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On Nuclear Dipole Vibrations

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The high frequency resonances recently observed for (γ,n) reactions as well as photo-fission are interpreted in analogy with the "reststrahl frequencies" of polar crystals. The estimated frequencies are in good agreement with the experimental results. An interesting consequence of this interpretation is the conclusion that strong resonance scattering of γ-rays should take place at a frequency characteristic of the scattering nucleus.

IT has been observed recently[1] that some nuclear photo-disintegrations, (γ,n) reactions as well as photo-fission, exhibit a behavior which has the appearance of a high frequency resonance. Such a resonance was observed in carbon at 30 Mev,[1] in copper at 22 Mev,[1] in tantalum at 16 Mev,[2] and for photofission in thorium and in uranium at 16–18 Mev.[1] The suggestion was made that this phenomenon may be due to the interplay of increasing level density and increasing competition from other processes such as, for instance, the $(\gamma,2n)$ process. Recent experiments[3] show, however, that in the case of Cu^{63} as well as other nuclei the decrease in the (γ,n) cross section in the high energy region is not compensated to any considerable extent by an increase in the $(\gamma,2n)$ cross section. It is, furthermore, desirable to explain why such different processes as (γ,n) reactions and photo-fission should have maximum cross sections at so similar γ-energies.

We propose to interpret these frequencies as resonances, somewhat different from those caused by definite nuclear levels, and analogous to the "reststrahl frequencies" of polar crystals. We assume that the γ-rays excite a motion in the nucleus in which the bulk of the protons move in one direction while the neutrons move in the opposite direction. We shall call this motion the "dipole vibration." Such a vibration will have a high frequency as a result of the partial separation of the protons from the neutrons to which they are strongly bound. The maximum in the γ-absorption cross section will give rise to similar maxima for several nuclear processes in which γ-absorption is the first step. We shall see below that the magnitude of the integrated cross sections deduced from experiments is too great to be explained by the motion of a single proton or of a small fraction of the protons contained in the nucleus.

The breadth of the resonance—so far only crudely known—is probably due to the transfer of energy from the orderly vibration described above into other modes of nuclear motion. Thus, the breadth is due to a process analogous to damping by friction. One may use a nuclear model in which the ordered dipole vibration of protons and neutrons in opposite directions correspond to well defined quantum states. Coupling with other degrees of freedom will broaden these states. Our resonance will then correspond to the transition of the nucleus from the lowest state to the first excited state of the dipole vibration. A more complete nuclear model will include from the beginning the coupling between the dipole vibration and the other nuclear motions. In this case we shall obtain a great number of nuclear levels each of which contains to some extent the dipole vibration. If, however, the coupling is weak, only such levels will contain considerable contributions from the first excited state of the dipole vibration whose energy does

[1] G. C. Baldwin and G. S. Klaiber, Phys. Rev. **71**, 3 (1947) and Phys. Rev. **73**, 1156 (1948).
[2] J. McElhinney and A. O. Hanson, private communication.
[3] M. L. Perlman and G. Friedlander, Phys. Rev. **74**, 442 (1948). We are indebted to Dr. Friedlander for informing us of these results before publication.

not differ greatly from the energy of this first excited state. Absorption of γ-rays by an energy level is due to the contribution from the dipole vibration. Thus, a great number of nuclear levels actually contribute to the γ-absorption, but they all cluster around the first excited state of the idealized dipole-vibration. In the following we shall discuss this idealized vibration.

In order to gain some insight into the change of the resonance frequency with nuclear properties we shall consider the dipole vibration on the basis of three simple models.

I. We assume that the displacements of the average position of a proton from its original location in the nucleus requires a force which is proportional to the displacement. The proportionality constant shall be the same for every kind of nucleus and for every proton within the nucleus. These assumptions should be well approximated in nuclei consisting of α-particles. It is clear that in this model the frequency of the dipole vibration will be the same for all nuclei.

II. We assume that the protons and neutrons on the surface of the nuclei have fixed positions with respect to each other. Motion of protons and neutrons inside the nucleus causes density changes in the proton-fluid and neutron fluid. The restoring forces per unit mass will be proportional to the gradients of these densities. For a given displacement in the nucleus the maximum density change is inversely proportional to the nuclear radius R and the gradient will be proportional to $1/R^2$. Thus, the frequency, which must vary as the square root of the restoring force, is proportional to $1/R$ or to the inverse cube root of the nuclear mass.

III. We assume that the protons and the neutrons behave as two inter-penetrating incompressible fluids. During the dipole vibration the two fluids suffer a relative displacement so that near the nuclear surface the two fluids no longer overlap. The total restoring force will be proportional to the surface, that is, to R^2. For small displacements we may again assume proportionality between displacement and force. The frequency ω of the resulting harmonic motion is proportional to the square root of force over mass. Thus we have $\omega\sim(R^2/R^3)^{\frac{1}{2}}\sim R^{-\frac{1}{2}}$, and the frequency varies inversely as the sixth root of the nuclear mass.

The experimental variation of the frequency seems to be best described by hypothesis III. It is of interest to explore this model in a more quantitative way even though it is clear that the picture used is too idealized. Indeed we shall further simplify our calculation by assuming that the number of protons and neutrons is equal.

The relative displacement of the neutrons and protons shall be denoted by ξ. We consider the case $\xi\ll R$, where R is the nuclear radius. We shall, however, not assume at first that ξ is infinitesimal. Because of this displacement a volume $\pi R^2|\xi|$ of the proton-fluid no longer overlaps the neutron-fluid and a similar volume of the neutron-fluid extrudes on the other side of the nucleus. If ρ is the density of protons (and neutrons) within the nucleus there will be $\pi R^2|\xi|\rho$ protons separated from the neutron-fluid (and a similar number of neutrons separated from the proton fluid). The total increase in potential energy will then be $2\pi R^2|\xi|\rho\varphi$, where φ is the energy needed to extract one proton from its neutron environment (or one neutron from its proton environment). It is to be noted that φ is not the binding energy of protons (or neutrons). This latter quantity is the difference of the potential energy and of the "zero-point" kinetic energy, whereas φ is that part of the potential energy which is due to the neutron-proton interaction.

For infinitesimal ξ-values the potential should vary as ξ^2 rather than as $|\xi|$. We write $k\xi^2/2$ +const. for the potential at small ξ-values. At a certain point $\xi=\epsilon$ we shall require the two approximate potentials to join smoothly. Here ϵ is the length within which the interaction between a neutron and a nucleus changes from a zero-value outside the nucleus to a high value inside. Setting derivatives equal at $\xi=\epsilon$ we get

$$2\pi R^2\rho\varphi=k\epsilon,$$

or $k=2\pi R^2\rho\varphi/\epsilon$. The reduced mass to be used in the vibration is one-half of the mass of the protons (or neutrons), that is $(2\pi/3)R^3\rho m$, where m is the mass of a single nucleon. Vibrations with zero or one quantum of excitation have amplitudes small compared to ϵ so that the frequency of the vibration is that of a harmonic oscillator. Taking the square root of the force constant divided by the mass we get for the angular fre-

quency

$$\omega=\left(\frac{3\varphi}{\epsilon Rm}\right)^{\frac{1}{2}},$$

and for the resonance energy

$$\hbar\omega=\left(\frac{3\varphi\hbar^2}{\epsilon Rm}\right)^{\frac{1}{2}}.$$

Setting $\epsilon=2\times10^{-13}$ cm, $\varphi=40$ Mev and using $R=10^{-12}$ cm for uranium, we obtain $\hbar\omega=16$ Mev. For copper we get $\hbar\omega=20$ Mev and for carbon $\hbar\omega=26.5$ Mev. The agreement with the experiments is good. It must be remembered, however, that the assumed values of ϵ and φ are quite uncertain and thus the agreement obtained above is significant only in that the correct order of magnitude of the resonance energy is found.

For the vibration described above one finds that the total absorption is given by

$$\int\sigma d\omega=\frac{\pi^2Ae^2}{2mc}$$

where A is the mass-number of the nucleus. For copper this gives $\hbar\int\sigma d\omega=1$ Mev×barn. Comparison with experiment is difficult, since Baldwin and Klaiber do not give absolute values of the cross section in this case. The cross section of the $Cu^{63}(\gamma,n)$ reaction for 17.5 Mev photons was, however, determined by Bothe and Gentner,[4] who find $\sigma=5\times10^{-26}$ cm^2. Using their value and the relative curve given by Baldwin and Klaiber one obtains 1 Mev×barn. The value deduced from these two experiments may easily be in error by a factor of about 5. The theoretical formula would change somewhat, if model III should be abandoned or if the difference in proton and neutron numbers should be taken into account. Furthermore, the value $\int\sigma d\omega$ was derived under the assumption that the wave-length of the γ-ray is long compared to the nuclear radius. In the present case this condition is not very well satisfied. Finally, the effect of reactions competing with the (γ,n) process has been neglected. All these effects would result in a decrease in the theoretical $\int\sigma d\omega$-values. Thus it seems clear that the observed large values for this integral can be explained only if it is assumed that a considerable fraction of the nuclear charge participates in the absorption process.

In the case of photo-fission in uranium, the experimental value of $\int\sigma d\omega$ is about 15 times smaller than the theoretical figure. This may well be due to competition from the (γ,n) reaction. In thorium the experimental value of $\int\sigma d\omega$ for photo-fission is about 30 times smaller than the calculated value for the absorption cross section.

The dependence of the frequency of the dipole vibration on the nuclear radius has further interesting consequences. For light nuclei the (γ,n) reaction will occur with comparatively small probability, if a nucleus is exposed to a continuous x-ray spectrum with a fairly high upper limit. The nucleus absorbs only a narrow band of photon energies and, in light nuclei, these energies are so high that they are apt to produce other reactions than the (γ,n) process. For the lightest nuclei up to $Z\approx7$, it seems most likely that the excited nucleus will first emit an α-particle. This may be followed by further disintegration. For heavier nuclei the decrease in the resonance energy may cause the (γ,n) process to become more probable, while for the heaviest nuclei the decrease in the binding energy of the neutron may favor the $(\gamma,2n)$ process.

Photo-fission for bismuth and lighter elements is likely to have small cross sections for the following reason. The theoretical photo-fission threshold, according to Frankel and Metropolis,[5] is 16 Mev for bismuth and it is higher for lighter elements. According to our present picture bismuth does not absorb photons strongly above 17 Mev. Thus the cross section is low at high energies because the γ-ray is not absorbed, whereas the resonance energy is so close to the threshold that competition from the (γ,n) reaction becomes strong. For lighter nuclei the frequency of the dipole vibration will be actually lower than the threshold. It is not clear how small the absorption of γ-rays above the resonance will be and whether there are other reasons that make photo-fission an unlikely process for bismuth and lighter nuclei. Baldwin and Klaiber

[4] W. Bothe and W. Gentner, Zeits. f. Physik **106**, 236 (1937).

[5] S. Frankel and N. Metropolis, Phys. Rev. **72**, 914 (1947).

find that the photo-fission cross section of bismuth is at least 10^3 times smaller than the cross section of uranium.

It would be interesting to investigate the resonance scattering of photons by the dipole vibration. The value of $\int \sigma_{\text{scatt}} d\omega$ for this process should be a considerable fraction of the theoretical value derived above for the absorption cross section. The resonance scattering will be, therefore, comparable in intensity to the Compton scattering. The two kinds of scattering processes can be distinguished by their angular dependence. The Compton scattering will be observed in the forward direction. The resonance scattering has, in the first approximation, an intensity proportional to $\sin^2\vartheta$, where ϑ is the angle included by the direction of the scattered radiation and the polarization direction of the primary γ-ray. Deviations from the $\sin^2\vartheta$ law are to be expected due to the finite radius of the nucleus. The scattered radiation may turn out to be a good source of fairly monochromatic γ-rays. The investigation of the spectrum of these scattered rays may open a convenient way to study the breadth of the resonance, while a detailed study of the angular distribution may yield information about the current distribution in the dipole vibration.

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Coincidence Experiments on Sc^{46}, Ga^{72}, Au^{198}, and Rb^{86}

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Gamma-gamma- and beta-gamma-coincidences have been investigated in the disintegration of the active isotopes Sc^{46}, Ga^{72}, Au^{198}, and Rb^{86}. Gamma-gamma-correlation was found in Sc^{46} and in Ga^{72} but not in Au^{198} or Rb^{86}. The beta-ray spectra of Sc^{46} and Au^{198} are found to be simple on the basis of beta-gamma-coincidence measurements, while those of Ga^{72} and Rb^{86} are complex. Gamma-ray energies and beta-ray maximum energies were determined by absorption methods.

I. INTRODUCTION

COINCIDENCE experiments, as described by Mitchell,[1] frequently are a great aid in establishing decay schemes for radioactive isotopes. Experiments on the radiations from Sc^{46}, Ga^{72}, Au^{198}, and Rb^{86} have been done, and the results are to be reported herein. The coincidence amplifier, which has a resolving time of about two microseconds, and its associated counting apparatus have been described previously.[2]

II. SCANDIUM 46

Peacock and Wilkinson[3] have measured the energies of the beta- and gamma-rays from Sc^{46} with a 180°-type magnetic spectrometer and find gamma-ray energies of 0.89 Mev and 1.12 Mev and beta-ray groups with end-point energies of 0.36 Mev and 1.49 Mev, with the 1.49 Mev beta-ray group very weak.

A source was prepared on the Indiana University cyclotron according to the reaction $Sc^{45}(d,p)Sc^{46}$, and the resulting Sc^{46} was purified chemically. Two silver-cathode glass counters were arranged with thick aluminum radiators to obtain gamma-gamma-coincidences. For the particular geometrical arrangement which was used, the ratio of the gamma-gamma-coincidence rate to the rate of counting gamma-rays in one of the counters was measured to be $(0.72\pm0.08)\times10^{-3}$. From this it can be concluded that cascaded gamma-rays occur in the disintegration.

Figure 1 shows that the ratio of the beta-gamma-coincidence rate to the beta-ray counting rate is independent of the thickness of absorber

[1] A. C. G. Mitchell, Rev. Mod. Phys. **20**, 296 (1948).
[2] E. T. Jurney and A. C. G. Mitchell, Phys. Rev. **73**, 1153 (1948).
[3] C. L. Peacock and R. G. Wilkinson, Phys. Rev. **74**, 297 (1948).