IV. CONCLUSION

From the above preliminary studies on the Zeeman effect in the absorption spectra of gas molecules, it is seen that a knowledge of the nuclear and molecular g-factors can be obtained from the analysis of Zeeman splitting, if there exists a spin-rotation coupling. If one g-factor is known, the other g-factor can be determined from the experimental data, on the basis of known spin and rotational quantum numbers. By studying the Paschen-Back effect at strong fields in conjunction with the analysis of the Zeeman effect, it will be possible to evaluate the two g-factors separately.

If the spin-rotation coupling is absent, the Zeeman splitting of the spectral lines of a molecule will yield direct information on the molecular g-factor. This information bears a close relationship to the charge distribution in a molecule and may throw some light on the nature of molecular bonds. The same knowledge should be particularly useful for the study of the interaction between the nuclear magnetic moment and the magnetic moment resulting from molecular rotation. Also, if the spectral lines of a molecule differ among themselves in their characteristic Zeeman patterns, the Zeeman studies can lend much help to the identification and assignment of molecular lines.

A new magnet-cavity assembly has now been built which is capable of yielding a magnetic field of the order of 10,000 oersteds. It is planned that more intensive investigations will be carried out for a large class of molecules in both the Zeeman and Paschen-Back spectra.

ACKNOWLEDGMENTS

I am deeply indebted in this work to Professor E. L. Chaffee for much encouragement, Mr. S. P. Cooke for technical advice, and Professors J. H. Van Vleck and E. B. Wilson, Jr. for very valuable suggestions.

PHYSICAL REVIEW VOLUME 74, NUMBER 10 NOVEMBER 15, 1948

Relation between Apparent Shapes of Monoenergetic Conversion Lines and of Continuous β-Spectra in a Magnetic Spectrometer*

G. E. OWEN AND H. PRIMAKOFF
Department of Physics, Washington University, St. Louis, Missouri
(Received August 2, 1948)

A method has been worked out for correcting distortions which affect the apparent shape of a continuous β-ray spectrum. Internal conversion lines are studied in the β-spectrometer and found to have a finite width and in the case of 180° focusing an unsymmetrical shape.

On the assumption that these conversion lines are actually monoenergetic, it is possible to calculate the distortion produced by the spectrometer and source, and thus to determine the approximate true shape of the continuous β-ray spectrum from the experimental data.

The Kurie plots for a continuous β-ray spectrum corrected in this way are, in general, straight over a greater range than the plots of the uncorrected data. The correction has been applied to the ratio of the number of positrons to the number of negatrons in Cu^{64} and gives results in better agreement with the Fermi theory than if there has been no correction.

1. INTRODUCTION

In recent years one of the most important problems in the investigation of the shapes of the continuous β-spectra has been the study of their low energy regions. In the course of such study, discrepancies have been reported between experiment and the Fermi theory, in particular, for the supposedly allowed β^+- and β^--spectra from Cu^{64}.[1] In the present note, a systematic method is described for correcting

* Assisted by the joint program of the Office of Naval Research and Atomic Energy Commission.

[1] C. S. Cook and L. M. Langer, Phys. Rev. **73**, 601 (1948); J. Backus, Phys. Rev. **68**, 59 (1945).

the directly observed shapes of the continuous spectra for distortions of various kinds that occur in a given spectrometer with a given source. These distortions, arise from various factors, e.g., the finite width of an actually monoenergetic line due to the focussing properties of the spectrometer, the elastic and inelastic scattering of beta-particles in the source and its backing, by the slits, chamber walls, and residual gases, etc.

The nature of the corrections is based upon a study of the apparent shapes of monoenergetic internal conversion lines taken with the same spectrometer and with sources of the same thickness as used in the continuous spectrum work.[2] The magnitude of the corrections is in general not negligible, and is in a direction tending to decrease the above mentioned (low energy) discrepancies.

II. METHOD OF CORRECTION

Let $\mathfrak{N}(H)$ be the number of electrons recorded by the detector (counter) of the spectrometer[4] from some continuous β-spectrum at a magnetic field setting H. The maximum electron path radius of curvature and the counter slit width are $\bar{\rho}$ and $\Delta\rho$, respectively.[5] Then, if $N(p')dp'$ is the number of electrons emitted by the nuclei of the source with magnetic rigidity between p' and $p'+dp'$,[6] and if $P(p', p)dp$ is the probability that, because of the various distortions, an electron emitted with magnetic rigidity p' appears at the counter slit at a field H appropriate to a magnetic rigidity between p and $p+dp$, one has,

$$\frac{\mathfrak{N}(H)}{H\Delta\rho}=\frac{\mathfrak{N}(p/\bar{\rho})}{p\Delta\rho/\bar{\rho}}=\frac{1}{\Delta\rho}\int_{\bar{\rho}-\Delta\rho}^{\bar{\rho}}d\rho\left\{\int_0^\infty N(p')P(p', p'')dp'\right\},$$
$$\frac{\mathfrak{N}(p/\bar{\rho})}{p\Delta\rho/\bar{\rho}}=\int_0^\infty dp' N(p')\left\{\frac{1}{H\Delta\rho}\int_{H\bar{\rho}-H\Delta\rho}^{H\bar{\rho}}P(p', p'')dp''\right\}, \quad (1)$$

where $H\bar{\rho}=p$, $H\rho=p''$.

Thus, it is seen that the directly observed $\mathfrak{N}(H)/H\Delta\rho$, the quantity usually considered to give the electron momentum distribution and so compared with theory by means of the Kurie plot, is not equal to the true momentum distribution $N(H\bar{\rho})$ unless $P(p', p'')=\delta(p'-p'')$[3] and $\Delta\rho/\bar{\rho}\approx 0$, i.e., unless the scattering and focusing distortions are absent and the counter slit is very narrow. In the presence of such distortions, one is first confronted with the physical problem of discovering the probability function $P(p', p'')$ or rather the integral

$$\int_{H\bar{\rho}-H\Delta\rho}^{H\bar{\rho}} P(p', p'')dp'',$$

and then with the mathematical problem of deducing

$$N(H\bar{\rho})=N(p),$$

with

$$\int_{H\bar{\rho}-H\Delta\rho}^{H\bar{\rho}} P(p', p'')dp''$$

and $\mathfrak{N}(H)$ being considered known. Now

$$\int_{H\bar{\rho}-H\Delta\rho}^{H\bar{\rho}} P(p', p'')dp''$$

may, for a given spectrometer and source geometry and thickness, be found by studying the observed shapes of monoenergetic lines (with sources of the same geometry and thickness as used in the continuous spectrum work); thus for a monoenergetic line with momentum p_0, one has $N_{p_0}(p)=C(p_0)\delta(p-p_0)$,[7] whence, from Eq. (1),

[2] Corrections to the apparent shapes of β-spectra deduced from the theoretical focusing properties of 180° spectrometers have been discussed by W. H. Henderson, Camb. Phil. Soc. **31**, 285 (1935) and by J. L. Lawson and A. W. Tyler, Rev. Sci. Inst. **11**, 7 (1940).

[3] δ is the usual Dirac delta-function.

[4] To fix ideas, a 180° focusing spectrometer, provided with a fixed source and counter, is envisaged. It will be clear from the ensuing discussion in this and the following sections that the considerations developed apply, with obvious changes, to other types of spectrometers as well.

[5] $\bar{\rho}$ is the distance from the center of the source to the far edge of the counter.

[6] $N(p')dp'$ is the *true* magnetic rigidity or momentum distribution of the continuous β-spectrum.

[7] N_{p_0} is the true momentum distribution of the monoenergetic line; one has $C(p_0)=\int_0^\infty N_{p_0}(p)dp=$ total number of electrons emitted in the conversion.

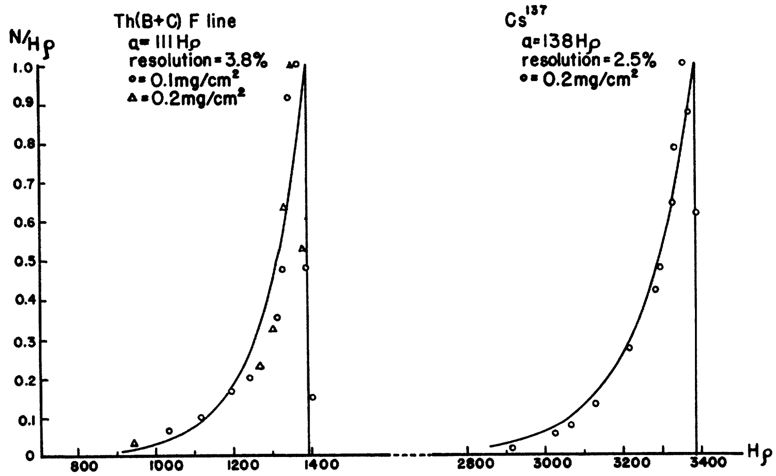

FIG. 1.

$$\frac{\mathfrak{N}_{p0}(H)}{H\Delta\rho}=\frac{\mathfrak{N}_{p0}(p/\bar{\rho})}{p\Delta\rho/\bar{\rho}}=\frac{C(p_0)}{H\Delta\rho}\int_{H\bar{\rho}-H\Delta\rho}^{H\bar{\rho}}P(p_0,p'')dp''. \quad (2)$$

Combining Eqs. (2) and (1), one has,

$$\frac{\mathfrak{N}(p/\bar{\rho})}{p\Delta\rho/\bar{\rho}}=\int_0^\infty dp'N(p')\left\{\frac{\mathfrak{N}_{p'}(p/\bar{\rho})}{C(p')p\Delta\rho/\bar{\rho}}\right\}, \quad (3)$$

so that if the shapes of a sufficient number of monoenergetic lines are observed, one needs only to solve the integral equation,

$$M(p)=\int_0^\infty dp'N(p')K(p',p),$$
$$M(p)=\frac{\mathfrak{N}(p/\bar{\rho})}{p\Delta\rho/\bar{\rho}};\quad K(p',p)=\frac{\mathfrak{N}_{p'}(p/\bar{\rho})}{C(p')p\Delta\rho/\bar{\rho}}, \quad (4)$$

in order to find the true momentum distribution $N(p)$. This last can always be done, at least formally, by writing,

$$N(p)=\int_0^\infty dp'M(p')L(p',p), \quad (5)$$

with, L, the "kernel" reciprocal to the "kernel" P, satisfying,

$$\int_0^\infty dpK(p',p)L(p,p'')=\delta(p'-p'')=\int_0^\infty dpL(p',p)K(p,p''). \quad (6)$$

Thus in principle, and in simple cases in practice (see below) the true momentum distribution of the continuous spectrum, $N(p)$, may be found from the observed number of counts (as a function of $H\bar{\rho}$) divided by $H\Delta\rho$, and from the observed shapes of a sufficient number of monoenergetic lines. An example of the procedure is given in the next section.

III. APPLICATION TO A SMALL 180° FOCUSING SPECTROMETER[8]

In this instrument the monoenergetic internal conversion lines of Cs^{137} (3380 gauss cm) and Th B+C (1385 gauss cm) were studied. The sources were mounted upon a Formvar backing of 0.03 mg/cm² and covered with a Formvar film of 0.005 mg/cm². The total source thickness was 0.2 mg/cm². It was found that the apparent shape of the Thorium line was independent of the thickness below 0.2 mg/cm² by comparing the line shapes for sources of 0.1 mg/cm² and 0.2 mg/cm² thickness. Comparable backing and source thickness were used in a companion study of the β^+- and β^--spectra of Cu^{64}, the Cu^{64} being prepared by an (n,p) reaction on Zn^{64}.[9] The Cu was separated chemically and was found to have a very high specific activity.

The observed shapes of the monoenergetic lines could be reasonably well fitted by a con-

[8] This instrument was constructed in the Washington University Physics Department under the direction of F. N. D. Kurie and F. Rasetti. It has $\bar{\rho}=5.72$ cm and $\Delta\rho=0.2$ cm.

[9] Suggested by F. N. D. Kurie.

venient empirical formula,

$$\frac{\mathfrak{N}_{p_0}(H)}{C(p_0)H\Delta\rho}=\frac{1}{a}\exp\left\{-\frac{(p_0-p)}{a}\right\}, \tag{7}$$

where a is the average spread in $H\bar{\rho}=p$. In Fig. 1 the experimental points of the Cs^{137} line and the Th F line are given with the exponential fit:

Thorium $a=111$ gauss cm,
Cs^{137} $a=138$ gauss cm.

As a very rough approximation, one may then consider three limiting forms for $K(p', p)$.

$$K(p',p)=\frac{1}{a}\exp\left\{-\frac{(p'-p)}{a}\right\},\quad \text{if}\quad p'\geqslant p,$$
$$K(p',p)=0,\quad \text{if}\quad p'<p, \tag{8}$$

for all p' and p, and with

I: $a(p')=\text{const.}$,
II: $a(p')=(\text{const.})p'+\text{const.}$,
III: $a(p')=(\text{const.})p'$.

Using Eq. (8) for $K(p', p)$ and I for $a(p')$ the integral Eq. (4) for $N(p)$ can be solved easily by Fourier analysis. Writing

$$M(p)=\int_{-\infty}^{+\infty}\mu(q)\exp(2\pi ipq)dq,$$

$$N(p)=\int_{-\infty}^{+\infty}\nu(q)\exp(2\pi ipq)dq,$$

and substituting into Eq. (4), one gets

$$\mu(q)=\nu(q)[1-2\pi iaq]^{-1}$$

so that,

$$N(p)=M(p)-a(dM(p)/dp). \tag{9}$$

The form of Eq. (9) maintains equal areas under the $N(p)$ and the $M(p)$ momentum distribution curves.

Using II and III for $a(p')$, Eq. (9) (with $a=(\text{const.})p+\text{const.}$, or $a=(\text{const.})p$) is still a valid approximation provided that

$$da(p')/dp'\ll 1.$$

The true momentum distribution $N(p)$, given by Eq. (9), may now be compared with experiment. Calculations have been made for all three cases of $a(p')$. The corrections for cases I, II, and III are based on the following $a(p')$:

I: $a(p')=(111+138)/2$ gauss cm $=125$ gauss cm,
II: $a(p')=[0.0135p'+92]$ gauss cm,
III: $a(p')=0.05p'$ gauss cm.

Corrections based on I, II, and III give essentially the same results down to the low energy region below 200 kev. Since the experimental measurement of the internal conversion lines gives $a(p')=[0.0135p'+92]$ gauss cm as a rough approximation, this $a(p')$ of case II is used for the graphical presentation of the correction.[10]

We use first in Eq. (9) the directly observed

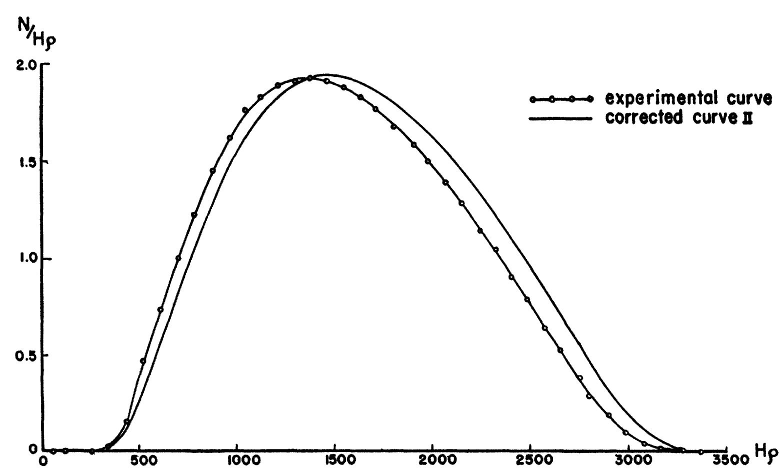

FIG. 2. Momentum distribution of the negatrons from Cu^{64}.

[10] One might expect the $a(p')$ to be of case II for 180° instruments since the apparent width of a monoenergetic line is the sum of a contribution arising from the focusing which is proportional to p', and a contribution arising from the scattering which may be roughly independent of p'.

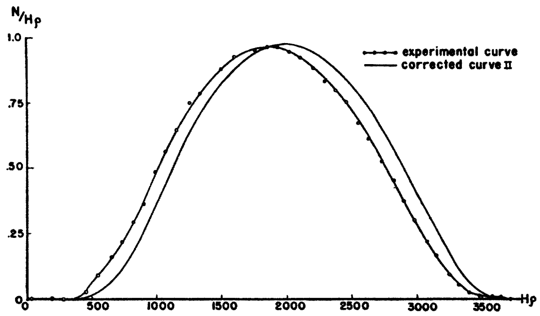

FIG. 3. Momentum distribution of the positrons from Cu^{64}.

$\mathfrak{N}(H)/H\Delta\rho=M(p)$ and then take the correction:

$$a(p')_{II}\frac{dM(p)}{dp}.$$

Figures 2 and 3 show that the experimental momentum distribution is shifted in the direction of the higher momenta after this correction is applied.

Figures 4 and 5 then show the Kurie plots for the β^+- and β^--spectra of Cu^{64}. An apparent decrease in the number of particles in the low energy region as a result of the correction is observed in both cases. In Figs. 6 and 7 the logarithm of the ratio of the number of positrons to the number of negatrons is given for the experimental data $(M(p))$ and the corrected data $(N(p))$.[11] The superior agreement of the latter with the theory is apparent. The limiting cases I and III are shown in Fig. 8 and it is noted that for $a(p')=\text{const.}=125$ gauss cm, the best agreement with theory is obtained.

A shift in the end point of about 5 percent to a higher energy is noted on the corrected Kurie plots. In principle this corrected end point is no more dubious than the uncorrected one since both involve theoretical extrapolations of experimental data.

From the present discussion it is evident that these corrections have no meaning in the region where Geiger window distortions are present. A study of window transmission characteristics is planned in order that corrections for the entire spectrum can be applied.

IV. DISCUSSION

It is clear that while the corrections to the observed momentum distribution discussed in

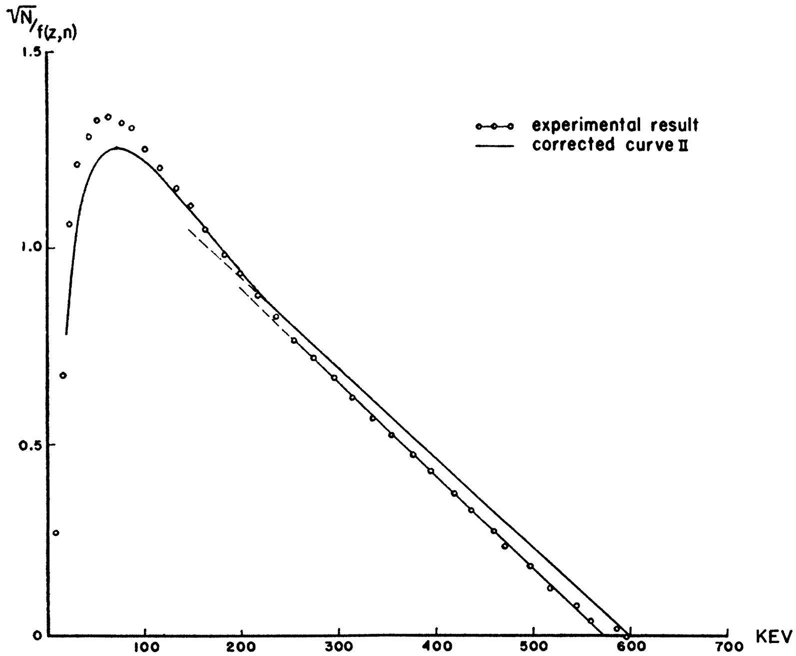

FIG. 4. Kurie plot of the negatrons from Cu^{64}.

[11] Qualitatively, our uncorrected β^+- and β^--distributions agree with those of Cook and Langer, reference 1, though the deviations from the Fermi theory in our original experimental data are greater than in theirs. This is no doubt due to the poorer resolution and greater scattering of our instrument.

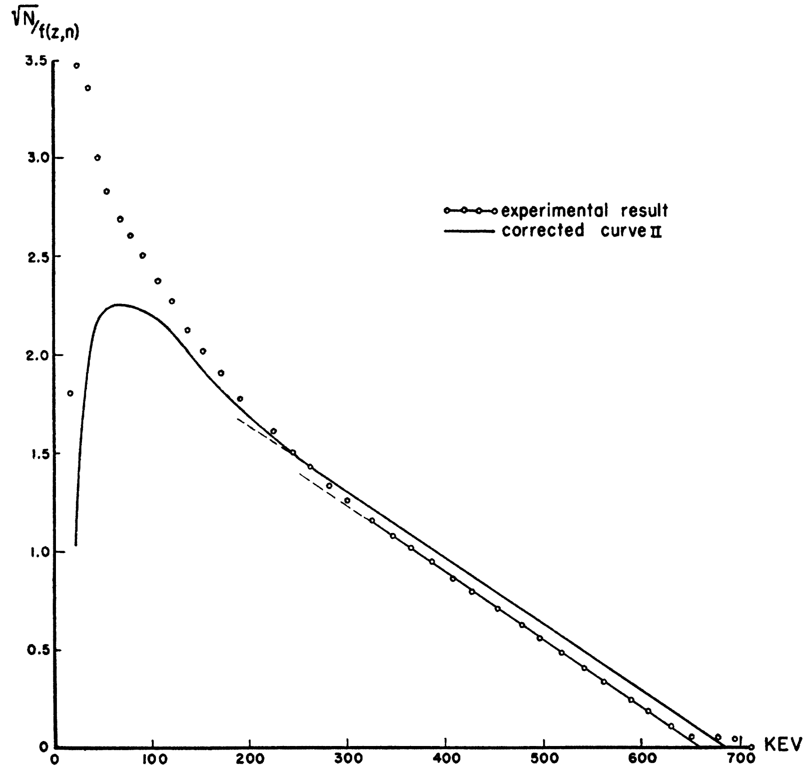

FIG. 5. Kurie plot of the positrons from Cu^{64}.

the present note always exist, it is possible that with low scattering and high resolution instruments they may be quite negligible. For example, if in Eq. (9), $a(p')=0.005p'$, the Kurie plots constructed from the $M(p)$ and the $N(p)$ distributions are very similar. On the other hand, considerable caution must always be exercised in ensuring that the difference between the directly observed $M(p)$ and the corrected $N(p)$ distribution is indeed small with a particular instrument and source; this should be done by an investigation of the apparent shapes of a sufficient number of strategically situated monoenergetic lines[12] and by the subsequent application of Eqs. (4) and (5).

One remark should be added in conclusion. It may be expected (and has indeed been observed in this laboratory)[13] that a thin magnetic lens β-spectrometer gives monoenergetic line shapes which are much more symmetrical in $p'-p$ than those given by a 180° focusing spectrometer. The line shapes given by the thin magnetic lens spectrometer are roughly repre-

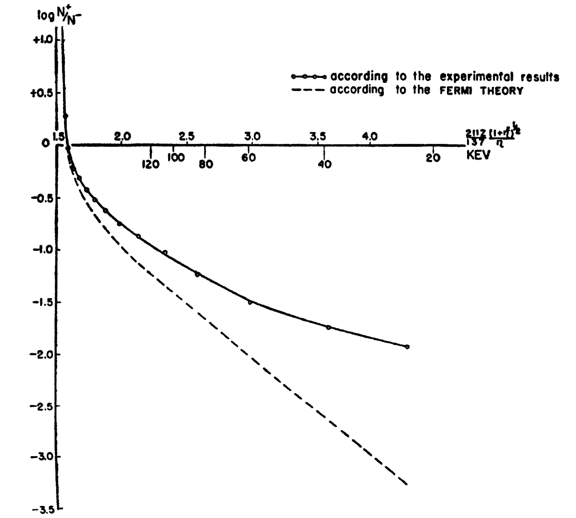

FIG. 6. The log of the ratio of the number of positrons to the number of negatrons in the low energy region for Cu^{64}.

[12] Using sources of the same geometry and stopping power as in the continuous spectrum work.

[13] J. Townsend, Washington University, St. Louis, Missouri.

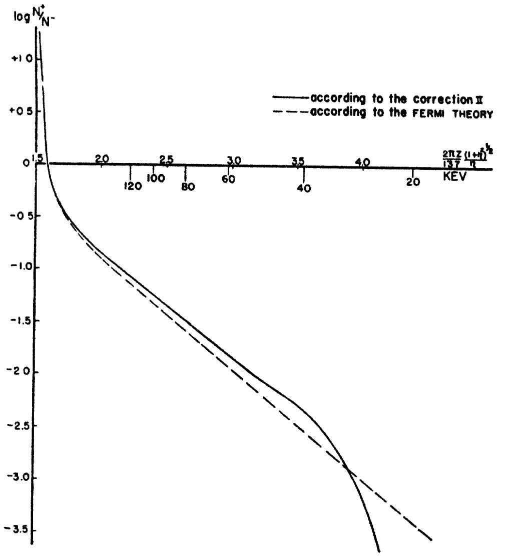

FIG. 7. The log of the ratio of the number of positrons to the number of negatrons in the low energy region for Cu^{64}.

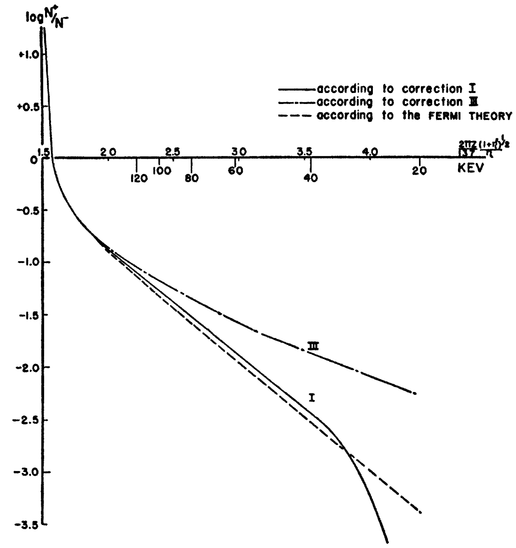

FIG. 8. The log of the ration of the number of positrons to the number of negatrons in the low energy region for Cu^{64}.

sentable by a Gaussian error function:

$$K(p', p) = \frac{1}{a(\pi)^{\frac{1}{2}}} \exp\left\{ -\frac{(p'-p)^2}{a^2} \right\}. \quad (10)$$

In such instruments, using Eq. (4),

$$M(p) = \int_{-p/a}^{\infty} dx N(p+ax) \frac{e^{-x^2}}{(\pi)^{\frac{1}{2}}} \approx N(p) + \frac{a^2}{4} \frac{d^2 N(p)}{dp^2},$$

whence

$$N(p) \approx M(p) - \frac{a^2}{4} \frac{d^2 M(p)}{dp^2}, \quad (11)$$

so that, in general, a much smaller difference between N and M (for a given a) is to be expected for a thin magnetic lens spectrometer than for a 180° spectrometer. It would therefore be highly interesting to carry out a study of the Cu^{64} spectra with a thin magnetic lens spectrometer. Experiments along these lines are to be undertaken in this laboratory.

We are grateful to Professor F. N. D. Kurie and Professor A. L. Hughes for their stimulating comments and discussion; we also wish to thank Mr. David Moe for his aid in obtaining experimental data.