for several unidentified lines observed by A. Adel,[3] in Flagstaff, between 7.2 and 8.1μ.

Details concerning the identification of the 7.7μ fundamental band of CH_4 in the solar spectrum will be published in the Astrophysical Journal.

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On the Intrinsic Moment of the Electron

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In a recent paper Foley and Kusch[1] have reported observations which give, for the g value of the electron,

$$g_S = 2.00244 \pm 0.00006.$$

It is perhaps of interest to mention that in a paper[2] dealing with an attempt to seek the solution of the problem of elementary particles with the help of a non-Euclidian geometry, I obtained for the g value (reference 1, Eqs. (10) and (8)), if the proper mass defect of the electron is also considered (Eqs. (14), (12)):

$$g_S = 2(\mu)_s^* = (\beta_s I_s/S_s)(m_s^*/m_s) = 2.00343.$$

This value is in agreement with the experimental result, and it does not involve the existence of an intrinsic moment of the electron.

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On a Refinement of the W.K.B. Method

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As is well known, the W.K.B. method is a very powerful mathematical tool in the application of quantum mechanics. Moreover, it can also be applied with success to problems in other fields of mathematical physics, e.g., propagation of electromagnetic waves, high speed flow of gases, etc. The method is essentially concerned with the asymptotic solution of the second-order differential equation:

$$d^2\Phi/dx^2 + k^2P(x)\Phi = 0 \quad \text{for} \quad k \rightarrow \infty. \tag{1}$$

According as $P(x)$ is positive or negative, the solution Φ is known to behave like a sine or an exponential function. The connection between the regions where $P(x)$ is positive and negative can be established by the various procedures due to Kramers,[1] Zwaan,[2] Langer,[3] and Furry.[4] But none of these procedures except Langer's provides a reliable information as to the behavior of Φ near the turning point, at which $P(x)$ changes the sign.

In an attempt to apply the W.K.B. method to the two-dimensional flow of a compressible fluid, the present writer developed a procedure which turned out to be essentially the same as that adopted by Langer. Recently a further refinement of the procedure has been achieved so as to give better approximation to Φ in the neighborhood of the turning point.

If we introduce the variables z and Ψ, defined by

$$z = \int P^{\frac{1}{2}}dx, \quad \Phi = P^{-\frac{1}{4}}\Psi, \tag{2}$$

Eq. (1) becomes

$$d^2\Psi/dz^2 + (k^2 - Q)\Psi = 0, \tag{3}$$

where

$$Q = P^{-\frac{1}{4}}d^2(P^{\frac{1}{4}})/dz^2 = -P^{-\frac{3}{4}}d^2(P^{-\frac{1}{4}})/dx^2. \tag{4}$$

Except in the immediate neighborhood of the turning point Q is finite, so that for $k \rightarrow \infty$ (3) may be asymptotically solved by

$$\Psi \sim Ae^{ikz} + Be^{-ikz} \quad \text{or} \quad A'e^{k|z|} + B'e^{-k|z|}, \tag{5}$$

according as P is positive or negative.

Next, taking $x = 0$ to be a turning point, let us assume that $P(x)$ can be developed in a power series near $x = 0$,

$$P = a_1x + a_2x^2 + a_3x^3 + \cdots. \tag{6}$$

Then, we have, by (2) and (4),

$$z = (2/3)a_1^{\frac{1}{2}}x^{\frac{3}{2}}\{1 + (3a_2/10a_1)x - \cdots\}, \tag{7}$$

$$\begin{aligned} Q = -(5/36)z^{-2}\{1 &+ (48/175)(3a_2^2 - 5a_1a_3)a_1^{-4}(3a_1z/2)^{4/3} \\ &- (64/375)(14a_2^3 - 35a_1a_2a_3 \\ &+ 25a_1^2a_4)a_1^{-6}(3a_1z/2)^2 + \cdots\}. \end{aligned} \tag{8}$$

Now, it can be shown without difficulty that the differential equation

$$d^2\Psi_1/dz^2 + \{\kappa^2 + (5/36)z^{-2} + \lambda(3z)^{-\frac{2}{3}}\}\Psi_1 = 0 \tag{9}$$

is satisfied by

$$\Psi_1 = z^{1/6}\xi^{\frac{1}{2}}Z_{\frac{1}{3}}(3^{-1}\kappa\xi^{\frac{3}{2}}), \quad \xi = (3z)^{\frac{2}{3}} + \lambda\kappa^{-2}, \tag{10}$$

where $Z_{\frac{1}{3}}$ is the Bessel function of order $\frac{1}{3}$.

Comparison of (3), (8), and (9) shows that Ψ_1 provides a very good approximation to Ψ, if we take

$$\lambda = (12/35)(3a_2^2 - 5a_1a_3)(2a_1^2)^{-4/3},$$
$$\kappa^2 = k^2 - (4/75)(14a_2^3 - 35a_1a_2a_3 + 25a_1^2a_4)a_1^{-4}.$$

Thus

$$\Phi_1 = P^{-\frac{1}{4}}z^{1/6}\xi^{\frac{1}{2}}Z_{\frac{1}{3}}(3^{-1}\kappa\xi^{\frac{3}{2}}) \tag{11}$$

is a very good approximation to Φ in the neighborhood of $x = 0$. It may be interesting to note that Φ_1 is a one-valued function of x.

If k and hence κ are large, (10) reduces to

$$\Psi_1 = z^{\frac{1}{2}}Z_{\frac{1}{3}}(kz), \tag{12}$$

which is just the same as Langer's result. It should be remarked, however, that the above reduction is valid only so long as z and hence x are not too small except when λ vanishes exactly. In fact, Langer's expression (12) satisfies Eq. (9) with $\lambda = 0$, which can be an approximation to (3) only to the order of z^{-2}. Further it may be mentioned that the connection formulas can be readily obtained from (11) by use of the asymptotic expression for the Bessel function.

Details of the analysis as well as applications will be given elsewhere.

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