for Po-α-energies. It definitely does not occur even at 16 Mev α-energy.[2]

5. $Mg^{25}(\alpha,n)Si^{28}$ process is exoergic by a mass defect of $+2.31\pm0.8$ TMU. It is energetically possible that the γ-radiation could belong to this process. The question could be decided by exact experimental determination of the excitation function and the absolute yield of this transmutation.

6. $Mg^{26}(\alpha,n)Si^{29}$, this transmutation having a mass defect of -1.5 ± 0.9 TMU may be connected with the γ-radiation as well, but it is as little investigated experimentally as process (5). We are intending in the near future to carry out measurements about the yield and the excitation function of the neutron emission of Mg, when bombarded by Po-α-particles.

SUMMARY

Mg hemispheres were bombarded by a very pure Po-α-source of small diameter from the center. The excitation function of the short-living artificial radioactivity belonging to the process $Mg^{25}(\alpha,p)Al^{28}$ has been investigated (Fig. 3). The absolute yield of this transmutation has been determined very carefully. It has a value of 1.4×10^{-7} transmutations/bombarding α-particle of 5.3 Mev energy, in a thick Mg layer consisting of the natural isotope mixture of the Mg isotopes.

The excitation function of the γ-radiation, which is excited in Mg by Po-α-particles, was investigated (Fig. 5).

The absolute yield of the γ-radiation has been determined very reliably by direct comparison with the ThC″-γ-radiation of a ThC+C′+C″ preparation of exactly known strength. The absolute yield of the γ-radiation excited in a thick Mg layer of natural isotopic composition under bombardment of full energy Po-α-particles amounts to 5.2×10^{-7} γ-quanta/α-particle.

A discussion of the origin of the γ-radiation is given. Four of the six possible processes can be excluded by considerations of energy, or by the comparison of the absolute yields as well. Two remain as possible origins of the γ-radiation, $Mg^{25}(\alpha,n)Si^{28}$ being the most probable by considerations of energy.

Further investigations in these processes are intended at this Institute.

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New Aspects of the Photon Self-Energy Problem

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A finite but non-vanishing value for the self-energy of the photon, corresponding to a finite rest-mass, can be deduced from the new invariant formulation of quantum electrodynamics developed by Tomonaga and Schwinger, in the e^2 order approximation. The implications of this result are discussed.

INTRODUCTION

THE new development in quantum electrodynamics has led to the conviction that the anomalies of the hydrogen $2S$ level and of the magnetic moment of the electron can be explained in terms of field-dependent parts of the electronic self-energy.[1] Accordingly, it seems that the concept of electromagnetic self-energy now acquires a more than merely mathematical significance. However, the field-dependent terms can, at best, be defined as finite parts of the still diverging, total self-energy of the electron which has to be eliminated from the Hamiltonian by a formal readjustment. Therefore, there is still but little hope for a final and satisfactory solution of the self-energy problems within the framework of the conventional quantum theory of fields.

[1] J. Schwinger and V. Weisskopf, Phys. Rev. **73**, 1272 (1948); J. Schwinger, Phys. Rev. **73**, 416 (1948).

Nevertheless, just these fundamental questions ought to be examined with particular care.

Even more involved than the electron self-energy problem is that of the photon. It is well known that, according to Dirac's theory of the positron, a photon can virtually create and re-absorb electron-positron pairs, which process gives rise to a self-energy that is divergent even in Heisenberg's elaborate subtraction formalism.[2] On the other hand, Schwinger claims that, in his new formulation of quantum electrodynamics, the photon self-energy vanishes identically.[3] However, the proof which Schwinger presented in his report at the Pocono Manor Conference, March 30–April 2, 1948, is highly objectionable.* Indeed, as we wish to show, if the same method of calculation is applied as is used by Schwinger in computing the electron self-energy, the self-energy of the photon turns out to be finite but not zero, whereas other methods of calculation yield infinite values.

A non-vanishing self-energy, or rest-mass, of the photon, formally appearing in the theory, is, of course, a very undesirable feature since, even if it can be subtracted, it will destroy the gauge-invariance of the theory at least at some stage of the mathematical development, so that the electromagnetic potentials will appear as being observable quantities. Moreover, the subtraction of the lowest order self-energy terms ($\sim e^2$) will entail higher order corrections whose physical significance is rather doubtful.

THE FUNDAMENTAL EQUATIONS

The quantum theory of the electromagnetic field, as developed by Schwinger, is based on the generalized multiple-time formulation of the quantum theory of fields which was first proposed by Tomonaga.[4] Considering the electrons as specified particles $n=1, 2, \cdots$, Dirac, Fock, and Podolsky[5] introduced a state functional Ψ depending on the individual time coordinates t_n of the electrons as well as on their space and spin coordinates. In Tomonaga's theory, where all particles are described by quantized wave fields, every volume element (at x, y, z) is associated with an individual time $t(xyz)$, the function $t(xyz)$ defining a surface σ in the four-dimensional x, y, z, t space. The state functional Ψ is supposed to be a function of σ, i.e., of the function $t(xyz)$, and the Schrödinger equation determines the change of $\Psi(\sigma)$ in the event that the surface σ is displaced infinitesimally only at a particular point of the surface.

Let the surface be shifted at the point $x=(ct, x, y, z)$ over a volume $d^4x=cdtdxdydz$, and the corresponding change of $\Psi(\sigma)$ be denoted by $d^4x\partial\Psi(\sigma)/\partial\sigma(x)$, then the generalized Schrödinger equation reads:

$$i\hbar c(\partial\Psi(\sigma)/\partial\sigma(x))=\mathcal{H}(x)\Psi(\sigma), \qquad (1)$$

where $\mathcal{H}$ is the Hamiltonian density representing the interaction of electrons and the electromagnetic field:

$$\mathcal{H}(x)=-(1/c)j_\mu(x)A_\mu(x), \qquad (2)$$

j_μ = electric current density produced by electrons and positrons, A_μ = electromagnetic potentials explicitly depending on space and time coordinates according to the wave equations[6]

$$(\partial^2/\partial x_\nu^2)A_\mu(x)=0, \qquad (3)$$

and the invariant commutation rules

$$[A_\mu(x), A_\nu(x')]=(\hbar c/i)\delta_{\mu\nu}D(x-x'). \qquad (4)$$

Both j_μ and A_μ are operators, operating on the field variables (occupation numbers) of which $\Psi(\sigma)$ is a function. In order that (1) be integrable, $\mathcal{H}(x)$ must commute with $\mathcal{H}(\bar{x})$ taken at any other point $\bar{x}$ on σ. This condition is satisfied for all spacelike surfaces (i.e., when each two surface points are spacelike to each other).

In addition, we have the supplementary conditions

$$(\partial A_\mu(x)/\partial x_\mu)\Psi(\sigma)=0, \qquad (5)$$

for all points x on the surface σ. As in the Dirac-Fock-Podolsky formalism, it is convenient to require more generally

$$\Omega(x', \sigma)\Psi(\sigma)=0, \qquad (6)$$

where x' is now an independent space-time point,

[2] W. Heisenberg, Zeits. f. Physik **90**, 209 (1934).

[3] J. Schwinger, Phys. Rev. (to be published).

* *Note added in proof, October 1, 1948:* Professor Schwinger kindly informed me of a new derivation of his previous result. But this new argument again involves an integration by parts which, in my opinion, is not legitimate mathematically.

[4] S. Tomonaga, Progress Theor. Phys. **1**, 27 (1946).

[5] P. A. M. Dirac, V. Fock, and B. Podolsky, Physik. Zeits. Sowjetunion **2**, 468 (1932).

[6] $x_4=ix_0=ict$. Similarly for all four-vectors.

not necessarily lying on σ, Ω being an operator reducing to $\partial A_\mu/\partial x_\mu$ for surface points, satisfying the wave equation

$$(\partial^2/\partial x_\nu'^2)\Omega(x',\sigma)=0, \qquad (7)$$

and the compatability conditions

$$[\Omega(x',\sigma),\Omega(x'',\sigma)]=0, \qquad (8)$$

$$i\hbar c(\partial\Omega(x',\sigma)/\partial\sigma(x))+[\Omega(x',\sigma),\mathcal{H}(x)]=0. \qquad (9)$$

The last condition is sufficient (but not necessary) to ensure $\partial(\Omega(x',\sigma)\Psi(\sigma))/\partial\sigma(x)=0$, in agreement with (6). An operator Ω satisfying all requirements is

$$\Omega(x',\sigma)=\frac{\partial A_\mu(x')}{\partial x_\mu'}+\frac{1}{c}\int_\sigma d\sigma_\mu j_\mu(x)D(x'-x), \qquad (10)$$

($d\sigma_\mu$ = four-vector surface area at x: in a particular frame of reference $d\sigma_1=d\sigma_2=d\sigma_3=0$, $d\sigma_4=-idxdydz$).

The perturbation method, as applied by Schwinger, starts with a unitary transformation

$$\Psi(\sigma)=e^{-iS(\sigma)}\phi(\sigma), \qquad (11)$$

$$S(\sigma)=\frac{1}{2\hbar c}\int d^4x'\epsilon(x',\sigma)\mathcal{H}(x'). \qquad (12)$$

Here

$$\epsilon(x',\sigma)=\begin{cases}+1 \text{ for points } x' \text{ earlier than } \sigma,\\ -1 \text{ for points } x' \text{ later than } \sigma,\end{cases}$$

and the x' integration is meant to be extended over the entire four-dimensional space (between two fixed surfaces in the infinite past and the infinite future). Inserting (11), with

$$e^{-iS}=1-iS-\tfrac{1}{2}S^2+\cdots,$$

into (1), the terms linear in e cancel, because of

$$\hbar c(\partial S(\sigma)/\partial\sigma(x))=\mathcal{H}(x),$$

and the Schrödinger equation for ϕ, up to second-order terms in e, reduces to

$$i\hbar c(\partial\phi(\sigma)/\partial\sigma(x))=\tfrac{1}{2}i[S(\sigma),\mathcal{H}(x)]\phi(\sigma)+\cdots. \qquad (13)$$

According to (2) and (12) the new second-order Hamiltonian is

$$\frac{i}{2}[S(\sigma),\mathcal{H}(x)]=\frac{i}{4\hbar c^3}\int d^4x'\epsilon(x',\sigma)\times[j_\mu(x')A_\mu(x'),j_\nu(x)A_\nu(x)]. \qquad (14)$$

THE SELF-ENERGY OF THE PHOTON

Disregarding terms independent of the A_μ's, i.e., neglecting the commutators (4), the essential terms in (14) may be written

$$\frac{i}{4\hbar c^3}\int d^4x'\epsilon(x',\sigma)[j_\mu(x'),j_\nu(x)]A_\mu(x')A_\nu(x). \qquad (15)$$

Since we want to study the energy of photons, in the absence of electrons and positrons, the operator $[j_\mu(x'),j_\nu(x)]$ may be replaced by its vacuum expectation value which is easily derived by expressing the current densities in terms of the electron wave operators:

$$\langle[j_\mu(x'),j_\nu(x)]\rangle_{\text{vac}}=4ie^2c^2\Bigg\{\frac{\partial\Delta(x'-x)}{\partial x_\mu}\frac{\partial\Delta_1(x'-x)}{\partial x_\nu}+\frac{\partial\Delta(x'-x)}{\partial x_\nu}\frac{\partial\Delta_1(x'-x)}{\partial x_\mu}-\delta_{\mu\nu}\Bigg[\frac{\partial\Delta(x'-x)}{\partial x_\lambda}\frac{\partial\Delta_1(x'-x)}{\partial x_\lambda}+\mu^2\Delta(x'-x)\Delta_1(x'-x)\Bigg]\Bigg\}, \qquad (16)$$

where Δ and Δ_1 denote the two invariant delta-functions involving $\mu=mc/\hbar$, as specified below (cf. (20), (21)). Subtracting, finally, from (15), the vacuum value (no photons present), we obtain the operator corresponding to the photon self-energy density

$$\mathcal{H}_{\text{self}}=\frac{i}{4\hbar c^3}\int d^4x'\epsilon(x',\sigma)\langle[j_\mu(x'),j_\nu(x)]\rangle_{\text{vac}}\times\{A_\mu(x')A_\nu(x)-\langle A_\mu(x')A_\nu(x)\rangle_{\text{vac}}\}. \qquad (17)$$

Let us calculate the expectation value $\langle\mathcal{H}_{\text{self}}\rangle_\kappa$ for a state in which we have only one photon of momentum $\hbar\kappa_\lambda(\kappa_\lambda^2=0)$. Then

$$\langle A_\mu(x')A_\nu(x)\rangle_\kappa-\langle A_\mu(x')A_\nu(x)\rangle_{\text{vac}}=C_{\mu\nu}(\kappa)\cos\kappa_\lambda(x_\lambda'-x_\lambda). \qquad (18)$$

Hence, putting

$$x_\lambda'-x_\lambda=\xi_\lambda,\quad\text{and}\quad \epsilon(x',\sigma)=-\xi_0/|\xi_0|$$

(which is justified because the function (16) vanishes identically for space-like vectors ξ):

$$\langle\mathcal{H}_{\text{self}}\rangle_\kappa=\frac{e^2}{\hbar c}C_{\mu\nu}\int d^4\xi\,\cos\kappa_\lambda\xi_\lambda\cdot\frac{\xi_0}{|\xi_0|}\times\left\{\frac{\partial\Delta(\xi)}{\partial\xi_\mu}\frac{\partial\Delta_1(\xi)}{\partial\xi_\nu}+\frac{\partial\Delta(\xi)}{\partial\xi_\nu}\frac{\partial\Delta_1(\xi)}{\partial\xi_\mu}-\delta_{\mu\nu}\left[\frac{\partial\Delta(\xi)}{\partial\xi_\lambda}\frac{\partial\Delta_1(\xi)}{\partial\xi_\lambda}+\mu^2\Delta(\xi)\Delta_1(\xi)\right]\right\}.\quad(19)$$

If the ξ-space integration is carried out by introducing polar coordinates and integrating first over the invariant hyperboloids $\xi_\lambda^2=\text{const.}(<0)$, the integral proves to be divergent even if the singularities of the invariant delta-functions on the light-cone ($\xi_\lambda^2=0$) are removed (smeared out over a narrow region). The strongest singularities are of the type

$$\left.\begin{aligned}\Delta(\xi)&=D(\xi)+\cdots=\frac{1}{2\pi}\frac{\xi_0}{|\xi_0|}\delta(\xi_\lambda^2)+\cdots,\\ \Delta_1(\xi)&=D_1(\xi)+\cdots=1/2\pi^2\xi_\lambda^2+\cdots.\end{aligned}\right\}\quad(20)$$

Only by introducing integral representations of the delta-functions and carrying out the integrations in a particular order will one be able to arrive at finite values for the self-energy.

Following Schwinger's procedure in his calculation of the electron self-energy, we insert in (19)

$$\left.\begin{aligned}\Delta(\xi)&=\frac{1}{4\pi^2}\frac{\xi_0}{|\xi_0|}\int_{-\infty}^{+\infty}d\alpha\,\exp\left(-i\alpha\xi_\nu^2+\frac{i\mu^2}{4\alpha}\right),\\ \Delta_1(\xi)&=\frac{i}{4\pi^2}\int_{-\infty}^{+\infty}d\beta\frac{\beta}{|\beta|}\exp\left(-i\beta\xi_\nu^2+\frac{i\mu^2}{4\beta}\right),\end{aligned}\right\}\quad(21)$$

and integrate first over the ξ-space, while keeping α, β fixed. Observing that

$$\int d^4\xi\,\exp(i\gamma\xi_\nu^2)=\int d\xi_0\exp(-i\gamma\xi_0^2)\times\prod_\kappa\int d\xi_\kappa\exp(+i\gamma\xi_\kappa^2)=\frac{i\pi^2}{\gamma|\gamma|},\quad(22)$$

we obtain

$$\langle\mathcal{H}_{\text{self}}\rangle_\kappa=\frac{1}{16\pi^2}\frac{e^2}{\hbar c}C_{\mu\nu}\int d\alpha\int d\beta\frac{\beta}{|\beta|}\frac{1}{(\alpha+\beta)|\alpha+\beta|}\times\exp\left(\frac{i\mu^2}{4}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right)\cdot\left[8\alpha\beta\frac{\partial^2}{\partial\kappa_\mu\partial\kappa_\nu}-\delta_{\mu\nu}\left(4\alpha\beta\frac{\partial^2}{\partial\kappa_\lambda^2}+\mu^2\right)\right]\exp\left[\frac{i\kappa_\lambda^2}{4(\alpha+\beta)}\right].$$

We note

$$\frac{\partial^2}{\partial\kappa_\mu\partial\kappa_\nu}\exp\left[\frac{i\kappa_\lambda^2}{4(\alpha+\beta)}\right]=\exp\left[\frac{i\kappa_\lambda^2}{4(\alpha+\beta)}\right]\times\left(\frac{i}{2(\alpha+\beta)}\delta_{\mu\nu}-\frac{1}{4(\alpha+\beta)^2}\kappa_\mu\kappa_\nu\right),$$

and here we may put $\kappa_\lambda^2=0$ and drop the term $\sim\kappa_\mu\kappa_\nu$[7] because $\kappa_\mu\kappa_\nu C_{\mu\nu}=0$ on account of the supplementary condition (5) (vanishing of the longitudinal components). If we symmetrize the integrand in α and β

$$\left(\frac{\beta}{|\beta|}\rightarrow\frac{1}{2}\left(\frac{\alpha}{|\alpha|}+\frac{\beta}{|\beta|}\right)\right),$$

the result is

$$\langle\mathcal{H}_{\text{self}}\rangle_\kappa=\frac{1}{32\pi^2}\frac{e^2}{\hbar c}C_{\nu\nu}\int d\alpha\int d\beta\times\left(\frac{\alpha}{|\alpha|}+\frac{\beta}{|\beta|}\right)\frac{1}{(\alpha+\beta)|\alpha+\beta|}\times\exp\left[\frac{i\mu^2}{4}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right]\times\left(-\frac{4i\alpha\beta}{\alpha+\beta}-\mu^2\right).\quad(23)$$

In order to carry out the remaining integrations, let us introduce new variables

$$\frac{\alpha-\beta}{\alpha+\beta}=y,\quad\frac{\mu^2}{4}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=z,$$

$$\frac{d\alpha d\beta}{(\alpha+\beta)^2}=\left|\frac{dydz}{2z}\right|,\quad\frac{\alpha}{|\alpha|}+\frac{\beta}{|\beta|}=\begin{cases}2\dfrac{z}{|z|} & \text{for } |y|<1,\\ 0 & \text{for } |y|>1.\end{cases}$$

[7] This was also done by Schwinger in his Pocono Manor lecture, but he apparently overlooked the remaining term $\sim\delta_{\mu\nu}$.

From (23) we obtain

$$\langle \mathcal{H}_{\text{self}} \rangle_\kappa = \frac{1}{32\pi^2} \frac{e^2}{\hbar c} \mu^2 C_{vv} \times \int_{-1}^{+1} dy \int_{-\infty}^{+\infty} dz \frac{1}{|z|} e^{iz} \left(\frac{i}{z} - 1 \right).$$

The imaginary part of the integrand has a singularity at $z=0(\sim \pm z^{-2})$, but since it is an odd function of z, it may be regarded as giving no contribution to the integral. Keeping only the real part,

$$\int_{-\infty}^{+\infty} \frac{dz}{|z|} e^{iz} \left(\frac{i}{z} - 1 \right) = 2 \int_0^\infty dz \frac{d}{dz} \left(\frac{\sin z}{z} \right) = 2,$$

$$\langle \mathcal{H}_{\text{self}} \rangle_\kappa = \frac{1}{8\pi^2} \frac{e^2}{\hbar c} \mu^2 C_{vv}. \tag{24}$$

$C_{\mu v}$, as defined by (18) (with $x' = x$), is equal to the electromagnetic energy density of the photon state considered, divided by κ_0^2. Therefore, a photon of momentum $\hbar \boldsymbol{\kappa}$, having the total energy $\hbar c \kappa_0 = \hbar c |\boldsymbol{\kappa}|$ in the zero-order approximation, appears in the second-order approximation as having the energy

$$\hbar c \kappa_0 \left(1 + \frac{1}{8\pi^2} \frac{e^2}{\hbar c} \frac{\mu^2}{\kappa_0^2} + \cdots \right) = c \left[(\hbar |\boldsymbol{\kappa}|)^2 + \frac{1}{4\pi^2} \frac{e^2}{\hbar c} (mc)^2 \right]^{\frac{1}{2}}. \tag{25}$$

This corresponds to a "photon rest-mass" amounting to one electron mass divided by $(137\pi)^{\frac{1}{2}}$ (because, in the Heaviside units used here, $e^2/4\pi\hbar c = 1/137$).

ELIMINATION OF THE PHOTON SELF-ENERGY

In order to restore Maxwell's equations, one might try to replace the Hamiltonian (2) by

$$\mathcal{H}(x) = -\frac{1}{c} j_\mu(x) A_\mu(x) - \tfrac{1}{2} \gamma A_\mu^2(x),$$

and to determine the constant γ such that the new term in $\mathcal{H}$ cancels the self-energy term. If the result (24) or (25) is accepted, this would be achieved by

$$\gamma = \frac{1}{4\pi^2} \frac{e^2}{\hbar c} \mu^2.$$

However, the new Schrödinger equation would no longer be compatible with the supplementary conditions (6). In order to ensure the compatibility, it is easiest to construct the amended Hamiltonian and supplementary conditions by a unitary transformation from the old ones.

The transformation

$$\Psi(\sigma) = e^{-iU(\sigma)} \Psi'(\sigma) \tag{26}$$

leads to the new Schrödinger equation

$$i\hbar c (\partial \Psi'(\sigma) / \partial \sigma(x)) = \mathcal{H}'(x, \sigma) \Psi'(\sigma), \tag{27}$$

with

$$\mathcal{H}'(x, \sigma) = e^{iU(\sigma)} \mathcal{H}(x) e^{-iU(\sigma)} - i\hbar c e^{iU(\sigma)} \frac{\partial e^{-iU(\sigma)}}{\partial \sigma(x)}. \tag{28}$$

Defining

$$\Omega'(x', \sigma) = e^{iU(\sigma)} \Omega(x', \sigma) e^{-iU(\sigma)}, \tag{29}$$

the conditions (6) become

$$\Omega'(x', \sigma) \Psi'(\sigma) = 0, \tag{30}$$

and the compatibility requirements are, of course, fulfilled; indeed

$$[\Omega'(x', \sigma), \Omega'(x'', \sigma)] = 0,$$

and, on account of (9),

$$\frac{\partial}{\partial \sigma(x)} \{ \Omega'(x', \sigma) \Psi'(\sigma) \} = \left\{ \frac{\partial e^{iU(\sigma)}}{\partial \sigma(x)} e^{-iU(\sigma)} + e^{iU(\sigma)} \frac{\partial e^{-iU(\sigma)}}{\partial \sigma(x)} + \frac{1}{i\hbar c} \mathcal{H}'(x, \sigma) \right\} \Omega'(x', \sigma) \Psi'(\sigma),$$

which vanishes according to (30).

If we choose

$$U(\sigma) = \frac{\gamma}{4\hbar c} \int d^4 x' \epsilon(x', \sigma) A_\mu^2(x'), \tag{31}$$

with the same meaning of the symbols as in (12), so that

$$\hbar c \frac{\partial U(\sigma)}{\partial \sigma(x)} = \tfrac{1}{2}\gamma A_\mu{}^2(x),$$

and if we consider γ to be small of the second order ($\sim e^2$), the transformed Hamiltonian (28) becomes, up to third-order terms inclusively,

$$\mathcal{H}'(x, \sigma) = \mathcal{H}(x) - \tfrac{1}{2}\gamma A_\mu{}^2(x) + i[U(\sigma), \mathcal{H}(x)] + \cdots. \quad (32)$$

The second term is the one that will be cancelled by the photon self-energy term, after the transformation (11), (12), now applied to Ψ' instead of Ψ. The first and third terms may be written

$$-(1/c) j_\mu(x) B_\mu(x), \quad (33)$$

with

$$B_\mu(x) = A_\mu(x) + i[U(\sigma), A_\mu(x)], \quad (34)$$

or, according to (4) and (31),

$$B_\mu(x) = A_\mu(x) + \tfrac{1}{2}\gamma \int d^4x' \epsilon(x', \sigma) \times A_\mu(x') D(x'-x). \quad (35)$$

Here σ is meant as a spacelike surface passing through the point x; since $D(x'-x)$ vanishes for all spacelike vectors $x'-x$, $B_\mu(x)$ is independent of σ. If x is varied, σ has to be varied accordingly. From (34) and (31) it is easily derived that $B_\mu(x)$ obeys, up to terms $\sim e^2$, the equation

$$(\partial^2/\partial x_\nu{}^2) B_\mu(x) = \gamma B_\mu(x); \quad (36)$$

thus $B_\mu(x)$ may be represented as a superposition of plane waves $\exp(ip_\lambda x_\lambda)$, with $p_\lambda{}^2 = -\gamma$. (This follows also from (35), if plane waves $\exp(i\kappa_\lambda x_\lambda)$ are inserted for A_μ.) Accordingly,

$$[B_\mu(x), B_\nu(x')] = (\hbar c/i)\delta_{\mu\nu}\Delta_\gamma(x-x'), \quad (37)$$

where Δ_γ is the modified D-function obeying the differential equation (36). This result makes it clear, that the third term in $\mathcal{H}'(32)$, as a correction to the first, reflects a wrong rest-mass of the photon. Of course, this term is again unwanted, since it would affect mainly the low frequency phenomena, and should be transformed away.

The introduction of the rest-mass term also destroys the gauge invariance. It is true that the Schrödinger equation (27) is formally gauge invariant in the sense that the change of gauge $A_\mu \rightarrow A_\mu + \partial\Lambda/\partial x_\mu$ is equivalent to a unitary transformation, so that the inverse transformation restores the Schrödinger equation in its original form. But the operators corresponding to observable quantities, such as the field strengths $\partial A_\mu/\partial x_\nu - \partial A_\nu/\partial x_\mu$, are not invariant under these transformations. One may, however, expect that in the second approximation, where Maxwell's field equations are re-established, a gauge-invariant formulation will become possible.

CONCLUSION

The results of the last section are hardly encouraging in view of higher approximations. We have tried to take the quantum theory of fields seriously, without admitting any *ad hoc* subtractions inconsistent with the principles of quantum mechanics. The outcome shows that the empirical fact, that the photon has no rest-mass, does not fit naturally into the framework of quantum electrodynamics. It seems questionable to what extent the predictions of such a theory in higher order effects are trustworthy.

Finally, it should be remembered that the pair creation of other charged particles (mesons, protons) is likely to contribute to the photon self-energy. Therefore, the phenomena involving electrons, positrons, and photons only, can hardly be expected to be quite independent, in the higher order effects, of the existence of other particles and their nature.