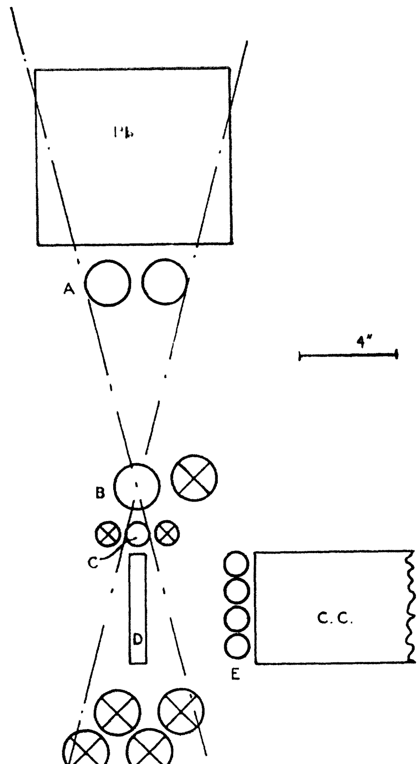

FIG. 1. Cosmic-ray telescope. Counters ABC and E are in coincidence. Crossed counters are anticoincidence. D is 2-cm carbon absorber. C.C. is cloud chamber.

steradians; hence, the expected number of expansions was about one in two hours. The number actually occurring was one per hour. Two hundred hours of operation were selected during which the apparatus was performing properly. Only 17 pictures were obtained which could in any way be interpreted as decay electrons. Of these, 12 penetrated only the first or second compartments of the cloud chamber and are classified as doubtful. Two more tracks appear to be electrons which left the chamber without stopping or scattering. Descriptions of the other three tracks are given below.

A track was identified as a decay electron by its ionization and scattering. Neon lights, photographed with the track, indicated which of the side counters was discharged by the decay electron and served as a further check on the identification of the particle. The energy of the electron was determined from its range by the Bethe-Block formula,[1] including radiation loss.

Track F2-9 stops in the third plate and shows some scattering. Its range is 1.58 ± 0.32 cm Al $+ 0.16$ cm Cu $+ 1 \pm 1$ cm carbon. This is consistent with an energy of 13 ± 3 Mev.

Track F2-11 penetrates two plates and shows large scattering in the gas of the second compartment. It was assumed that as it left the chamber through the side wall, it had negligible energy. Its range is 1.96 cm Al $+ 0.25$ cm Cu $+ 1 \pm 1$ cm carbon, which corresponds to an energy of 18 ± 4 Mev.

Track F4-5 penetrates eight plates showing scattering and leaves the chamber without stopping. From the scattering formula of Williams,[2] $E\bar{\alpha}_1$ is a constant for a given plate, where $\bar{\alpha}_1$ is the average projected angle of scattering and E the energy. The values of $E\alpha_1$ for this track were computed from seven observations of the scattering. Comparison of the average $E\alpha_1$ with the Williams value gave 15 ± 5 Mev for the most probable energy with which the electron left the chamber. This corresponds to an initial energy for the electron of 50 ($+15$, -10) Mev.

A particle detected only in the first compartment was counted as natural background and not identified as a decay electron. The average energy for a particle to penetrate the second compartment where the criteria for decay electrons could be applied was 14 Mev for most of these experiments. If all decay electrons had energies greater than this value, we should have observed at least 50 tracks. This estimate is based on the solid angle subtended by the third plate at the carbon absorber and the calculated value of $\frac{1}{2}$ percent for the number of mesons stopped. Actually, only five possible, and twelve additional, doubtful decay tracks were found.

If the Fermi density correction[3] is appreciable, the ranges of the decay electrons for a given disintegration energy would be increased, and the number expected in the chamber on the basis of the Bethe-Bloch formula is presumably a lower limit. The energies given here would then have to be corrected downward for this effect.

The results here cited would appear difficult to reconcile with a monochromatic energy for μ-meson electron decay. A possible explanation is an energy distribution for decay electrons with a maximum frequency below 14 Mev.

We gratefully acknowledge the assistance of Professor I. S. Lowen, who suggested this problem.

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[1] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1936).
[2] E. J. Williams, Proc. Roy. Soc. **169**, 531 (1939).
[3] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).

Lines of Methane at 7.7μ in the Solar Spectrum*

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IN an earlier publication,[1] the presence of 13 lines of methane was reported in the 3.3μ region of the solar spectrum. Many lines were also found in the 7.7μ region, showing a possible correspondence between the laboratory data obtained on CH_4 by A. H. Nielsen and H. H. Nielsen[2] and the solar observations of A. Adel[3] published in 1941. However, identifications in this region are complicated by the presence of water vapor lines.

Recently, we have obtained improved spectra of water vapor, under laboratory conditions, between 7.2 and 8.2μ by using a more sensitive detector.

The introduction of methane into the optical path gave spectra which permitted a direct comparison with previously obtained solar spectra in the 7.7μ region.

This comparison has shown that methane is responsible for several unidentified lines observed by A. Adel,[3] in Flagstaff, between 7.2 and 8.1μ.

Details concerning the identification of the 7.7μ fundamental band of CH_4 in the solar spectrum will be published in the Astrophysical Journal.

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[1] Marcel V. Migeotte, Phys. Rev. **73**, 519 (1948); Ap. J., in print.
[2] A. H. Nielsen and H. H. Nielsen, Phys. Rev. **48**, 864 (1935).
[3] A. Adel, Ap. J. **94**, 451 (1941).

On the Intrinsic Moment of the Electron

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IN a recent paper Foley and Kusch[1] have reported observations which give, for the g value of the electron,

$$gs=2.00244\pm 0.00006.$$

It is perhaps of interest to mention that in a paper[2] dealing with an attempt to seek the solution of the problem of elementary particles with the help of a non-Euclidian geometry, I obtained for the g value (reference 1, Eqs. (10) and (8)), if the proper mass defect of the electron is also considered (Eqs. (14), (12)):

$$gs=2(\mu)_s{}^*=(\beta_s I_s/S_s)(m_s{}^*/m_s)=2.00343.$$

This value is in agreement with the experimental result, and it does not involve the existence of an intrinsic moment of the electron.

[1] H. M. Foley and P. Kusch, Phys. Rev. **73**, 412 (1948).
[2] J. Barnóthy, Papers of Terrestrial Magnetism, Hungary, No. 2, (1947). See review in Nature **160**, 847 (1947).

On a Refinement of the W.K.B. Method

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AS is well known, the W.K.B. method is a very powerful mathematical tool in the application of quantum mechanics. Moreover, it can also be applied with success to problems in other fields of mathematical physics, e.g., propagation of electromagnetic waves, high speed flow of gases, etc. The method is essentially concerned with the asymptotic solution of the second-order differential equation:

$$d^2\Phi/dx^2+k^2P(x)\Phi=0 \quad \text{for} \quad k\rightarrow\infty. \tag{1}$$

According as $P(x)$ is positive or negative, the solution Φ is known to behave like a sine or an exponential function. The connection between the regions where $P(x)$ is positive and negative can be established by the various procedures due to Kramers,[1] Zwaan,[2] Langer,[3] and Furry.[4] But none of these procedures except Langer's provides a reliable information as to the behavior of Φ near the turning point, at which $P(x)$ changes the sign.

In an attempt to apply the W.K.B. method to the two-dimensional flow of a compressible fluid, the present writer developed a procedure which turned out to be essentially the same as that adopted by Langer. Recently a further refinement of the procedure has been achieved so as to give better approximation to Φ in the neighborhood of the turning point.

If we introduce the variables z and Ψ, defined by

$$z=\int P^{\frac{1}{2}}dx, \quad \Phi=P^{-\frac{1}{4}}\Psi, \tag{2}$$

Eq. (1) becomes

$$d^2\Psi/dz^2+(k^2-Q)\Psi=0, \tag{3}$$

where

$$Q=P^{-\frac{1}{4}}d^2(P^{\frac{1}{4}})/dz^2=-P^{-\frac{3}{4}}d^2(P^{-\frac{1}{4}})/dx^2. \tag{4}$$

Except in the immediate neighborhood of the turning point Q is finite, so that for $k\rightarrow\infty$ (3) may be asymptotically solved by

$$\Psi\sim Ae^{ikz}+Be^{-ikz} \quad \text{or} \quad A'e^{k|z|}+B'e^{-k|z|}, \tag{5}$$

according as P is positive or negative.

Next, taking $x=0$ to be a turning point, let us assume that $P(x)$ can be developed in a power series near $x=0$,

$$P=a_1x+a_2x^2+a_3x^3+\cdots. \tag{6}$$

Then, we have, by (2) and (4),

$$z=(2/3)a_1^{\frac{1}{2}}x^{\frac{3}{2}}\{1+(3a_2/10a_1)x-\cdots\}, \tag{7}$$

$$Q=-(5/36)z^{-2}\{1+(48/175)(3a_2^2-5a_1a_3)a_1^{-4}(3a_1z/2)^{4/3} -(64/375)(14a_2^3-35a_1a_2a_3 +25a_1^2a_4)a_1^{-6}(3a_1z/2)^2+\cdots\}. \tag{8}$$

Now, it can be shown without difficulty that the differential equation

$$d^2\Psi_1/dz^2+\{\kappa^2+(5/36)z^{-2}+\lambda(3z)^{-\frac{4}{3}}\}\Psi_1=0 \tag{9}$$

is satisfied by

$$\Psi_1=z^{1/6}\xi^{\frac{1}{2}}Z_{\frac{1}{3}}(3^{-1}\kappa\xi^{\frac{3}{2}}), \quad \xi=(3z)^{\frac{2}{3}}+\lambda\kappa^{-2}, \tag{10}$$

where $Z_{\frac{1}{3}}$ is the Bessel function of order $\frac{1}{3}$.

Comparison of (3), (8), and (9) shows that Ψ_1 provides a very good approximation to Ψ, if we take

$$\lambda=(12/35)(3a_2^2-5a_1a_3)(2a_1^2)^{-4/3},$$
$$\kappa^2=k^2-(4/75)(14a_2^3-35a_1a_2a_3+25a_1^2a_4)a_1^{-4}.$$

Thus

$$\Phi_1=P^{-\frac{1}{4}}z^{1/6}\xi^{\frac{1}{2}}Z_{\frac{1}{3}}(3^{-1}\kappa\xi^{\frac{3}{2}}) \tag{11}$$

is a very good approximation to Φ in the neighborhood of $x=0$. It may be interesting to note that Φ_1 is a one-valued function of x.

If k and hence κ are large, (10) reduces to

$$\Psi_1=z^{\frac{1}{2}}Z_{\frac{1}{3}}(kz), \tag{12}$$

which is just the same as Langer's result. It should be remarked, however, that the above reduction is valid only so long as z and hence x are not too small except when λ vanishes exactly. In fact, Langer's expression (12) satisfies Eq. (9) with $\lambda=0$, which can be an approximation to (3) only to the order of z^{-2}. Further it may be mentioned that the connection formulas can be readily obtained from (11) by use of the asymptotic expression for the Bessel function.

Details of the analysis as well as applications will be given elsewhere.

[1] H. A. Kramers, Zeits. f. Physik **38**, 518 (1926).
[2] A. Zwaan, Thesis, Utrecht (1929).
[3] R. E. Langer, Trans. Am. Math. Soc. **33**, 23 (1931); **34**, 447 (1932); Phys. Rev. **51**, 669 (1937).
[4] W. H. Furry, Phys. Rev. **71**, 360 (1947).