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Further Remarks on the Magnetic Moments of H^3 and He^3

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Arguments for the existence of exchange moments in these nuclei depend on an accurate evaluation of the spin and orbital contributions to the moments. Assumptions made in previous evaluations are re-examined. Although the results do *not* establish the existence of the exchange moment conclusively, arguments are presented which make the alternative appear to be less reasonable. Methods for obtaining further information on this question are discussed.

I. INTRODUCTION

THE recent measurement[1] of the magnetic moment of He^3 has been interpreted[2] as a strong indication of the existence of exchange moments in the nuclei of H^3 and He^3. The term exchange moment is meant to refer to any contribution to the magnetic moment of a system which cannot be accounted for by simply adding the spin and orbital moments of the nucleons making up the nucleus. On the basis of rather general assumptions concerning the nuclear wave functions, formulae have been given[3] for the spin and orbital contributions to the moments of these nuclei, and the failure of the formulae to agree with the measured values has led to the aforementioned interpretation. In view of the importance that may be attached to the existence of an exchange moment with regard to the understanding of nuclear interactions,[4,5] it seems necessary to investigate more fully the assumptions made in reference 3 in order to verify their validity.

The failure to obtain agreement occurs by a very small margin. This can be seen from Fig. 1, which displays both the linear relationship between the ^2P, ^4P and ^4D state probabilities imposed by the sum of the observed moments (see Eq. (1)) and the relationship required by the H^3 moment alone (Fig. 1 of reference 3). Agreement would correspond to the intersection of the curves referring to the same value of the ^4P probability. Although the curves fail to intersect, they almost intersect in the neighborhood of zero probability for the ^4D state if the ^2P and ^4P state probabilities are greater than 20 percent and 15 percent respectively. Small corrections, such as the effect of coulomb repulsion between protons on the He^3 wave function or relativistic corrections,[6] may overcome the difference. Therefore the experimental evidence does not conclusively establish the existence of the exchange moment. The alternative description requires, however, that the moments be understood in terms of a ground state for which the amplitudes of the ^2P and ^4P functions are much larger than that of the ^4D function. Such a description is objectionable on aesthetic grounds while the concept of an exchange moment is most acceptable in view of current ideas concerning nuclear forces. But this argument does not constitute a proof.

A more likely argument may be based on the results of rough binding energy calculations for the nuclei in question. These calculations[7] indicate that the introduction of any amount of P state decreases the binding even in the presence of a tensor interaction. Therefore it would seem that the conventional two-body interaction would lead to a wave function containing little or no P state. The best choice of wave function appears[8] to consist of about 96 percent ^2S state and 4 percent ^4D state. Now information con-

[1] H. L. Anderson and A. Novick, Phys. Rev. **73**, 919 (1948). The value used here is that presented at the Washington meeting of the American Physical Society, 1948.
[2] H. L. Anderson, Phys. Rev. **73**, 919 (1948).
[3] R. G. Sachs, Phys. Rev. **72**, 312 (1947). Fig. (2) of this paper is in error and therefore should not be used for the interpretation of any results.
[4] F. Villars, Phys. Rev. **72**, 256 (1947), F. Villars, Helv. Phys. Acta **XX**, 476 (1947), A. Thellung and F. Villars, Phys. Rev. **73**, 924 (1948).
[5] R. G. Sachs, Phys. Rev. **74**, 433 (1948).
[6] H. Primakoff, Phys. Rev. **72**, 118 (1947); G. Breit and I. Bloch, *ibid.* 135 (1947); and R. G. Sachs, *ibid.* 91 (1947).
[7] M. Goeppert-Mayer and R. G. Sachs, Phys. Rev. **73**, 185 (1948).
[8] E. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942).

cerning the ^2S, ^2P, ^4P, and ^4D state probabilities may also be obtained from the sum of the magnetic moments of the two nuclei by application of the formula[9]

$$\mu(\mathrm{H}^3)+\mu(\mathrm{He}^3)=\mu_p+\mu_n-2(\mu_p+\mu_n-\tfrac{1}{2})\times(3^4D-{}^4P+2^2P)/3, \quad (1)$$

where ^2P, ^4P, and ^4D represent the probabilities in question. This formula is probably valid even in the presence of exchange currents since the exchange moments appear to be equal and opposite for the two nuclei.[5,10] If we insert the experimental value[11]

$$\mu(\mathrm{H}^3)/\mu_p=1.066636\pm0.00001 \quad (2)$$

into Eq. (1) we obtain

$$\mu(\mathrm{He}^3)/\mu_p=-0.7514+0.0906(3D-{}^4P+2^2P). \quad (3)$$

Now taking the values $^2S=0.96$, $^4D=0.04$, which are suggested by the binding energy calculations, we find

$$\mu(\mathrm{He}^3)/\mu_p=-0.7623 \quad (4)$$

which is in agreement with the experimental[1] value

$$\mu(\mathrm{He}^3)/\mu_p=-0.7618\pm0.0008. \quad (5)$$

Of course this agreement may be fortuitous, but one can at least argue that it is a remarkable coincidence. This argument would appear to provide sufficient justification for rejecting the P function in favor of the simpler interpretation until contradictory evidence is obtained from other sources.

Having accepted this conclusion, it is ***not*** now possible to fit the formulas of reference 3 to the observed moments. However, there are still simplifying assumptions in reference 3 that could conceivably lead to a large error in the formulas and thereby invalidate the argument for an exchange current. In particular there is the assumption that the wave functions have no angular dependence other than the minimum required to provide the necessary symmetry properties of the ^2S, ^2P, ^4P, and ^4D functions (see Eq. (35) of reference 3). As a consequence the cross term between the ^2S and ^2P functions in the theoretical expression for the magnetic moment vanishes. Since the amplitude of the ^2S function is presumed to be large, this term might be important for a very small but finite ^2P amplitude in the event that the original assumption were in error.

II. THE S-P CROSS TERM

The reason for making the particular choice of wave function which leads to no cross term is that the wave function always adjusts itself in such a way as to minimize the total energy. Any extra angular dependence would seem to increase the energy by adding to the kinetic energy. In order to demonstrate that this is indeed the case, we now consider the expression for the magnetic moment of H^3 which is obtained under the three assumptions:

(1) There is no exchange moment.
(2) The ground state is predominantly (96 percent) a ^2S function with a small (4 percent) admixture of ^4D function and an even smaller admixture of ^2P function.
(3) There is no limitation on the angular dependence of the ^2S and ^2P functions.

If the coefficients in the wave function are chosen in such a way as to give a maximum importance to the $^2S-{}^2P$ cross term, the expression for

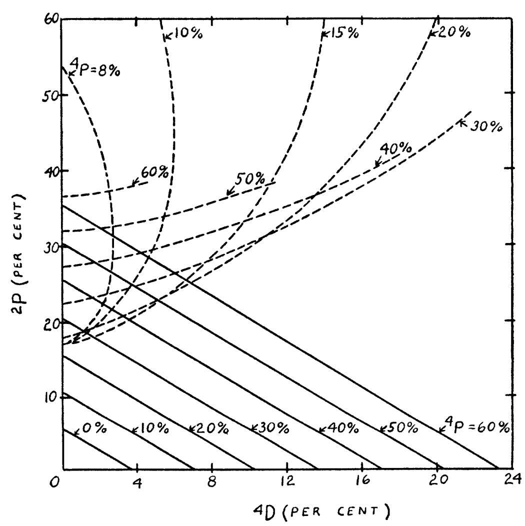

FIG. 1. Solid lines give relation between ^2P, ^4P, and ^4D state probabilities required to account for the sum of the observed moments of H^3 and He^3. Dashed lines give relation between probabilities required by H^3 moment alone on the basis of Eq. (43), reference 3.

[9] R. G. Sachs and J. Schwinger, Phys. Rev. **70**, 41 (1946).
[10] P. Morrison, Bull. Am. Phys. Soc. **23**, 28 (1948).
[11] H. L. Anderson and A. Novick, Phys. Rev. **71**, 372 (1947); Bloch, Graves, Packard and Spence, Phys. Rev. **71**, 373 and 551 (1947).

the H^3 moment becomes[3]

$$\mu(H^3)=\mu_p-(4/3)^2P\mu_p-(2/3)D(2\mu_p+\mu_n)+(2/9)^2P+(1/3)D+I(^2PS)^{\frac{1}{2}}, \quad (6)$$

where

$$I=2/9\int\int\int(1-q^2)qf_3f_1'r^4\rho^4drd\rho dq. \quad (7)$$

$\boldsymbol{\rho}$ is the distance between the neutrons, $\mathbf{r}$ the distance from the center of the neutrons to the proton and $q=(\mathbf{r}\cdot\boldsymbol{\rho})/r\rho$. The functions f_1 and f_3 are the radial ^2S and ^2P functions and $f_1'=df_1/dq$. Since we are interested only in the case of a very small P state probability, the condition has been imposed that $^2P\leq 0.04$. In order that Eq. (6) lead to the observed value of $\mu(H^3)$ we find

$$I\geq 2.13. \quad (8)$$

It can be shown that if f_3 does not depend on q, the condition Eq. (8) cannot be satisfied by virtue of the normalization conditions on the radial functions. In order to obtain a maximum value of I with minimum angular dependence, we assume that f_3 has the same angular dependence as f_1, which has been taken to be

$$f_1=c_1[\exp(-\gamma^2q^2)f_0(r,\rho)], \quad (9)$$

where f_0 is a Gauss function. It is then found that $I\approx 4\gamma/9$. Thus if Eq. (8) is to be satisfied, then

$$\gamma\geq 4.7. \quad (10)$$

Now the contribution of the S state to the kinetic energy may also be determined on the basis of Eq. (9). The result is

$$\text{K.E.}=6\gamma^2\alpha\hbar^2/M, \quad (11)$$

where $2/\alpha$ is the mean square distance characteristic of the Gauss function f_0. Taking[7] $\alpha=0.7(mc^2/e^2)^2$ we find a kinetic energy greater than 400 Mev, a value which is too great by at least a factor of ten.

From this it is clear that no reasonable wave function consisting predominantly of an S function will produce agreement between the observed moment and Eq. (6). Therefore the exchange moment appears to be required to account for the observations if the ground state of H^3 contains only the S and D functions to an appreciable extent.

III. FURTHER EXPERIMENTAL POSSIBILITIES

Although the arguments for the existence of the exchange current appear to be rather convincing, they cannot be said to be conclusive. Corroborative experiments would be very desirable. One experiment which would appear to be most promising for this purpose is the accurate measurement of the hyperfine splitting of the ground state of tritium. In view of the explanation by A. Bohr[12] of the corresponding anomaly[13] for deuterium, the anomaly for tritium would be expected to depend to some extent on the nature of the moment, i.e., on the relative contributions of orbital and exchange moments. However, a rough estimate of the effect indicates that either explanation of the triton moment (i.e., exchange moment or large orbital moment) could lead to an anomaly in the hyperfine splitting of the order of five percent of the deuteron anomaly. Therefore a high degree of accuracy would seem to be required of both theory and measurements before this experiment will settle the question.

Additional information could be obtained from the photo-disintegration cross sections of H^3 and He^3 or the neutron and proton capture cross sections of the deuteron. These cross sections will depend on the nature of the H^3 or He^3 wave function.[14] Furthermore, the interaction of a nucleus with radiation is modified by the presence of exchange currents.[5] Consequently the cross sections for these radiative processes will depend in a detailed way on the nature of the exchange current.

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[12] A. Bohr, Phys. Rev. **73**, 1109 (1948).

[13] J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947); D. E. Nagle, R. S. Julian, and J. R. Zacharias, Phys. Rev. **72**, 971 (1947).

[14] This point was called to the authors' attention independently by L. Schiff and H. Primakoff.