

$$\frac{V-V_s}{V_B-V_s}=R=\log\frac{\epsilon}{\theta}.$$

We may thus express the behavior of m above $m=1$ in terms of the ratio R of the overvolts to the overvolts required to give the break in the q curve since,

for $R<1$ $\qquad m=R,$

for $R>1$

$$m=1+\frac{\bar{E}_i(1+\log\log\xi)-\bar{E}_i(1)}{e}, \quad (21)$$

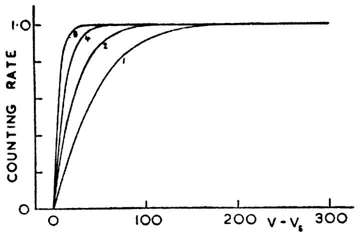

FIG. 6. Calculated plateau curves for the liberation of 1, 2, 4 or 8 initial electrons.

where

$$\xi \log \xi = e^R. \tag{22}$$

This gives

R	m
0	0
0.5	0.5
1.0	1.0
1.5	1.23
2.0	1.45
2.5	1.63
3.0	1.85

This should be the same for all internally quenched counters. The relation between m and R is almost linear above $m=1$, but that between q and R is more so. In Fig. 5 are plotted q and m as functions of V for our typical counter, in the same way as in Fig. 4.

The ratio of the slopes above and below $m=1$ is given by differentiation of (21) with respect to R, when, using (22) one finds a ratio of exactly $\frac{1}{2}$.

A. G. Fenton working in Birmingham has made 20 independent determinations of this slope ratio for counters with values of p from 80 to 180 mm Hg and a values of 0.005, 0.007 and 0.01 cm. The ratio ranged from 0.4 to 0.67 with an average value of 0.50. There was no significant difference in the ratio for various pressures and wires.**

(A rather simpler expression for m may be obtained by a slight further approximation. It is, for $m>1$

$$m = 1 + \log \log \xi$$

giving results almost identical with those tabulated above, and the same slope-ratio.)

It thus seems that all points concerned with pulse size are satisfactorily dealt with by the theory.

One point has been overlooked. The above calculation refers only to the total ionization generated. Since this is greater than Q_0 the question arises, how can all the positive ions traverse the counter, since reverse fields seem to be present? This is not so, however, since the electrons cannot all attain the wire either, and remain trapped in the space-charge sheath, the separation occurring gradually as the more remote positive ions begin to move across the counter, reducing the induction on the wire.

THE SHAPE OF THE PLATEAU CURVE

When the threshold V_s is passed, the counting rate in a Geiger counter climbs rapidly to a constant value at which it remains until multiple discharges and so on mark the end of the plateau. It is the purpose of this section to inquire into the nature of the initial rapid climb and to see whether it depends on any variable.

At some $V > V_s$ we have an average of

$$Z = N(1)\epsilon = \frac{\epsilon}{\theta}$$

avalanches breeding from the first. Now $Z>1$ is the condition for a divergent chain of avalanches, that is, a count, but statistical fluctuations in the number of daughter avalanches may extinguish the discharge by breaking off the chain before enough charge has been generated to give a count. Thus, the probability that there should be not even one daughter avalanche is, for one initial electron in the counter,

$$p(1) = e^{-Z}$$

assuming a Poisson distribution.

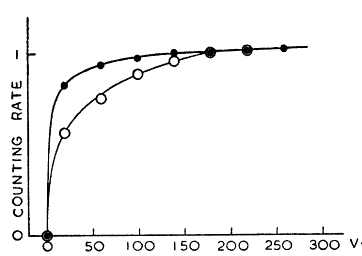

FIG. 7. Experimental plateaus for CuO cathode counter. Open circles: ultraviolet irradiation; solid circles: γ-ray irradiation.

** I am grateful to Mr. Fenton for permission to make use of these results prior to their publication.

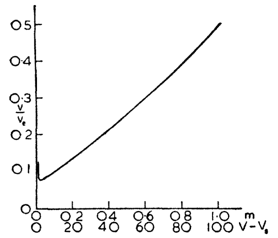

FIG. 8. Relation between v and m or $V-V_s$.

If now $p(w)$ represents the probability of the discharge converging at or before the wth stage of multiplication, it is easy to show that

$$p(w+1)=\exp(-Z(1-p(w))). \tag{23}$$

The probability of a convergent chain $p(\infty)$ is thus given by

$$p(\infty)=\exp(-Z(1-p(\infty)))$$

or

$$\log p(\infty)=Zp(\infty)-Z \tag{24}***$$

assuming Z to be constant, which is reasonable since we are effectively interested only in the first few multiplication stages.

Now the plateau curve is given simply by $1-p(\infty)$ and so may be calculated as a function of Z or $V-V_s$. If there are n intial electrons liberated by the ionizing particle, the probability of a convergent discharge is $p(\infty)^n$ and the plateau curve is given by $1-p(\infty)^n$. The plateau curve has been calculated for our typical counter for $n=1$, 2, 4 and 8 and is shown in Fig. 6.

It is seen from Fig. 6 that the plateau curve, for the release of single electrons should be detectably worse than for the release of four or five. To check this, the inside of the cathode of a copper counter was irradiated with ultra-violet light, liberating single electrons, and then with γ-rays, giving a few electrons at a time. The results are given in Fig. 7 where it is seen that the plateau is indeed much worse for the single electrons.

It seems probable that 100 percent counting is being approached for an overvoltage of about 180 (the plateaus have been normalized over their upper regions). Thus when Geiger counters are used to monitor ultra-violet radiation, or in, say, specific ionization studies where the liberation of a single electron is generally considered enough to initiate the discharge, the overvoltage should always be as high as possible.

*** Since this work was completed I have seen a copy of Nawijn's book (see reference 9), in which this problem is discussed, and Eq. (24) derived in a simple way. For this reason I have thought it worth while to give the intermediate probabilities expressed in (23).

THE PROPAGATION OF THE DISCHARGE DOWN THE COUNTER WIRE

The last important problem to be tackled with the aid of our theory is that of the velocity of propagation of the discharge down the wire (v). The theory presented here is based on the remark that if at any instant a length x of counter wire is "burning" in the sense that dense avalanche activity is taking place in it, and if it "burns" for a time T, then

$$v=\frac{x}{T}. \tag{25}$$

We thus require to calculate x and T. Matters are simplified by the observation that the number of stages w_f of multiplication of avalanches required to burn out the length x of counter wire (achieving the n_f avalanches) does not depend much on x—logarithmically, as will be seen.

x is now the distance which the region of intense avalanche activity propagates in both directions combined along the wire during the w_f stages of multiplication. If now we represent by t_0 the average distance, measured along the wire, which an ultra-violet photon moves before being absorbed and giving its daughter avalanche, we can find x in terms of it. If $Z=1$, the problem is

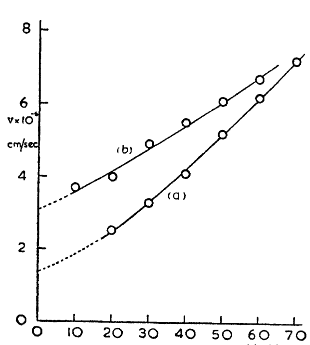

FIG. 9. Miss Freeman's results for the relation between v and $V-V_s$ for the counter fillings of (a) 9.5 cm Hg argon plus 0.5 cm Hg alcohol, (b) 9.5 cm Hg argon plus 2.8 cm Hg alcohol.

that of the random walk with w_f steps. The solution is that the probability of a final displacement ψ (measured in units of the step length) is proportional to

$$\exp\left(-\frac{\psi^2}{2w_f}\right)$$

a Gaussian distribution of width $2(w_f)^{\frac{1}{2}}$. The region of intense avalanche concentration would thus have a length $x=2(w_f)^{\frac{1}{2}}\times t_0$, being quite well defined because of the rapid fall of the Gaussian distribution. For $Z>1$ the problem is more complicated, but one may approximate by increasing the step length proportionally with the probability $(1-(\frac{1}{2})^Z)$ that a step will be taken in a given direction in any single multiplying event. t_0 may be calculated from ν, the mean free path for photon absorption, and is

$$t_0=\frac{\nu}{2}. \tag{26}$$

Thus

$$x=2(w_f)^{\frac{1}{2}}\times 2(1-(\tfrac{1}{2})^Z)\frac{\nu}{2}. \tag{27}$$

We have now limited x by the Gaussian distribution, and imagine the new length of wire to start burning at the end of the first, the peaks of the Gaussians being placed one half-width apart. Thus the time of burning on any one spot on the wire is

$$T=2w_f\tau \tag{28}$$

where τ is the time for a single stage of multiplication. τ depends on various factors—the migration time of the electron from its point of birth to the wire, the excitation time of the (say argon) atoms responsible for the photon emission, and the photon's transit time. These times are,

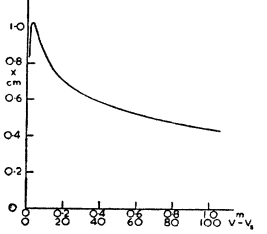

FIG. 10. Relation between x and m or $V-V_s$ ($\nu=1$ mm).

respectively, a few times 10^{-9}, 2×10^{-10} and 3×10^{-12} second, the first being derived as explained below and the second taken from the paper of Alder, Baldinger, Huber, and Metzger.[4] We thus may probably ignore all but the electron migration time. The average distance of birth of the photoelectrons from the wire is

$$r_e=\frac{\pi\nu}{4} \tag{29}$$

((26) and (29) are calculated on the assumption that $\nu\gg a$).

Thus, if the mean electron velocity in the immediate vicinity of the wire is v_e, we have

$$\tau=\frac{\pi\nu}{4v_e}. \tag{30}$$

Thus the problem is reduced to the calculation of w_f.

Considering the length x of counter wire, suppose that at stage w of multiplication there are j photoelectrons present, n avalanches having already taken place, then

$$j(w+1)=j(w)N(n)\epsilon,$$

and

$$\frac{dn}{dw}=j(w);$$

using (13) we obtain the relation between n and w

$$\frac{d^2n}{dw^2}=\frac{dn}{dw}\left(\frac{\epsilon}{(en/Q_0x)+\theta}-1\right),$$

so

$$\frac{dn}{dw}=\frac{Q_0x\epsilon}{e}\log\left(\frac{en}{Q_0x\theta}+1\right)-n+1. \tag{31}$$

Noting that the greater part of the build-up process is spent at $n\ll n_f$ we write the log term as $en/Q_0x\theta$, integration yielding

$$w_f=\frac{\log\left(\dfrac{xQ_0}{e}\dfrac{(\epsilon-\theta)^2}{\epsilon}\right)}{\dfrac{\epsilon}{\theta}-1}. \tag{32}$$

(A numerical integration of (31) shows the approximation leading to (32) to be accurate to

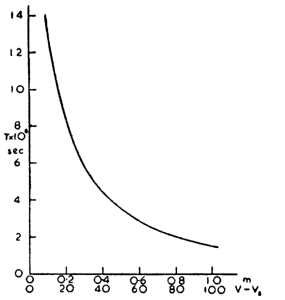

FIG. 11. Relation between T and m or $V-V_s$ ($\nu=1$ mm).

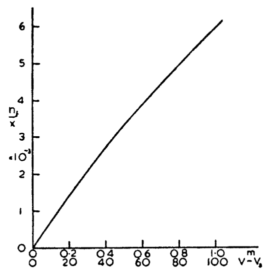

FIG. 12. The number of avalanches per cm of counter wire (n_f/x) as a function of overvoltage.

about 20 percent over the interesting range of m.)

w_f is seen to depend only logarithmically on x and, as $x\sim 0.5-1.0$ cm, it is set $=1$ and effectively omitted from (32). Thus, combining (25), (27), (28), (30) and (32) we have

$$v=\frac{4}{\pi}(1-(\tfrac{1}{2})^{e^m})\left(\frac{e^m-1}{\log((Q_0\epsilon/e)(1-e^{-m})^2)}\right)^{\frac{1}{2}}\times v_e. \quad (33)$$

Having now obtained our expression for v, we may examine the facts which it has to explain. These are derived chiefly from the work of Alder, Baldinger, Huber, and Metzger,[4] of Hill and Dunworth,[12] and from the unpublished results of Miss Freeman obtained in this laboratory.†

(i) The relation between v and $V-V_s$ is roughly linear, the curve being slightly concave towards the v axis.

(ii) A lowering of noble gas pressure, keeping the quenching gas pressure constant, results in a roughly inverse increase of v for a given value of $V-V_s$.

(iii) v depends on the nature of the noble gas, all other factors being constant.

(iv) The v *versus* $V-V_s$ curve does not pass through the origin, but may be extrapolated to a finite positive value of v at $V-V_s=0$.

(v) v does not depend very strongly on quenching agent pressure for a given value of $V-V_s$, though at low $V-V_s$ a higher quenching gas pressure gives a higher value of v.

Alder, Baldinger, Huber, and Metzger[4] have put forward a theory of v, but it is unsatisfactory because it requires the fixing of three constants, is not in very good accord with the facts, and makes no mention of any role of the space charge sheath. It also does not allow for the fact that before a pulse can be observed the local discharge is extinguished.

Figure 8 shows v plotted as a function of $V-V_s$ or m, being taken from (33) appropriately for our typical counter. v is plotted only as far as $m=1$ where a break must occur. It is seen that the result satisfies fact (i) above.

The second and third facts are explained with reference to the v_e of expression (33). A decrease of p will produce a roughly inverse increase of v_e, hence of v, and v_e will obviously depend on the nature of the noble gas. (Hill and Dunworth[12] find v values in the ratio of about 3:1 for helium and argon counters. This is just the ratio if the v_e's in the pure gases for measured values of X/p, though this is probably coincidental, considering the profound effect on v_e of admixed gases.) The fourth fact is seen to be correctly explained, the curve of Fig. 8 giving a finite intercept at $V-V_s=0$. For our typical counter, the ratio of v at $V-V_s=50$ to that at $V-V_s=0$ (extrapolated value) is 0.3. Experimental values range from 0.2 to 0.5. It is seen that the theoretical curve goes off to infinity for very small $V-V_s$ values of the order of one volt. This divergence is probably mathematical, and, in any case, at such tiny overvoltages the assumption of a uniform space charge sheath can no longer hold. Examination of fact (v) brings to us the necessity of deciding in which constituent of the filling mixture the absorption of the photons to give the secondary avalanches takes place. It has generally been assumed that the absorption is in the quenching agent, and Alder, Baldinger,

[12] J. M. Hill and J. V. Dunworth, Nature **158**, 833 (1946).

† I am very grateful to Miss Freeman for her kind permission to quote and discuss her results prior to their publication.

Huber, and Metzger,[4] for example, have found an absorption coefficient of 640 cm^{-1} for normal alcohol vapor. Liebson,[10] however, claims that the noble gas is responsible for the absorption. It is almost certainly a mixed effect, but we shall make the more plausible assumption—that the quenching agent is responsible. One thus sees that v should not depend much on the partial pressure of the quenching agent, since there is no ν in the expression (33) for v. However, v_e will depend somewhat on ν, a bigger ν giving a smaller v_e since the mean X for the electron path will be smaller. Thus increasing the quenching gas pressure should increase v. One thing tends to hold v_e constant however, and that is the space charge, whose effect is to tend to equalize the fields near the wire in the manner calculated above. Thus, the bigger m, the less should v depend on ν. This is shown in Fig. 9, where Miss Freeman's results are displayed.

It is seen that at low overvoltages, the bigger alcohol pressure does indeed give the bigger v, but that the curves approach for higher overvoltages.

It may be remarked that if the assumption of a negligible excitation time for the argon atom is incorrect, we must write

$$\tau=\frac{\pi\nu}{4v_e}+\phi,$$

ϕ being the mean excitation time. The v_e in expression (33) then becomes

$$\frac{v_e}{1+(4\phi v_e/\pi\nu)},$$

and v does not change so rapidly with ν, both numerator and denominator increasing with decreasing ν.

All five facts having been adequately explained, it remains to compare the absolute values of v predicted by expression (33) with those experimentally determined. This comparison is rendered difficult by our lack of knowledge of electron mobilities in very high fields. Den Hartog, Muller, and Verster[13] have measured electron mobilities in a counter gas (9 cm Hg of argon plus 1 cm Hg of alcohol), finding

$$v_e'=k_3X,$$

k_3 being a true mobility constant of value 15,600 cm/sec./volt/cm at $p=100$ mm Hg. Sherwin[14] has found a conflicting result, namely a parabolic relation

$$v_e'=k_4\left(\frac{X}{p}\right)^{\frac{1}{2}},$$

k_4 equalling 4.5×10^6 for a 92 percent argon 8 percent amyl acetate mixture. If these results could be extrapolated down to the X/p values of interest in the present problem, they would yield, for an alcohol pressure of 0.5 cm Hg and an argon pressure of 6.5 cm Hg, using Alder, Baldinger, Huber, and Metzger's[4] value for the effective absorption coefficient, τ values of 4×10^{-9} and 8×10^{-9} sec., respectively. This corresponds to effective v_e's of 4×10^7 and 2×10^7 cm/sec. These values are not widely different, and are of the order to be expected from cloud-chamber photographs of electron avalanches at high X/p values such as those taken by Raether,[15] and Kerr cell studies such as those of White.[16] We thus average, and take $v_e=3\times10^7$ cm/sec. This yields, for our typical counter, a value of v at $V-V_s=50$ of 8×10^6 cm/sec. The alcohol pressure used by Hill and Dunworth[12] was also 0.5 cm Hg and they found, for a counter containing 4.5 cm Hg of argon and $V-V_s=50$ a value of v of 7×10^6 cm/sec. The uncertainties in v_e and the C value of the counter used make wider comparisons unprofitable, but it is seen that the predicted value of v is at any rate roughly correct.

THE MEAN FREE PATH OF THE PHOTONS

We may remark here on the assumption made above that all photoelectrons are born outside r_c. Under the alcohol absorption hypothesis, the value of ν would be a little over 1 mm for 1 cm Hg pressure of alcohol. As noted, $r_c/a\sim5$, hence for $a=0.01$ cm, as large a wire as normally used, $r_c-a=0.04$ cm, and the majority of the photoelectrons will indeed be born outside r_c. The

[13] H. Den Hartog, F. A. Muller, and N. F. Verster, Physica 13, 251 (1947).

[14] C. W. Sherwin, Phys. Rev. 71, 479 (A) (1947).
[15] H. Raether, Zeits. f. Physik 107, 91 (1937).
[16] H. J. White, Phys. Rev. 46, 99 (1934).

smaller a and the alcohol pressure the better is the assumption. On Liebson's picture of argon absorption, almost the same ν would result for a 10-cm Hg argon pressure, so the two views should give roughly the same result for the majority of the effects calculated above. If the absorption is in the argon, the small effect of alcohol pressure on v is more readily understood, but greater difficulty would be encountered in explaining the dependence of v on total pressure p. In any case, the photons mainly responsible for spreading the discharge may have a very small ν and not be observed in the experiments of Liebson and Alder, Baldinger, Huber, and Metzger. The agreement of observed and calculated v however, seems to tell against this. If ν were less than r_c one effect would be to impose another straggling on that of numbers of ions in individual avalanches. In the present state of uncertainty it is not profitable to modify the calculations for the $\nu < r_c$ case.

SOME MAGNITUDES OF THE DISCHARGE

It is of interest to evaluate x, the "burning length" of the wire, T the time of "burning" at any one spot on the wire, and n_f, the number of avalanches taking place per cm of the wire. These are obtained from the appropriate relations above.

x is shown in Fig. 10 for our typical counter (setting $\nu = 1$ mm).

It is seen to vary but slowly with $V - V_s$, being 0.5–1.0 cm in the usual range of counter operation.

T is shown in Fig. 11, being of the order of a few times 10^{-8} sec. n_f is shown in Fig. 12.

This is the only quantity which depends much on ϵ (being proportional to it) and so is liable to considerable error. Only its order is of any interest fortunately. All other quantities are of the $\log \epsilon$ type.

It has not been thought worth while to calculate the behavior above $m = 1$, since these quantities are more interesting for their rough magnitude than for their exact dependence on $V - V_s$.

DISCUSSION

It is seen that all features of the Geiger discharge considered here depend strongly on m, which should therefore be measured when counter behavior of any kind is being studied. It may be enough simply to find V_B, and hence the C value (relation (15)), but a direct measurement would always be preferable. The C value of a counter is of great importance in determining other aspects of counter behavior—the dead time, for example, which drops sharply above V_B.

Note added June 26, 1948: Sherwin[17] has recently published experimental results which may be interpreted as evidence in favor of the dependence on overvolts and absolute magnitude of T as calculated above and shown in Fig. 11 He finds in Geiger counters a delay between the entry of the ionizing particle and the manifestation of the pulse which can be only partially interpreted in terms of the time of drift of the initial electrons to the wire. There remains a residual delay of the form and magnitude shown in Fig. 11. It is clear that a delay of order T must occur after the initial electrons have arrived at the wire, as sufficient charge must be generated to manifest the pulse. Because of the exponential growth of n with w the delay will certainly be of order T.

[17] C. W. Sherwin, Rev. Sci. Inst. 19, 111 (1948).