# A Quantitative Determination of the Magnetic Moment of the Neutron in Units of the Proton Moment*

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A precision determination of the ratio of the magnetic moments of neutron and proton is presented. The method consists in determining the Larmor precession frequencies of neutrons and protons simultaneously in the same extremely homogeneous magnetic field. In this manner the ratio of the two Larmor frequencies gives directly the ratio of the two magnetic moments. The precession frequency of the neutrons is obtained by the occurrence of non-adiabatic transitions of a polarized neutron beam, the precession frequency of the protons by nuclear induction. Measurements are made so that only small differences in frequency have to be measured in terms of a constant but otherwise unknown standard. The value of the ratio of the two moments is found to be $|\mu_N|/\mu_P=0.685001\pm0.00003$. By use of this value in connection with the recent determination of the ratio of the moments of proton and deuteron by Bloch, Levinthal, and Packard, the deviation of the deuteron moment from additivity is found to be $0.02230\pm0.00009$ nuclear magnetons.

## INTRODUCTION

AMONG the moments of atomic nuclei, those of the elementary constituents, the proton and the neutron, are of central significance. Their values are not only basic in order to discuss the moments of more complex nuclear structures, but they are also of greatest interest in themselves in view of their bearing upon the constitution of fundamental particles.

As a most important first step towards the knowledge of these elementary moments, Stern and his collaborators have shown that the magnetic moment $\mu_P$ of the proton[1] is about 2.5 times larger than the nuclear magneton, $\mu_n=(m/M)\mu_B$, defined through the Bohr magneton $\mu_B$ and the mass ratio $m/M$ of electron and proton. From a later measurement of the magnetic moment $\mu_D$ of the deuteron[2] and the assumption of the simple law of additivity, $\mu_D=\mu_N+\mu_P$, they have also been led to the conclusion that the neutron has a finite and negative moment $\mu_N$ of about 2 nuclear magnetons. Both results are equally significant since they indicate that neither the proton nor the neutron is adequately described through the relativistic wave equation of Dirac, which explains the magnetic moment of the electron but would lead here to the wrong values $\mu_P=\mu_n$, $\mu_N=0$. The ingenious new methods of Rabi and his collaborators have subsequently led to the far more accurate value

$$\mu_P=(2.7928\pm0.0008)\mu_N \qquad (1)^{**}$$

for the magnetic moment of the proton.[3]

The direct proof that the free neutron possesses a magnetic moment has originated from the effect of magnetic scattering.[4] The interaction of the neutron moment with the atomic moments in a ferromagnetic substance causes polarization of an originally unpolarized neutron beam and, as one of its consequences, an increase of the transmission upon magnetization of the substance. While this change in single transmission has been actually demonstrated,[5] it was not possible to infer from the measurement more than the expected order of magnitude of the neutron moment since the effect depends not only upon the neutron but also upon various complicating features of the scattering substance.

The quantitative determination of the neutron moment requires, besides the polarization of neutron beams, another independent principle which is closely related to the one first described

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[1] R. Frisch and O. Stern, Zeits. f. Physik **85**, 4 (1933).

[2] I. Estermann and O. Stern, Zeits. f. Physik **85**, 17 (1933); Phys. Rev. **45**, 665 (1934).

** This value differs from the originally given value 2.7896 by the factor $(1+\alpha/2\pi)$, as a result of a recent radiative correction for the magnetic moment of the electron. (See Section V.)

[3] S. Millman and P. Kusch, Phys. Rev. **60**, 91 (1941).

[4] F. Bloch, Phys. Rev. **50**, 259 (1936).

[5] P. N. Powers, H. Beyer, and J. R. Dunning, Phys. Rev. **51**, 371 (1937).

by Rabi[6] as a means for the sign determination of nuclear moments and which was simultaneously conceived by Gorter and one of us (F.B.). It is based upon the resonance of a rotating or oscillating magnetic field with the Larmor frequency of the nuclear moment in a constant field at right angles. The initial use of this magnetic resonance method has been made by Rabi and his collaborators[7] in their famous experiments on molecular and atomic beams.

Alvarez and Bloch[8] have succeeded in applying the method to the neutron. In their experimental arrangement a beam of thermal neutrons passes through a polarizing and analyzing plate of magnetized iron. Between these two plates the neutrons are exposed to a magnetic field which has a constant component $H_0$ and at right angles to it an oscillating component with circular frequency $\omega$ and amplitude $H_1$. Under resonance conditions, there occurs in this field a reorientation of the neutron moments and thus a partial depolarization of the beam. Consequently, the total transmission through the analyzer is lowered so that resonance depolarization manifests itself by a drop in the transmitted intensity detected as one passes through resonance. From the measured value $H_0$ of the constant field at which resonance is observed and the frequency $\omega$ of the oscillating field, one obtains

$$\frac{\mu_N}{\hbar I_N}=\gamma_N=\frac{\omega}{H_0} \tag{2}$$

for the gyromagnetic ratio $\gamma_N$, defined as the magnetic moment of the neutron $\mu_N$, divided by its angular momentum $\hbar I_N$. Assuming the spin of the neutron to be $I_N=\frac{1}{2}$, the absolute value of the neutron moment was determined to be

$$|\mu_N|=(1.93\pm 0.02)\mu_n. \tag{3}$$

The fact that the sign of the neutron moment is actually negative has been shown in an earlier experiment of Powers.[9]

The limitation in accuracy of the result (3) was due to two major causes. In the first place the depolarization effects, although clearly established, were rather small. The largest observed drop of intensity amounted to about only 1.5 percent, so that the location of the exact resonance condition was only possible to within the width of the resonance line. A higher percentage accuracy could have been obtained with the same line width by carrying out the experiment at larger values of the constant field $H_0$ than those of about 600 gauss, which were actually used. This, however, would not have eliminated the second comparable error, originating in the determination of the resonance value of $H_0$. Although this field was determined by two independent procedures, one using flip coils directly, the other based upon a comparison with the field of a proton accelerating cyclotron, the consistency and accuracy of these two methods did not exceed $\frac{1}{2}$ percent.

A vast improvement of the experimental conditions has been made possible through our recent work. To a large extent, the two previously mentioned handicaps were eliminated, and thereby a far more accurate determination of the neutron moment has been achieved. The experiments have been extended over a considerable period, although preliminary results were obtained at an early stage and could easily have been trusted to within 0.1 percent. The indispensable part of a precision measurement which consists in the examination of detailed features required a more time-consuming effort which, however, seemed well justified in view of the importance of obtaining a reliable result of the highest accuracy compatible with the experimental conditions.

## II. METHOD

With the principle of our method, to observe resonance depolarization of a collimated neutron beam between the polarizer and analyzer, being the same as in the experiment of Alvarez and Bloch,[8] we shall restrict ourselves to the description of those features which represent essential innovations.

An important improvement of the experimental conditions has been achieved by a very close approach to magnetic saturation in the polarizing and analyzing plates of iron. Our previous investigations[10] of the transmission

[6] I. I. Rabi, Phys. Rev. **51**, 652 (1937).
[7] Rabi, Zacharias, Millman, and Kusch, Phys. Rev. **55**, 526 (1939).
[8] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).
[9] P. N. Powers, Phys. Rev. **54**, 827 (1938).
[10] F. Bloch, M. Hamermesh, and H. Staub, Phys. Rev. **64**, 47 (1943); F. Bloch, R. I. Condit, and H. H. Staub, Phys. Rev. **70**, 972 (1946).

effect caused by magnetic scattering have shown that saturation to within less than 0.1 percent is necessary in order to approach the full effect in a single plate of hot-rolled steel of 3.8-cm thickness. By the use of strong magnetizing fields of the order of 10,000 gauss, we were able to observe an increase of the transmission by magnetization of as much as 22 percent, while the largest effects observed before did not exceed 6 percent. The dependence of the effect upon saturation was found to be in full agreement with the theory of Halpern and Holstein,[11] who investigated the influence of magnetic deviations from the field direction as a result of the microcrystalline structure. They have shown that such deviations, even if they are small, can cause an appreciable depolarization of the neutrons in the steel and thereby result in a reduction of the single transmission effect; besides, they have pointed out that this reduction must become even more pronounced if one deals with double transmission, as in our arrangement, where the combined result of the passage through polarizer and analyzer is observed.

This can best be seen by expressing the observed effect for double transmission in terms of likewise observable single transmission effects: Let $f_0(v)dv$ be the number of neutrons with velocity between $v$ and $v+dv$ which are recorded in the detection chamber per unit time after having passed through the polarizer and analyzer plate, if both are unmagnetized. This number will be increased by a different characteristic factor $C(v)$ if either or both of the two plates are magnetized without any change of orientation of the neutrons occurring between them, and one has to consider the following cases:

(a) Polarizer and analyzer both magnetized; $C(v)=C_D(v)$.
(b) Polarizer magnetized; analyzer unmagnetized; $C(v)=C_P(v)$.
(c) Analyzer magnetized; polarizer unmagnetized; $C(v)=C_A(v)$.

Assuming the spin of the neutron to be $\frac{1}{2}$, we shall now consider the case that there exists a finite probability $P(v)$ for a neutron of velocity $v$ to change its orientation between polarizer and analyzer. The total recorded intensity can then be expressed in the form

$$I=\int f_0[(1-2P)C_D+2PC_PC_A]dv, \qquad (4)$$

where $f_0$, $C_D$, $C_P$, $C_A$, and $P$ are all functions of the velocity. The proof of this formula requires merely that the recorded neutrons have not changed their velocity in their passage through the apparatus, and that the steel plates of the polarizer and analyzer are sufficiently thick so that an originally polarized beam of neutrons would emerge completely depolarized upon passage through the unmagnetized plates. Both conditions are certainly well satisfied under our experimental conditions. Let the recorded intensities corresponding to the three cases $a$, $b$, and $c$ previously mentioned be

$$I_D=\int f_0C_Ddv, \qquad (5a)$$

$$I_P=\int f_0C_Pdv, \qquad (5b)$$

$$I_A=\int f_0C_Adv, \qquad (5c)$$

and, furthermore, let

$$I_0=\int f_0dv \qquad (6)$$

represent the intensity if polarizer and analyzer are both unmagnetized. Then we shall define the observable effect $E_T$ resulting from transitions as

$$E_T=\frac{I-I_D}{I_D}=-2\frac{\int f_0(C_D-C_AC_P)Pdv}{\int f_0C_Ddv} \qquad (7)$$

The case where one has complete depolarization for all neutron velocities, i.e., where $P(v)=\frac{1}{2}$, is of special practical interest. To obtain here a simple form for $E_T$, one must consider that for normal thickness of polarizer and analyzer the factors $C_A$ and $C_P$ will differ only little from unity so that the product $(C_A-1)(C_P-1)$ can safely be neglected. With this simplification expression (7) becomes

$$E_T^*=-\frac{[E_D-(E_A+E_P)]}{1+E_D}, \qquad (8)$$

[11] O. Halpern and T. Holstein, Phys. Rev. 59, 960 (1941).

where

$$E_D=\frac{I_D-I_0}{I_0}, \qquad (9a)$$

$$E_P=\frac{I_P-I_0}{I_0}, \qquad (9b)$$

$$E_A=\frac{I_A-I_0}{I_0} \qquad (9c)$$

measure the relative increase of intensity upon magnetization, corresponding to the three previously mentioned cases. It is thus possible, in the case of complete depolarization, to compare directly the effect $E_T{}^*$ with the independently observable transmission effects (9).

The importance of having polarizer and analyzer very close to magnetic saturation can be seen directly from (8). It follows from the theory of Halpern and Holstein,[11] and has been verified in our previous investigations, that the transmission effect for insufficient saturation is essentially proportional to the total thickness of the traversed magnetized iron. Thus, with $d_A$ and $d_P$ for the thicknesses of analyzer and polarizer, respectively, one has $E_A\sim d_A$, $E_P\sim d_P$, and $E_D\sim(d_A+d_P)$, so that the transition effect (8) vanishes in this approximation and will be greatly reduced as long as the experimental conditions are close to this limit. The fact that Alvarez and Bloch[8] with a transmission effect $E_D$ of 6 percent observed values of $E_T$ which did not exceed 1.5 percent can be quantitatively ascribed to this circumstance.

The maximum value for $E_T{}^*$ is obtained at complete saturation, where one has for normal thicknesses $E_A\sim d_A{}^2$, $E_P\sim d_P{}^2$, and $E_D\sim(d_A+d_P)^2$, and, therefore, for $d_A=d_P$

$$E_T{}^*=-\frac{1}{2}\frac{E_D}{(1+E_D)}. \qquad (10)$$

Actually, this formula for complete saturation is valid even for large thicknesses, where the previously made assumptions do not hold. One has then for $d_A=d_P=d$, $C_A=C_P=\cosh dK$ and $C_D=\cosh 2dK$, where $K=K(v)$ is a function of the neutron velocity. Also, $C_D-C_AC_P=(1/2)\times(\cosh 2dK-1)$, so that for $P(v)=\frac{1}{2}$, and with the definitions (5a) and (9a), one obtains (10) directly from the general expression (7).

It is essential, for a quantitative investigation of neutron resonances, not only to consider the limiting case of complete depolarization, but also to have an insight into the more general situation which one meets in tracing out a complete resonance curve and, hence, to have a theory of the expected line shape which must be tested experimentally. A completely general theory of the line shape, which involves the detailed actual geometry of the field in which the transitions occur, meets with considerable mathematical difficulties. It is sufficient, however, to base the discussion upon a simple geometry which could be used in principle, and which, in practice, closely describes the actual conditions of our experiment.

Disregarding the inevitable slight deviations from homogeneity, we shall consider the constant field $H_0$ as homogeneous over the distance $l$ of the path where the neutrons are exposed to the oscillating field, and we shall furthermore assume that the oscillating field has a single component at right angles to $H_0$, with an amplitude $H_1$ which is constant within and zero outside of $l$. We consider thus the transitions of the neutrons in a field, given by

$$H_x=H_1\cos\omega t,\quad H_y=0,\quad H_z=H_0. \qquad (11)$$

With the gyromagnetic ratio $\gamma_N$ for neutrons, and using

$$\Delta=\omega-\gamma_NH_0 \qquad (12)$$

for the deviation of the applied circular frequency from the resonance value $\gamma_NH_0$, one obtains[6] for the transition probability of neutrons with velocity $v$

$$P(v)=\frac{\sin^2\left(\dfrac{l}{2v}(\Gamma^2+\Delta^2)^{\frac{1}{2}}\right)}{1+(\Delta/\Gamma)^2}, \qquad (13)$$

where the frequency

$$\Gamma=\frac{\gamma_NH_1}{2} \qquad (14)$$

is assumed to be small compared to the resonance frequency $\gamma_NH_0$ (i.e., $H_1\ll H_0$), so that the oscillating field can be replaced by an effective rotating field of magnitude $H_1/2$.[12]

[12] F. Bloch and A. Siegert, Phys. Rev. 57, 522 (1940).

Substituting (13) into the general expression (7) for the transition effect, one obtains

$$E_T=-\frac{F((\Gamma^2+\Delta^2)^{\frac{1}{2}})}{[1+(\Delta/\Gamma)^2][1+E_D]}, \tag{15}$$

where

$$F(\xi)=\frac{2\int f_0(C_D-C_AC_P)\sin^2(l\xi/2v)dv}{\int f_0 dv} \tag{16}$$

This function can evidently not be evaluated without knowledge of $f_0$, $C_D$, $C_A$, and $C_P$ in their dependence upon the neutron velocity $v$. It is possible, however, to determine the function experimentally by measuring the dependence of the transition effect at resonance upon $\Gamma$, or through (14) its dependence upon the amplitude $H_1$ of the oscillating field, since for $\Delta=0$ one has $E_T=-F(\Gamma)/(1+E_D)$. Equation (15) then allows one to obtain the transition effect $E_T$ in its dependence upon $\Delta$. Denoting by $F_0(\Gamma)$ the function $F$ at resonance, one obtains

$$E_T(\Delta,\Gamma)=-\frac{F_0((\Gamma^2+\Delta^2)^{\frac{1}{2}})}{[1+(\Delta/\Gamma)^2][1+E_D]}, \tag{17}$$

which predicts the shape of the resonance line for fixed amplitude of the oscillating field and for varying deviation $\Delta$ of its frequency from resonance.

It is seen from (15) and (16) that the simple shape of a resonance line with $E_T\sim 1/[1+(\Delta/\Gamma)^2]$ can be expected only for large values of the oscillating field. Denoting by $\bar{v}$ an average velocity of the neutrons, the term $\sin^2(l\xi/2v)$ in (16) will become rapidly varying and can be replaced by its average value $\frac{1}{2}$ if

$$\Gamma=\frac{\gamma_N H_1}{2}\gg\frac{\pi\bar{v}}{l}. \tag{18}$$

The characteristic value of $\Gamma=\pi\bar{v}/l$ is that for which the neutrons of velocity $\bar{v}$ will with certainty, according to (13), undergo a transition at resonance. With the condition (18) being satisfied one has

$$F=\frac{\int f_0(C_D-C_AC_P)dv}{\int f_0 dv} \tag{19}$$

independently of $\Delta$ and $\Gamma$, and

$$E_T=\frac{E_T^*}{1+(\Delta/\Gamma)^2}, \tag{20}$$

where $E_T^*$ is the effect observed for complete depolarization of the beam between polarizer and analyzer, which has been previously discussed.

It must be remarked, however, that the condition (18), which leads to the simple line shape (20), is not conducive to a good determination of the resonance frequency, since large values of $H_1$ or $\Gamma$ lead at the same time to unnecessarily broad resonance lines. The extreme opposite case $\Gamma\ll\pi\bar{v}/l$, although it would lead to very sharp lines, is likewise unfavorable since it would also lead to an appreciable reduction of the resonance effect and consequently to difficulty in its observation. The optimum condition is obtained for values of $H_1$ in the neighborhood of

$$H_1=\frac{2}{\gamma_N}\Gamma=\frac{2\pi\bar{v}}{\gamma_N l}, \tag{21}$$

where, according to (13),

$$P(\bar{v})=\frac{\sin^2((\pi/2)(1+(\Delta/\Gamma)^2)^{\frac{1}{2}})}{1+(\Delta/\Gamma)^2}, \tag{22}$$

so that a neutron with the average velocity $\bar{v}$ will, with certainty, undergo a transition at resonance, where $\Delta=0$. For this case the analysis of the line shape cannot be carried out with the assumption of a constant $F$, but requires the use of the more refined formula (15).

The gyromagnetic ratio $\gamma_N$ of the neutron is measured by determining the resonance value $\omega^*$ of the circular frequency $\omega$ for which $\Delta=0$, so that from (12)

$$\gamma_N=\frac{\omega^*}{H_0}, \tag{23}$$

and it is worth while to obtain an estimate for the accuracy to be expected in ascertaining the resonance value $\omega^*$. Assuming that the resonance is established merely within the half-width of the line, which is of the order of $2\Gamma$ in frequency scale, a measure of the relative accuracy is $a=2\Gamma/\omega^*$ and one has

$$a=\frac{\bar{v}}{l\nu^*} \tag{24}$$

for the optimum value of $\Gamma$ given by (21) and where $\nu_N^* = \omega^*/2\pi$ is the neutron resonance frequency. Using as characteristic figures for our experimental conditions $\bar{v} = 2\times 10^5$ cm/sec., $l = 10$ cm, and $\nu_N^* = 3\times 10^7$ sec.$^{-1}$, one obtains from (24) $a = 6.7\times 10^{-4}$. The accuracy of a crude experiment without any analysis of the line shape would indeed be essentially limited by this number. It is clear, on the other hand, that such an analysis is necessary in a precision measurement; its completion has allowed us, at the same time, to improve appreciably upon the previously mentioned accuracy.

Before entering into a discussion of the finer features caused by the deviations of the actual geometry from the one which has led to (15), we shall give a qualitative derivation of the function $F(\xi)$, defined through (16). It is based upon a simple assumptions that the velocity distribution $f_0(v)$ of the neutrons has the quasi-Maxwellian form $f_0(v) = Av^n \exp(-\beta v^2)$, and that the factor $(C_D - C_A C_P)$ can be considered within the important velocity range to be independent of the velocity. It is particularly this last simplification which is certainly no more than qualitatively justified. With these assumptions one obtains from (16)

$$F(\xi) = -(1+E_D)E_T^*\phi_n(l\xi(\beta)^{\frac{1}{2}}), \qquad (25)$$

where

$$\phi_n(x) = \frac{\int_0^\infty u^n \exp(-u^2)\left(1-\cos\frac{x}{u}\right)du}{\int_0^\infty u^n \exp(-u^2)du}. \qquad (26)$$

The appearance in (25) of the factor $E_T^*$, representing the transition effect for complete depolarization, arises from the fact that $\phi_n$ approaches unity for large values of its argument, and that for $\Delta = 0$ and large values of $\Gamma$, the expression (15) becomes $E_T = E_T^*$.

The function $\phi_n(x)$ has been computed for $n = 2$ and $n = 3$. The corresponding curves, showing the dependence of $E_T$ on $H_1$, are plotted in Fig. 10, together with the experimental results. The curves were obtained from $\phi_2$ and $\phi_3$ by a suitable choice of the scale parameters $\beta$ and $E_T^*$. They show essentially the same characteristic behavior, i.e., a quadratic dependence for very small values of the argument followed by a rather large interval, in which they are almost linear with $x$; an outspoken maximum is reached with $\phi_2 = 1.46$ for $x = 3.3$ and $\phi_3 = 1.54$ for $x = 3.8$, followed finally by a strongly damped oscillation around the asymptotic value $\phi = 1$. The fact that the experimental results show the characteristic features of the calculated functions is most gratifying; the complete fit within the experimental error, obtained particularly for $n = 3$, must, however, be considered as accidental. It shall be remarked that a Fourier transformation of (16) allows one, in principle, to determine the velocity dependence of $f_0(C_D - C_A C_P)$ in the integrand of (16) from the experimentally determined function $F(\xi)$. It seemed more convenient, however, to follow the opposite procedure, i.e., to show that a not unreasonable assumption about the velocity dependence leads to a sufficient agreement with the experiment.

While the preceding analysis offers the essential features which enter in a discussion of the magnitude of the effect and of the line shape, it is not sufficiently rigorous to be used as a perfect guide for the establishment of the resonance frequency. Deviations of the actual field in which the transitions occur from the form assumed in (11) require certain corrections which have to be considered.

A first correction arises if $H_1$, the amplitude of $H_x$, although constant in time is not constant in space so that, dependent upon its velocity, a neutron is exposed to a field with a time-dependent amplitude of its $x$ component. It is, of course, possible in this case to obtain the transition probability $P(v)$ through a numerical integration by inserting the measured local variation of $H_1$, but it cannot be expected, in general, to be of any simple form. One can show, however, that the formulae (13) and (15) still remain valid in the special case of resonance ($\Delta = 0$) if the quantity $H_1 l$ is replaced by $\int_0^l H_1 dx$. A simple and, for practical purposes, sufficient correction for the case $\Delta \neq 0$ can be obtained by replacing $H_1$ by its average value and correspondingly by writing

$$\Gamma = \frac{\gamma_N}{2l}\int_0^l H_1 dx$$

in place of (14). Apart from these minor corrections the expression (15) will not be essentially invalidated by the spatial variation of $H_1$. Particularly, the important feature that the line is symmetrical around the resonance value $\Delta=0$ can be seen to be unaffected by this variation.

A more serious correction arises if the assumption of a vanishing $y$ component of the oscillating field made in (11) is not satisfied, and particularly if this component shows likewise a spatial variation. Writing generally,

$$H_x=H_1\cos(\omega t+\delta_1),\quad H_y=H_2\cos(\omega t+\delta_2),\quad H_z=H_0. \tag{27}$$

Hence, both $H_1$ and $H_2$ must be considered, for a neutron with given velocity, to be functions of time. The phases $\delta_1$ and $\delta_2$ might differ as a result of energy-dissipating elements. However, the high $Q$ value of our coil arrangement ($Q\sim 100$) justifies the assumption of equal phases. The effect of the field (27) upon a moving neutron can then be expressed by the modification of (11),

$$H_x'=H_1'\cos\omega t,\quad H_y'=0,\quad H_z'=H_0\mp\frac{d\theta}{dt}\Big/\gamma_N, \tag{28}$$

where $H_1'=(H_1^2+H_2^2)^{\frac{1}{2}}$, and $\theta=\arctan(H_2/H_1)$. The time derivative of $\theta$ in (28) is to be taken in a system moving with the neutron. It appears with the $-$ or $+$ sign, depending upon whether the sense of rotation of the Larmor precession which the neutron moment performs in the field $H_0$ is the same or opposite to that corresponding to a positive variation of $\theta$.

The effect caused by the variation of $\theta$ is seen to be the same as that caused by an additional field in the $z$ direction superimposed upon $H_0$. As long as $\theta$ can be considered as a linear function of the time, it causes the resonance curve to be shifted by the amount $d\theta/dt$ in frequency, thereby causing an error in the determination of the resonance value. A non-linear variation, on the other hand, has the same effect as an inhomogeneity of the field $H_0$, since both will manifest themselves in the frame of reference of the neutron as a time variation of the effective $z$ component $H_z'$.

It is clear that such an inhomogeneity will, in general, destroy the symmetry of the resonance curve around any point. As far as the field $H_0$ is concerned it is important to reduce the actual inhomogeneity as much as possible by careful shimming, both for minimizing this undesirable asymmetry and also in view of improving the total accuracy of the measurement. In order to eliminate any possible error attributable to the variation of $\theta$, it is essential to observe that it appears in (28) with the opposite sign upon reversal of the field $H_0$, and thereby upon reversal of the sense of rotation in which the neutron precession occurs. By observing the transition effect twice, i.e., leaving the magnitude of $H_0$ unchanged but reversing its polarity, any falsifying shift of the resonance curve, as well as its asymmetry due to this cause is thus avoided. It should be remarked that similar to the feature used by Millman[13] to determine the sign of nuclear moments, the sign of the neutron moment can be ascertained from the direction in which an observable shift of the resonance curve takes place upon reversing the polarity of $H_0$.

By applying the previous considerations to a sufficient set of experimental data it is possible to establish the resonance value $\omega^*$ of the applied frequency well within the width of the resonance line and, hence, to determine with high accuracy the gyromagnetic ratio $\gamma_N$ of the neutron from (23) in terms of the field $H_0$. The conventional methods of determining this field are, however, inadequate to obtain a precision which appreciably exceeds the order of one percent. It was, in fact, this circumstance which limited to a large extent the accuracy of the original result obtained by Alvarez and Bloch.

The method of nuclear induction is ideally suited to overcome this limitation in the measurement of the neutron moment. As pointed out in the original paper,[14] it was indeed the search for a suitable method of field comparison which led to the thought of nuclear induction, and preparations to apply it to the determination of the neutron moment were announced at the same time.

The essential improvement over the earlier methods of determining the field lies in the direct comparison of the conditions for reso-

[13] S. Millman, Phys. Rev. 55, 628 (1939).
[14] F. Bloch, Phys. Rev. 70, 460 (1946).

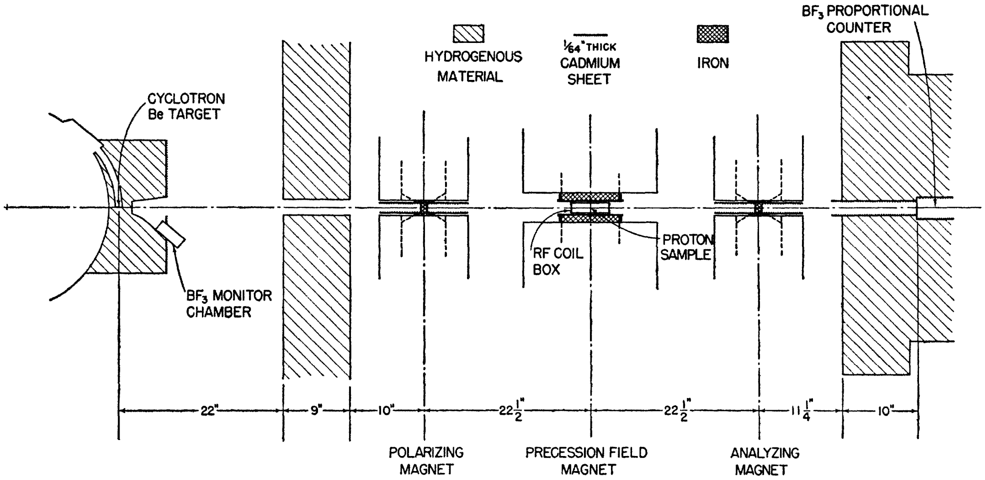


FIG. 1. Experimental arrangement.

nance observed simultaneously for neutrons and protons. By placing a sample containing protons into the same field $H_0$ in which the neutron transitions occur and by observing the proton resonance frequency $\nu_P=\omega_P/2\pi$ through nuclear induction, one obtains in analogy to (23) for the gyromagnetic ratio of the proton

$$\gamma_P=\frac{\omega_P}{H_0} \tag{29}$$

and, therefore, by division of (23) and (29)

$$\frac{\gamma_N}{\gamma_P}=\frac{\omega^*}{\omega_P}=\frac{\nu_N^*}{\nu_P}. \tag{30}$$

The determination of the gyromagnetic ratio of the neutron in terms of that of the proton is thus immediately obtained by a measurement of the ratio of two resonance frequencies $\nu_N^*$ and $\nu_P$ which can be carried out with high accuracy.

The actual value of $H_0$ does not enter at all in this determination. In fact, a stabilizing device was used in our experiment which held $H_0$ constantly at the value required for proton resonance so that the condition (29) was constantly and automatically fulfilled while establishing the neutron resonance (see below).

## III. APPARATUS

Figure 1 shows the arrangement of the apparatus used in our present measurements. Thermalized neutrons emerge from a $2\frac{1}{2}''\times 2\frac{1}{2}''$ hole in a block of paraffin surrounding the target of the cyclotron. They are collimated by a $2''\times 2''$ opening in the $9''$ thick hydrogenous wall in front of the target and enter and leave the polarizing magnet through cadmium channels $5\frac{3}{4}''$ long with openings $1\frac{1}{2}''\times 2''$. These channels sit directly in front and in back of the polarizing iron plate. After passing through the polarizer, the neutrons enter the gap of the magnet for the constant precession field $H_0$. A copper box containing the coil for the radiofrequency field is located between the poles of this magnet. The neutrons enter and leave this box through copper windows of 5-mil thickness which, as experiments showed, have no noticeable effect on the state of the polarization. Front and back of the box are covered with cadmium diaphragms with $\frac{3}{4}''\times 1\frac{1}{2}''$ openings, which prevent passage of the neutrons very close to the windings of the radiofrequency coil. While in the arrangement of Alvarez and Bloch the radiofrequency coil was a long solenoid with its axis perpendicular to the neutron beam, we used a shorter coil surrounding the beam. This choice results in a less homogeneous radiofrequency field, but it has the great advantage of avoiding neutron absorption in the coil windings. The axis of the precession field magnet is $21\frac{1}{2}''$ from the axes of both the polarizer and analyzer. The neutrons enter and leave the analyzer through an arrangement of cadmium channels similar to the one used for the polarizer. Finally, the neutrons enter the detector, located at a distance of about 16″ from the analyzing magnet. The detector for the present measurement was identical with the one used in the previous work of Bloch, Condit, and Staub,[10] except that enriched boron trifluoride, containing 96 percent $B_{10}$, furnished by the Atomic Energy Commission, was used. This filling resulted in an increase in counting efficiency from about 20 to 70 percent for thermal neutrons (2200 m/sec.). The detector is, therefore, not shallow for most polarized neutrons. It was imbedded in a large block of paraffin surrounded by oil-filled cans. A shield of $\frac{1}{4}$″ of boron carbide and cadmium enveloped the detector with the exception of an opening of 2-in. diameter at the front face.

With the two blocks of iron, each 1.9 cm thick, in the gaps of polarizing and analyzing magnet, the cadmium ratio of the recorded neutron beam was about 4. This ratio was regularly measured by placing a piece of cadmium sheet, 1/64″ thick and just slightly larger than the cross section of the beam, over the opening of the exit diaphragm of the radio frequency coil box.

A neutron beam monitor, using an integrating boron trifluoride chamber and a circuit described earlier,[15] was employed to change the conditions of the experiment in intervals of equal numbers of primary slow neutrons. The monitor chamber was located as close as possible to the region where the neutron beam originated, and the chamber was operated at a collecting potential of 5000 v. Checks showed only a very slight dependence of the monitor performance on variations in the position of the target spot as caused by changes in the deflecting voltage or resonance condition of the cyclotron. Saturation of the ionization current was quite high. For a change in deuteron beam intensity from 5 to 15$\mu$ amp., the number of counts per monitor interval increased by only 7 percent. All final measurements were taken with a beam current of 7$\mu$ amp., and this current was carefully kept as constant as possible.

The pulses of the detector were fed into a conventional amplifier, discriminator, and scale of 16, from which the output pulses alternately operated one of two different registers during each monitor interval.

[15] E. Fryer and H. Staub, Rev. Sci. Inst. **13**, 187 (1942).

The magnets for polarizer and analyzer are identical. One of them had been previously used for the measurements of polarization effects.[10] The plane pole faces of these magnets have an area of 2″×3″ tapered down by a 60° angle from pole pieces of 6-in. diameter. The gap is $1\frac{1}{2}$″ wide. The polarizing and analyzing blocks consisted, as in the previous work, of hot-rolled steel $1\frac{1}{2}$″×2″ and $\frac{3}{4}$″ thick in the direction of the neutron beam. For all the present experiments the magnets were operated at a magnetizing current which produces a field of 11,900 gauss. The direction of magnetization was always the same for polarizing, analyzing, and precession field magnet so as to avoid any region of weak field in which the polarization of the neutrons could be changed through non-adiabatic transitions.

The currents of polarizer and analyzer magnets, both about 34 amps., were kept constant to about 2 percent by manual control. Since the contribution of these two magnets to the precession field ($\sim 10^4$ gauss) was only about 100 gauss, a two percent variation of the current would, on account of saturation, produce only a variation of $\frac{1}{2}$ gauss. Moreover, the control mechanism which was employed for the precession field would also take care of these variations provided that they were uniform over the region occupied by the proton sample and the radiofrequency field. It was indeed verified that such small variations did not affect the necessary homogeneity of the precession field $H_0$.

The precession field magnet has circular plane pole faces of $7\frac{13}{16}$-in. diameter and a $1\frac{3}{4}$-in gap width. Circular brass plates $\frac{1}{16}$″ thick, which carried the system of shims to correct the field, were clamped onto the pole pieces. Two carefully machined brass spacers were tightly fitted between the two pole faces. They very effectively prevented major changes of the gap configuration upon magnetization of the pole pieces. The magnet was energized by two coils, each having 17,000 turns. A current of about 1.4 amp. was necessary to produce a field of about 9800 gauss (corresponding roughly to a Larmor frequency of the neutrons of 29 Mc, and it was supplied by a 3000 v full-wave, three-phase rectifier. The useful region of the precession field in which transitions can occur, defined by the length of

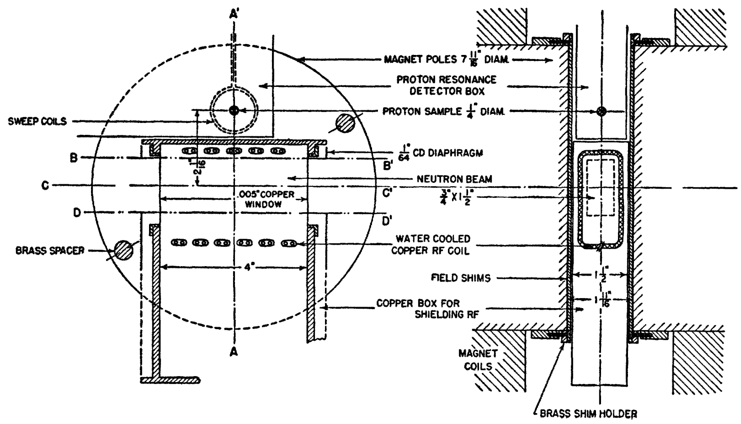


FIG. 2. Gap of precession field magnet, radiofrequency coil, and proton sample.

the copper box and the aperture of the cadmium diaphragms, was a volume 4″ long in the direction of the neutron beam, $\frac{3}{4}$″ wide across the gap, and $1\frac{1}{2}$″ high, and symmetric with respect to the center of the gap. The proton sample was approximately of spherical shape about $\frac{1}{4}$″ in diameter, with its center located $2\frac{1}{16}$″above the axis of the magnet (Fig. 2). Two small coils which were mounted coaxially with the proton sample on the walls of the box containing the sample and the rectifier, provided the modulation for the proton resonance. For measurements of the magnetic field (see below), the modulation frequency was 60 c.p.s. with an amplitude of about 3 gauss. For the actual measurements of the neutron resonance, 500 c.p.s. modulation at an amplitude of 0.3 gauss was employed. The effect of this alternating field in the region of the neutron passage is, of course, completely negligible. The magnet current passed through 8 parallel-connected 304 TH tubes in series with the magnet windings. The grids of these tubes were controlled by the output voltage of a differential amplifier with a gain of about 10,000. This differential amplifier compared a constant voltage derived from dry cells with the voltage drop of the magnet current across a 30-ohm resistor. The stabilization of the current was such that for a line-voltage fluctuation of 10 percent the field varied about 3 gauss at 10,000 gauss. This manner of stabilization was employed whenever measurements of the inhomogeneities of the precession field were performed. For the actual measurements of the neutron resonances, an additional stabilizer, designed and built by M. Packard,[16] was used. It injects into the differential amplifier a voltage derived from the proton resonance signal in such a manner as to keep the magnetic field at resonance with the proton excitation frequency. With this additional stabilization, the field varied about 0.02 gauss for a 10 percent line-voltage change, and, in addition, was stabilized by a factor of 150 against any other causes of field variation, such as temperature and extraneous fields.

The accuracy of the present experiment is largely determined by the degree of homogeneity of the precession field over the region occupied by the proton sample and the region in which neutron transitions can occur. Consequently, a great deal of time and effort was spent in making this field as perfect as possible.

Inhomogeneities of the field were measured by observing the change of the proton resonance frequency upon the displacement of the proton sample from the center of the magnet to the point under investigation. For these measurements the magnet current was stabilized in the manner described above. The proton resonance obtained by 60-cycle field modulation was observed on the screen of a cathode-ray tube, and the frequency adjusted until the maximum of the resonance peak coincided with a reference line. The accuracy of this procedure is limited mainly by the unavoidable fluctuations of the field caused by the finite stabilization against line-voltage fluctuations. At points near the outer boundaries of the useful region of the field, the proton signal becomes somewhat wider, since the

[16] M. E. Packard, Rev. Sci. Inst. 19, 435 (1948).

field is less homogeneous there than at the center. The accuracy of this measurement was about $\frac{1}{4}$ gauss for fields of about 10,000 gauss. All measurements were made with the fields of polarizing and analyzing magnets on so as to correct also for inhomogeneities caused by these fields. Within the above-stated accuracy, their contribution to the $H_0$ field was uniform over the useful region.

It was, of course, to be expected that a region of about $4'' \times 2''$ within the gap of the circular pole pieces of $7\frac{13}{16}$-in. diameter would not be nearly homogeneous enough for our measurements. Even if the pole pieces were perfectly symmetric and uniform, the fringing of the field would be considerable. Suitable shims were, therefore, mounted on the brass plates which held them directly against the pole faces. The first step in homogenizing the field consisted in using 90-degree sectors of annular rings cut from 5-mil shim steel, and of suitable radii so as to produce a slight increase of the field strength with increasing distance from the center. The second step was to place strips of 1-mil brass foil electroplated with nickel to thicknesses up to 0.3 mil wherever the field showed a dip. Every step was, of course, carefully checked by moving the proton sample along the four directions marked *ABCD* in Fig. 2 and by measuring the deviation of the resonance frequency from its value at the center of the magnet. A typical plot of the field variations along these four directions after completion of the shimming is shown in Fig. 3. The deviations given in kc represent a sixth of the actual frequency variation, since all proton frequencies were measured at one-sixth of the actual frequency (see below). Thus, a variation of 1 kc in Fig. 3 represents a variation of about 1 part in 7000 or about 1.4 gauss. Correction of the field was continued until no single point deviated by more than 0.5 kc, and the midpoint of the proton sample ($2\frac{1}{16}''$ vertically above the center) deviated not more than 0.2 kc from the average value. The largest deviations were always encountered at the end points ($\pm 2''$ from center) of the horizontal directions *BCD*, where their effect on the measurements is considerably reduced by the fact that at these points the effective radiofrequency field is quite small.

The rather severe requirements for the homogeneity of the precession field made it necessary to correct the field separately for every value of $H_0$ at which measurements were taken, since even changes of 5 percent in $H_0$ changed the configuration of the field entirely on account of saturation of the pole pieces and shims. A further complication, which was rather troublesome, was caused by variations of the field configuration with time. Although the magnet current and the cooling water were left on continuously, inhomogeneities developed over periods of several hours. It was found that the original situation could always be regained by alternately raising and lowering the current through a well defined cycle. The field configuration was, therefore, checked during the measurements at intervals of three hours and readjusted in the above manner if inhomogeneities could be detected. These field measurements also provided the information needed to correct the data for possible small differences in the value of the precession field $H_0$ for neutrons and protons.

The determination of the ratio of the resonance frequencies of protons and neutrons is greatly facilitated by the fact that the ratio of the neutron and proton magnetic moments is approximately $\frac{2}{3}$. The radiofrequency field of the protons was excited by the sixth harmonic of its master oscillator with frequency $\nu_P' = \nu_P/6$. Similarly, the fourth harmonic of another master oscillator with frequency $\nu_N' = \nu_N/4$ was used for the radiofrequency field of the neutrons. If one denotes the difference in the frequencies of the two master oscillators by $\Delta\nu_N'$, which has the

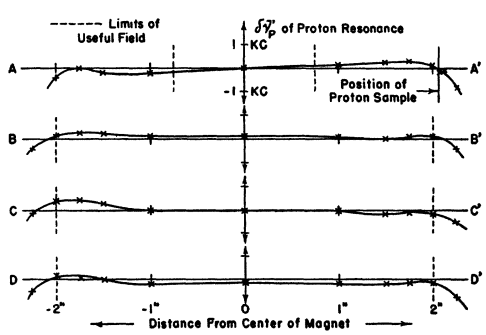


FIG. 3. Typical plot of the deviation of $H_0$ from value at center measured along the four directions *A*, *B*, *C*, *D* of Fig. 2. Deviations $\delta\nu_P'$ are measured in terms of one-sixth of the corresponding proton resonance frequency.

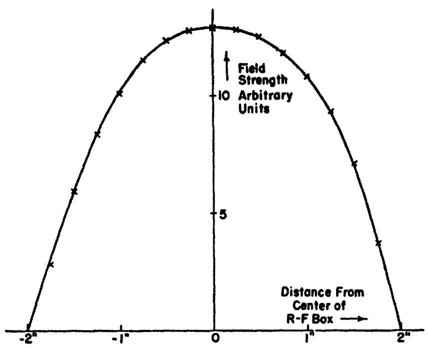


FIG. 4. $X$ component (direction of propagation of neutrons) of the radiofrequency field.

value $\Delta\nu_N'^*$ at the neutron resonance, and by $\nu_P'$, the frequency of the proton master oscillator which was always set for the proton resonance, the desired ratio is given by

$$R=\frac{|\mu_N|}{\mu_P}=\frac{\nu_N^*}{\nu_P}=\frac{2}{3}\left(1+\frac{\Delta\nu_N'^*}{\nu_P'}\right). \quad (31)$$

It follows from the approximately known values of the two moments that the relative error in $R$ is only about one-fortieth of the relative error of $\Delta\nu_N'^*/\nu_N'$. The two frequencies $\Delta\nu_N'$ and $\nu_P'$ were measured with a General Radio combined heterodyne-frequency meter and crystal-controlled calibrator. This device produces the fundamental and the harmonics of a crystal-controlled frequency of 100 kc as well as of frequencies exactly one-fifth and one-tenth of this basic frequency. A second generator with continuously variable frequency can be set to any desired frequency by visual interpolation or with the aid of an interpolation meter. This secondary frequency can thus be adjusted to zero beat with the unknown external frequency. From the working principle of the apparatus, it is apparent that the fundamental crystal frequency does not enter in our measurements, since it is only the ratio of two frequencies which determines $R$. In terms of the errors $\delta\Delta\nu_N'^*$ of $\Delta\nu_N'^*$ and $\delta\nu_P'$ of $\nu_P'$, the relative error in the ratio $R$ of the magnetic moments is given by

$$\frac{\delta R}{R}=\frac{\Delta\nu_N'^*/\nu_P'}{1+\Delta\nu_N'^*/\nu_P'}\left(\left(\frac{\delta\Delta\nu_N'^*}{\Delta\nu_N'^*}\right)^2+\left(\frac{\delta\nu_P'}{\nu_P'}\right)^2\right)^{\frac{1}{2}}. \quad (32)$$

$\nu_P'\cong 7$ Mc was determined to within 1 kc, and $\Delta\nu_N'^*\cong 200$ kc was obtained by beating $\nu_N'$ and $\nu_P'$ and measuring this beat frequency with the above-described frequency meter to about 0.2 kc. The value of $\Delta\nu_N'$ was checked continuously and kept at zero beat with the frequency meter by manual adjustment of $\nu_N'$. The proton frequency $\nu_P'$ was checked at frequent intervals during all measurements. Since $(\delta\nu_P'/\nu_P')^2 \ll (\delta\Delta\nu_N'^*/\Delta\nu_N'^*)^2$ and $\Delta\nu_N'^*/\nu_P' \ll 1$, one can, to a very good approximation, replace (32) by

$$\frac{\delta R}{R}=\frac{\delta\Delta\nu_N'^*}{\nu_P'}. \quad (33)$$

With the above figures this number amounts thus to 1/35,000.

The radiofrequency generator and receiver for the protons are of the usual design and are described elsewhere.[16] The generator for the neutron radiofrequency field consisted of an electron-coupled master oscillator with continuously variable frequency from 7 to 8 Mc, followed by a buffer and output stage, employing the second and fourth harmonics, respectively. The output stage is capable of delivering about 700 watts through a coaxial line and impedance-matching network to the coil. This coil (Fig. 2) was water cooled and consisted of 5 turns of $\frac{7}{16}''\times\frac{1}{8}''$ copper tubing wound with a rectangular cross section $2\frac{3}{8}''$ by $1\frac{1}{8}''$, and an over-all length of $3\frac{1}{2}''$. It was completely enclosed in a copper box 4″ long, located, as shown in Fig. 2, between the magnet poles. A tap at a short distance from

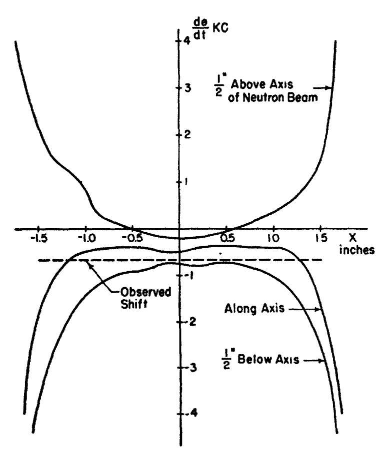


FIG. 5. Angular velocity $d\theta/dt$ of oscillating field $H_1$ as experienced by neutrons moving at a velocity of $2.2\times10^5$ cm sec.$^{-1}$ along the axis of the neutron beam and $\frac{1}{2}''$ above and below the axis, respectively.

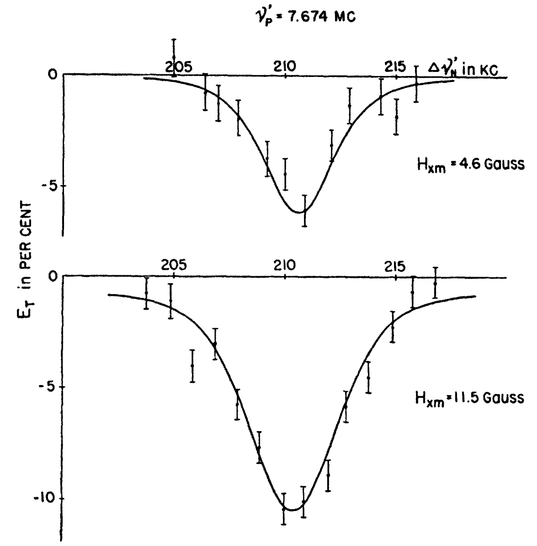


FIG. 6. Neutron resonances observed at $\nu_P'=7.674$ Mc. Solid curves computed from curve of Fig. 10.

the grounded end of the coil served as a connection for a radiofrequency current meter.

The radiofrequency magnetic field was carefully calibrated and its configuration measured, by means of a small $\frac{1}{4}$-in. diameter search coil connected to a radiofrequency voltmeter. The coil was introduced through a small hole in the window of the casing. The distribution of the radiofrequency field component in the $x$ direction (direction of propagation of neutrons) along the axis of the neutron beam is shown in Fig. 4. As expected, the field drops rather rapidly from its maximum value $H_{xm}$ at the center to zero at the walls. No great variation of this distribution throughout the area occupied by the neutron beam was found. As pointed out in Section II, a serious asymmetry and shift of the resonance curve can be caused if the moving neutrons encounter an effectively rotating field perpendicular to $H_0$. This shift can be expressed as an effective change of $H_0$ by means of (28). The shift of the resonance frequency caused by this effect was measured in the most obvious manner (see Sections II and IV) by inversion of the precession field. However, in order to ascertain whether the determined shift is compatible with the actual distribution of $H_x$ and $H_y$, the latter component was also measured as a function of $x$ along the axis of the neutron beam and along two parallel lines $\frac{1}{2}''$ above and below the axis. From these data the quantity $d\theta/dt$, representing the effective shift of the resonance frequency for neutrons of an average velocity, was computed and is represented in Fig. 5 as measured along the three lines in the $x$ direction. The frequencies refer again to one-quarter of the actual neutron resonance frequencies. The average velocity $\bar{v}$ was determined from the measurements of the dependence of the resonance effect on the amplitude of the radiofrequency field. Assigning a transition probability of unity for the average velocity neutrons at the maximum resonance effect, $\bar{v}$ is given by the relation (see Section II)

$$\bar{v}=\frac{\gamma_N}{2\pi}\int_0^l H_1 dx. \qquad (34)$$

Figure 5 shows clearly that with our experimental arrangement the average frequency shift differs markedly from zero. This is easily understood, since the axis of the neutron beam is located above the axis of the radiofrequency coil, and since, furthermore, the shielding is not sym-

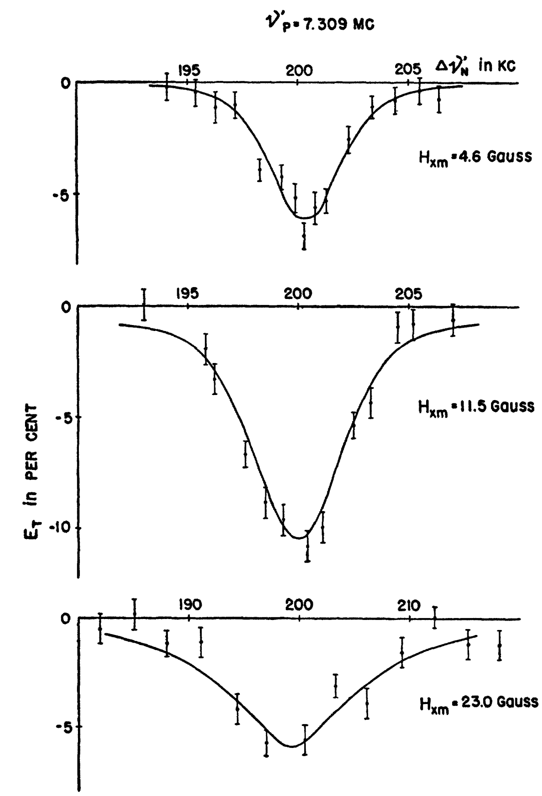


FIG. 7. Neutron resonances observed at $\nu_P'=7.309$ Mc.

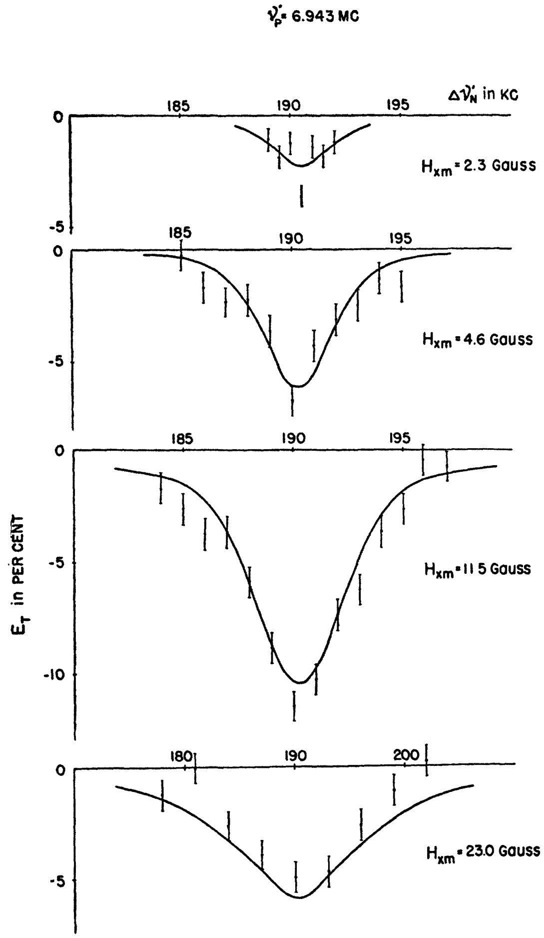


FIG. 8. Neutron resonances observed at $\nu_P' = 6.943$ Mc.

metric and consequently diverts the magnetic lines of force asymmetrically with respect to the axis. Also, as pointed out earlier, if one takes the geometrical features of the arrangement into account, the direction in which the shift occurs upon changing polarity of the precession field is directly connected with the sign of the neutron moment.

## IV. MEASUREMENTS

All measurements were performed with analyzer and polarizer operating at the highest possible field of about 11,900 gauss, corresponding to a magnetization within less than 0.02 percent of saturation. Both oscillators were always turned on several hours before the start of the measurements. With this precaution only small and slow drifts of the frequencies were encountered. The inhomogeneities of the $H_0$ field were checked as previously mentioned at regular intervals. The performance of the proton-controlled regulator for the precession field was continuously checked with the aid of a cathode-ray oscilloscope, as described by Packard.[16] Great care was taken to have always a pattern of pure second harmonic, or zero-average control voltage. Any deviation was corrected manually by changing the current setting of the automatic current control. It is estimated that no deviation from exact proton resonance amounting to an error in $\Delta\nu_N'$ of more than 0.1 kc was permitted. All measurements of neutron resonances were performed by counting detector pulses for each monitor interval with the neutron radiofrequency field alternately on and off. Each interval lasted for about 80 sec. After 20 intervals, conditions were usually changed by varying either the magnitude of the radiofrequency field or the neutron frequency, depending on the type of measurements. During the measurements of the neutron resonances, frequencies were changed in such a manner as to measure alternately at frequencies located approximately symmetrical to the expected resonance frequency. For each group of measurement the background was determined several times by placing a cadmium sheet over the exit diaphragm of the radiofrequency coil box. In every case, sufficient data were taken to make the contribution from the background measurement to the statistical error negligible. Cadmium ratios were always measured with the radiofrequency field alternately turned on and off. As it was expected, the neutron intensity recorded with the cadmium shield intercepting the beam did not exhibit any resonance feature. Likewise, no transmission effects could be detected in this case.

The measurement of the neutron resonance dip was carried out at three different proton frequencies $\nu_P' = 7.674$ Mc, 7.309 Mc, and 6.943 Mc. These frequencies were chosen because with these values for $\nu_P'$, $\Delta\nu_N'$ at resonance has values very near to 210, 200, and 190 kc, respectively; consequently, frequencies close to resonance could be determined without a large interpolation between two values of the crystal oscillator of the frequency meter. At every one of these frequencies several resonance curves were measured using different values of the radiofrequency field. Figure 6 shows the results at $\nu_P'=7.674$ Mc for peak values of $H_{xm}=11.5$ and 4.6 gauss at the center of the coil. Denoting by $I_{\text{on}}$ and $I_{\text{off}}$ the counting rates with and without the applied radiofrequency field, and by $I_{\text{Cd}}$ the counting rate with cadmium intercepting the beam, the ordinate represents the quantity

$$E_T=\frac{I_{\text{on}}-I_{\text{off}}}{I_{\text{off}}-I_{\text{Cd}}}. \quad (35)$$

The errors indicated are mean errors. The measurements were taken at frequency intervals of 1 kc and corrected for any measurable difference ($>0.1$ kc) of $H_0$ at the center and at the proton sample. The solid curves shown in Fig. 6 were calculated according to the procedure outlined in Sections II and V. Since the absolute values of $H_1$ involve the knowledge of the area of a small radiofrequency search coil, their accuracy is probably about 20 percent. No correction for the frequency shift caused by the spatial variation of the radiofrequency field was made for this and the following figures. Figure 7 shows the results of the measurements at $\nu_P'=7.309$ Mc taken with radiofrequency fields of $H_{xm}=23.0$, 11.5, and 4.6 gauss. These measurements were actually the first ones performed, and the $H_0$ field was not nearly so homogeneous as in the later measurements. Consequently, corrections for deviations of as much as 0.8 kc had to be applied for differences of $H_0$ for neutrons and protons. This caused the uneven spacing of the measured points. We, therefore, believe that these data are considerably less reliable than the ones at 7.674 and 6.943 Mc. Again, the solid curves were calculated as before.

Figure 8 finally shows the results of the measurements at $\nu_P'=6.943$ Mc. For this proton frequency the $H_0$ field was almost perfect; thus no correction at all had to be applied for field differences, and it was felt that with this field the most reliable data were obtained. Consequently, data were taken for $H_{xm}=23.0$, 11.5, 4.6, and 2.3 gauss, as shown in Fig. 8. Furthermore, an experiment to determine the shift of the resonance maximum caused by the presence of the $H_y$ component of the radiofrequency field was performed, as indicated in Section II, by reversing the polarity of the precession field. Upon reversal of the polarity, and after raising and lowering the magnet current according to the adopted procedure, it was found that no additional shimming was required. At the same time, the directions of polarizing and analyzing fields were inverted so as to have field reversal along the whole neutron path. The two measurements were taken with a radiofrequency field of 11.5 gauss, a value very near to that for which the maximum resonance effect occurs. The results are shown in Fig. 9. The points labeled "positive field" are those taken with the field in the same direction as for the preceding measurements. The points labeled "negative field" are those taken with the opposite field direction. The figure shows clearly a displacement between the two sets of data corresponding to about 1.3 kc. This difference represents, of course, twice the actual shift. The observed shift of 0.67 kc is indicated in Fig. 5 and is seen to be compatible with that expected from the measurements of the spatial variation $H_x$ and $H_y$.

A check was performed to ascertain whether the sign of this shift was in agreement with the negative sign of the neutron magnetic moment. This was done by determining the relative orientations of the precession field and the magnetic field of the cyclotron magnet. The sense of rotation of the positive ions in the cyclotron is known from the arrangement of the deflector. It follows from the geometrical arrangement that, in the average, the lines of force of the neutron radiofrequency field, ½ inch below the axis of the beam, curve downwards at the ends. The sense of rotation of the radiofrequency field thus seen

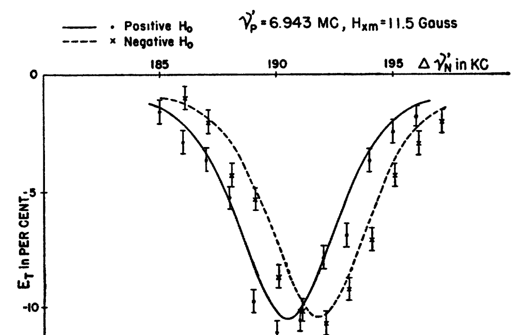


FIG. 9. Shift of resonance maximum caused by reversal of direction of precession field.

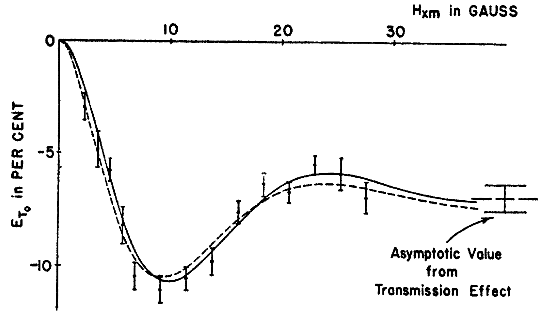


FIG. 10. Transmission effect at resonance *versus* amplitude of radiofrequency field. Values of $H_1$ are measured by $H_{xm}$, the $x$ component of $H_1$ at center of coil. Solid curve: best fit obtained with $v^3 \exp(-\beta v^2)$ distribution. Dotted curve: best fit obtained with $v^2 \exp(-\beta v^2)$ distribution.

by the moving neutrons and the direction of the observed shift were indeed found to indicate the negative sign of the neutron moment.

As shown in Section II, the shape of the neutron resonance dips can be predicted (assuming homogeneity of $H_0$) if the dependence of $E_T$ at resonance on the amplitude $H_1$ of the radiofrequency field is known (cf. (15) and (16)). Since the experimental results certainly do not exhibit a simple relation between $H_1$ and the observed width of the resonance dips, the values $E_{T0}$ of $E_T$ at resonance were carefully measured over the full available region of $H_1$ measured in terms of $H_{xm}$. The resonance frequencies for these experiments were obtained by inspection from the preceding measurements. Data were taken at the three proton resonance frequencies. The combined results are shown in Fig. 10. The errors are the mean statistical errors increased by an amount corresponding to a possible error $\delta\nu_P'$ in the resonance frequency of 0.5 kc. Whereas the dependence of $E_{T0}$ on $H_{xm}$ would obey a simple $\sin^2$ law if the neutrons were monoenergetic, the results show clearly the rapid damping and asymptotic approach to a finite value for a sufficiently large radiofrequency amplitude corresponding to a uniform transition probability $\frac{1}{2}$ for all neutrons of different velocity. This asymptotic value $E_T^*$ is related by expression (8) to the three transmission effects $E_D$, $E_A$, $E_P$, which were measured directly several times. In addition, a check measurement of the quantity $E_D - E_A$ was performed by leaving the analyzer magnetized and observing the intensity change upon magnetization of the polarizer. All these measurements were consistent and gave:†

$$E_A = E_P = (6.3 \pm 0.3) \text{ percent} \tag{36}$$

and

$$E_D = (21.1 \pm 0.5) \text{ percent.} \tag{37}$$

The asymptotic value $E_T^*$ is, therefore,

$$E_T^* = (-6.9 \pm 0.6) \text{ percent,} \tag{38}$$

as indicated in Fig. 10. The observed and calculated value of the asymptotic effect are in good agreement. The fact that the proper and quantitative relation between the single-transmission effects and the double-transmission resonance effect could be established must be considered as an important factor in evaluating the reliability of the data. The solid curve in Fig. 10 represents the best fit with $f_0(v) = Av^3 \exp(-\beta v^2)$ (cf. Section II), the dotted one with $f_0(v) = Av^2 \exp(-\beta v^2)$.

A last series of measurements was undertaken to ascertain that within a region extending on both sides of the established resonance over a one percent range in frequency scale no other resonance dips of real, spurious, or instrumental nature could be found. Measurements were taken at a proton frequency $\nu_P' = 7.309$ Mc, with a radiofrequency field of 11.5 gauss from $\Delta\nu_N' = 133$ kc to $\Delta\nu_N' = 267$ kc in steps of 6 kc, which is approximately the half-width of the previously observed resonance dip. The results of these measurements are shown in Fig. 11. Outside the resonance dip at 200 kc none of the observed points shows a deviation from zero towards negative values by more than about $(1 \pm 0.7)$ percent, or $\frac{1}{10}$ of the resonance maximum. It can, therefore, be safely stated that no other resonance dip with an intensity of more than $\frac{1}{3}$ of the observed main dip can be present within the explored region.

## V. RESULTS

As pointed out in Section II, it is not possible to predict the exact dependence of $E_{T0}$, the value of the transition effect at resonance, on the value of $H_1$ without the complete knowledge of the dependence of $C_D$, $C_A$, $C_P$, and $f_0$ upon the

† It should be noted that the values of the transmission effects are smaller than those previously quoted (see reference 10). These data were taken with the same arrangement but with the proportional counter filled with ordinary $BF_3$. The counter used in the present measurements favors fast neutrons and consequently the transmission effects are somewhat smaller.

velocity $v$. However, it was tried to match the observed points to a curve computed under reasonable assumptions. For this purpose $C_D$ and $C_A C_P$ in expression (16) were replaced by their average values, such that:

$$E_{T0} = E_T^* \frac{2\int_0^\infty f_0(v)\,\sin^2\left(\frac{l}{2v}\Gamma\right)dv}{\int_0^\infty f_0(v)dv}. \tag{39}$$

For $f_0(v)$, the two most obvious functions $f_0(v) = Av^2\exp(-\beta v^2)$ and $f_0(v) = Av^3\exp(-\beta v^2)$, were chosen and the expressions for $E_{T0}(H_1)$ integrated numerically. The appropriate values of $\beta$ and $E_T^*$ were determined by the best-fitting curves, as shown in Fig. 10. It should be emphasized, of course, that actually there is hardly any justification for using a Maxwellian distribution in conjunction with the average values of $C_A$, $C_P$, and $C_D$. However, the very fact that excellent agreement with the experimental results can be obtained, as shown particularly by the relative position and magnitude of the two extrema, can be taken as proof that the adopted procedure cannot be very far from being correct. Between the two distribution functions there is not too large a difference; however, the $v^3\exp(-\beta v^2)$ distribution gives definitely a better fit. Since the detector used in our experiment was rather thick, this fact agrees with the expectation. From the value of $\beta$ in the best-fitting curve one obtains a most probable velocity of the neutrons, $\bar{v} = 2200$ m/sec. For a distribution of the form $Av^3\exp(-\beta v^2)$, characteristic for thermal neutrons and a thick detector, the most probable velocity at room temperature is 2690 m/sec. Our result, therefore, indicates that the effective temperature of the polarized neutrons is lower than room temperature as one would expect, since the polarization cross section increases with decreasing velocity. The first maximum of the $E_{T0}$ curve occurs at a field $H_{xm} = 10.0$ gauss at the center. If one assigns a transition probability of one at this field to the neutrons of the most probable velocity, $\bar{v} = 2\mu_N H_1 l/h$, inserting for $H_1$ its measured average value over the length of the coil box, one obtains the former value of 2200 m/sec. for $\bar{v}$. The second parameter of $E_T^*$ for the best fit is

$$E_T^* = -7.0 \text{ percent}, \tag{40}$$

in excellent agreement with the value (38) as observed by the transmission experiments. It was, therefore, felt that the observed resonance dips were fully understood and that the calculation of the shape of the resonance curves could be safely based upon the $E_{T0}$ ($H_{xm}$) curve.

A difficulty in the calculation of the resonance curves arises from the fact that $H_1$ varies strongly over the region where transitions occur. As a first approximation, the value of $H_1$ entering in $\Gamma$ was taken as the average value of $H_x$ over the length $l$ of the radiofrequency coil box. For one case, the calculation was carried out by approximating $H_x(x)$ by four step-functions, and integrating the quantum-mechanical equations for neutrons of the most probable velocity over these steps. The result showed no noticeable difference from the one obtained with the average value of $H_x$. Thus the theoretical curves shown in Figs. 6, 7, 8, and 9 were obtained from (17) with the deviation $\Delta$ expressed in terms of frequency deviation, and through (14) with the average value of $H_1$ (instead of $H_x$) obtained from the measurements of $H_x$ and $H_y$ along the three lines $C$, $D$, $B$, on the axis and $\frac{1}{2}''$ below and above, respectively. The average value of $H_1$ thus determined is 0.74 $H_{xm}$, where $H_{xm}$ is the peak value of $H_x$ at the center.

The over-all agreement between observed points and theoretical curves is quite good, as shown in Figs. 6, 7, and 8. Significant dis-

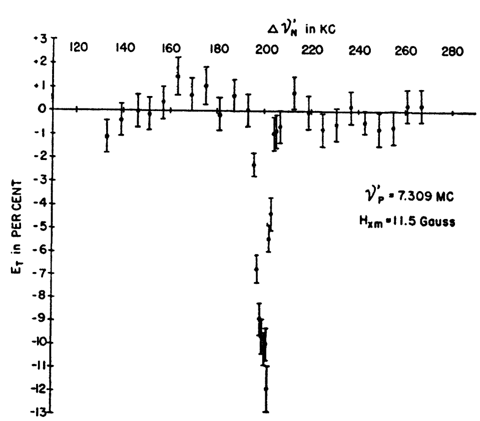


FIG. 11. Exploration of neighborhood of resonances at $\nu_P' = 7.309$ Mc.

Table I.

| $\nu_P'$ Mc | $H_0$ | $H_{xm}$ gauss | $\Delta\nu_N'^*$ kc | $\nu_N^*/\nu_P$ |
|---|---|---|---|---|
| 7.674 | + | 11.5 | 210.39±0.12 | 0.685003±0.000013 |
| | + | 4.6 | 210.62±0.20 | 0.685022±0.000019 |
| 7.309 | + | 23.0 | 199.30±0.40 | 0.684906±0.000037 |
| | + | 11.5 | 200.05±0.20 | 0.684975±0.000020 |
| | + | 4.6 | 200.33±0.14 | 0.685000±0.000015 |
| 6.943 | + | 23.0 | 190.19±0.40 | 0.684993±0.000039 |
| | + | 11.5 | 190.26±0.13 | 0.685000±0.000015 |
| | + | 4.6 | 190.25±0.19 | 0.684999±0.000020 |
| | + | 2.3 | 190.47±0.16 | 0.685020±0.000017 |
| | + | 11.5 | 190.46±0.12 | 0.685019±0.000014 |
| | − | 11.5 | 191.79±0.12 | |

crepancies can be found in the maximum value of $E_T$ at resonance for the lowest field $H_{xm}=2.3$ gauss. This discrepancy is probably caused by the inaccuracy of the radiofrequency field measurements to which this value is particularly sensitive. At the values $H_{xm}=11.5$ gauss the theoretical curves seem to be somewhat too narrow, and at 23.0 gauss somewhat too wide. However, in view of the approximations made in the calculation it is felt that a better agreement could hardly be expected. It is worth while to notice that for the relatively weak radiofrequency fields, i.e., fields for which $E_{T0}$ still deviates considerably from its asymptotic value, the half-width of the resonance curves differs considerably from proportionality with $H_1$. In fact, the half-width of the curves taken with $H_{xm}=4.6$ gauss and $H_{xm}=11.5$ gauss differ by only a factor 1.3. This is understood readily by considering Fig. 10, which shows that in the most important region near resonance and for increasing values of $\Delta$, $F(\xi)$ increases for the lower value of $H_1$ and decreases for the higher value. This results in an increase of the width for the 4.6-gauss curve and a decrease for the 11.5-gauss curve, in very good agreement with the observations.

For the determination of the exact resonance frequency, one could, in principle, find its value by adjusting the abscissa of the theoretical curves to the best fit with the observed points. However, it was felt that in view of the approximations made in the calculation of these curves a more general procedure which would not be based on special assumptions was indicated. Since no evidence for any asymmetry of the observed resonance curves could be found, the resonance frequencies were determined by assuming that the measured points represent a curve symmetric with respect to a resonance frequency. The axis of symmetry was then determined by pairing each of the observed points on the one side with a linearly interpolated point on the other side of the approximate resonance value. The average value of the resonance frequency was obtained by a properly weighted average of the resonance values obtained from each pair. This procedure was carried out by once choosing the interpolated values on the right and once on the left side of the axis of symmetry. In the same manner the mean error was determined. The final values represent the average of the two calculations, which in no case differed by more than their errors. For each measured point the following errors were considered:

(a) Statistical error in $E_T$;

(b) error in frequency measurement of $\Delta\nu_N'$, estimated to be ±0.2 kc;

(c) error caused by differences in the value of $H_0$ for neutrons and protons. This difference was always carefully measured, as stated in Section III, and the results accordingly corrected. The remaining error is equal to the accuracy of the magnetic field measurements and taken to be ±0.2 kc.

(d) error caused by drift of the neutron frequency, the proton frequency, and drift of the operating point of the magnetic field control from exact proton resonance. The difference of the two master oscillator frequencies, equal to $\frac{1}{4}$ and $\frac{1}{6}$ of the neutron and the proton frequency, respectively, was at no time allowed to drift by more than 0.1 kc. The absolute value of the proton frequency could drift or be mis-set by as much as 1 kc; this, however, would only cause a very small error in the ratio of the two frequencies (see Section III). The drift of the operating point of the proton-controlled field stabilizer, manually kept at exact resonance, is estimated to cause an error of ±0.1 kc.

Since all these errors are statistically independent, an over-all error of 0.3 kc was assumed for the values of $\Delta\nu_N'$ for all the measurements, except those at 7.309 Mc, for which the error was increased to 0.4 kc. Table I shows the values of

$\Delta\nu_N'^*$ for the resonance maximum as obtained for the various experiments. In the first column the values of one-sixth of the proton frequency and the direction of the $H_0$ field are listed. The second column contains the value of the radiofrequency field $H_{xm}$ at the center of the coil. In the third column the average value $\Delta\nu_N'^*$ of the axis of symmetry is given with its error. The last column finally shows the value of the ratio of $\nu_N^*/\nu_P$. These last values were obtained by correcting the values of $\Delta\nu_N'^*$ for the frequency shift caused by the asymmetry of the radiofrequency field. The magnitude of this shift, obtained from the last two measurements listed in the table and being equal to one-half the difference of their observed resonance frequencies, was found to be $+(0.67\pm0.09)$ kc for the positive field direction. Comparing this value with Fig. 5 shows that it represents a not unreasonable average value of $d\theta/dt$. It is worth while noticing that the actual spread of the values of $\nu_N^*/\nu_P$ in Table I is in excellent agreement with the errors computed for each value.

The final value of $\nu_N^*/\nu_P$ or of the ratio of the magnetic moments of neutron and proton was obtained by averaging the various values listed in Table I, excluding the last. In order to state the mean error of this final result, proper consideration should be given to systematic errors, such as the error in the frequency shift discussed above. The following other possible errors of this type were considered:

(a) A deviation of the fundamental standard frequency of the frequency meter cannot affect the result if the frequency subdivision to $\frac{1}{10}$ or $\frac{1}{5}$, respectively, is maintained, since only frequency ratios enter into the final value.

(b) An additional error could be introduced by a deviation of the average value of the field $H_0$ over the region in which transitions take place from its value at the center. Since in every case the field configuration was very similar, this error, as being systematic, was not included in the above evaluation, and due allowance has to be made for it in the final result. It is estimated that this error corresponds to a systematic deviation of $\pm0.2$ kc in $\Delta\nu_N'^*$.

(c) A correction of $\Delta\nu_N'^*$ because of the fact that the radiofrequency field is actually an oscillating rather than a rotating, field is too small to be important.[12]

(d) A final possible source of error arises from the fact that the neutrons were observed in air, while the protons were contained in an approximately 0.1-molar solution of $MnSO_4$ in water. The electron of the hydrogen atom, as well as the surrounding other molecules, could cause a modification of the effective resonance field to which the proton is exposed, and this effect would necessitate a correction in the measured ratio of the moments. While the effect caused by the diamagnetism of the water is far too small to be considered, the paramagnetism of the manganese ions would for a spherical sample raise the magnetic induction by about 1/90,000 over its value in air. It is clear, however, that even this small correction would represent a gross over-estimate of the paramagnetic effect, since a proton is on the average isotropically surrounded by manganese ions so that they effect merely a broadening of the resonance line, while its shift cancels out. The only effect which remains is the shielding of the applied field by the electron surrounding the proton, which was treated by Lamb.[17] Neglecting the distortion of the electron eigenfunction as a result of chemical binding, this shielding would cause a relative reduction of the effective field by $\alpha^2/3=1/57,000$ ($\alpha$=fine structure constant) which is negligible within our accuracy and again represents an overestimate because of the partial sharing of the electron with the oxygen of the $H_2O$ molecule. Summarizing, it therefore appears that even with our high accuracy, no correction for the resonance field of the proton has to be applied.

Thus the ratio of the two magnetic moments is found to be

$$\frac{|\mu_N|}{\mu_P}=0.685001\pm0.00003. \quad (41)$$

A previous and considerably less accurate determination by Arnold and Roberts[18] gave for this ratio a value $0.68479\pm0.0004$, which within its error agrees with the above value.

The absolute value of the neutron magnetic moment can be obtained from the value of the

[17] W. E. Lamb, Phys. Rev. **60**, 817 (1941).
[18] W. R. Arnold and A. Roberts, Phys. Rev. **71**, 878 (1947).

proton moment measured by Millman and Kusch[3] in terms of the electronic moment. The value given by these authors is

$$\mu_P=(2.7896\pm0.0008)\mu_n'.$$

The value of $\mu_n'$ deviates from a true nuclear Bohr magneton by a radiative correction, as pointed out by Schwinger.[19] According to his investigation, this correction raises the above value of $\mu_P$ by a factor $(1+\alpha/2\pi)=1.001162$ if referred to a true nuclear magneton $\mu_n=e\hbar/2Mc$, yielding thus the previously given value (1). In units of the present best known value of the nuclear magneton, and by using

$$\frac{\mu_D}{\mu_P}=0.3070126\pm0.000002, \tag{42}$$

as determined by Bloch, Levinthal, and Packard,[20] the values of the moments of the neutron, proton, and deuteron are

$$\begin{aligned}\mu_N&=-1.91307\pm0.0006,\\ \mu_P&=2.7928\pm0.0008,\\ \mu_D&=0.85742\pm0.0003.\end{aligned} \tag{43}$$

The value $|\mu_N|=(1.935\pm0.02)\mu_n$, which was found by Alvarez and Bloch,[8] agrees with the present more accurate determination.

## VI. DISCUSSION

The most outstanding information which has been gained from the investigation of nuclear moments is still contained in the result that the neutron has a finite magnetic moment and that the proton moment differs from the nuclear magneton. It clearly signifies that neither of the two particles is of the same nature as the electron, which has been successfully described by the famous relativistic wave equation of Dirac. The same theory, applied to the neutron and the proton, would give $\mu_N=0$, $\mu_P=\mu_n$, and it seems plausible that the different values, which have been observed for both moments, are of common origin. Using the values (41) and (43) for $|\mu_N|/\mu_P$ and $\mu_P/\mu_n$, respectively, one finds

$$\frac{\mu_P+\mu_N}{\mu_n}=\left(1-\frac{|\mu_N|}{\mu_P}\right)\left(\frac{\mu_P}{\mu_n}\right)=(0.8797\pm0.0003). \tag{44}$$

The fact that this number is close to unity suggests that the mechanism, which provides the magnetic moment of the neutron, adds, in the case of the proton, an approximately equal and opposite excess to the magnetic moment $\mu_n$, which one would expect to find as a direct consequence of relativity. Frohlich, Heitler, and Kemmer[21] have shown that such a mechanism is rather naturally found if one accepts Yukawa's hypothesis of the meson field for the nuclear forces. Both the negative neutron moment and the positive excess moment of the proton are here explained by the magnetic moment of the negative and positive mesons, respectively, which exist with a finite probability within the range of nuclear forces from the heavy particle. The calculations are based upon a weak coupling between the heavy particle and the meson field, and the relatively small difference of the value (44) from unity appears here as an effect resulting from the small but finite ratio of the masses of meson and heavy particles. Pauli and Dancoff[22] have shown, however, that in a theory with strong coupling, the magnetic moments of proton and neutron would appear as equal and opposite so that a vanishing result would be obtained instead of the value (44); the fact that this actual value lies between zero and unity might thus call for a theory with intermediate coupling.

Another interesting feature of the value (44) is found in the fact that it is very closely equal to the magnetic moment of the deuteron, measured in units $\mu_n$. Using here also the value (42), one has

$$\frac{\mu_P+\mu_N-\mu_D}{\mu_n}=\left(1-\frac{|\mu_N|}{\mu_P}-\frac{\mu_D}{\mu_P}\right)\left(\frac{\mu_P}{\mu_n}\right)=0.02230\pm0.00009. \tag{45}$$

A vanishing result instead of this small number would indicate that the magnetic moments of the proton and neutron are additive in the deuteron, and the first approximate data for the neutron moment were actually derived under this assumption. The value (45) represents the measure of a small but well established deviation

[19] J. Schwinger, Phys. Rev. **73**, 416 (1948).
[20] F. Bloch, E. C. Levinthal, and M. E. Packard, Phys. Rev. **72**, 1125 (1947).
[21] H. Frohlich, W. Heitler, and N. Kemmer, Proc. Roy. Soc. **166**, 154 (1938).
[22] W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).

from additivity which can be ascribed to various causes.

A first and in fact very accurate prediction of the non-additivity has been given by Rarita and Schwinger[23] in connection with the quadrupole moment $Q_D$ of the deuteron. Although the observed finite value of $Q_D$ is compatible with the spin $I_D=1$ of the deuteron, it would vanish, if the orbital angular momentum of the proton and the neutron were zero, i.e., if the ground state of the deuteron were a pure $S$ state. Such a vanishing result would not be accidental, but it would necessarily follow if the interaction potential between proton and neutron were spherically symmetrical. Through the assumption of a spin-dependent spherical asymmetry, it is found, however, that the ground state of the deuteron is not a pure $S$ state but rather a mixture of an $S$ and a $D$ state. With the simplest assumptions about the form of the interaction, Rarita and Schwinger were able, from the observed value of $Q_D$, to estimate the relative strength of the spin-dependent part.

Through the orbital contribution to the magnetic moment of the $D$ state, there enters a correction to the total magnetic moment of the deuteron. It is directly related to the probability of finding the deuteron in the $D$ state. With the form of interaction, which leads to the observed value of $Q_D$, this probability was found to be 0.039; introducing the observed values for $\mu_P$ and $\mu_N$ one obtains then from the theory of Rarita and Schwinger $(\mu_P+\mu_N-\mu_D)/\mu_n=0.022$. This excellent agreement with the observed value (45) is, however, largely coincidental; it could easily be spoiled by altering the simple but implausible assumption that the radial dependence of the interaction energy has the form of a square well.

The agreement seems even more accidental if one considers that another major cause of non-additivity has been entirely omitted in the calculations of Schwinger and Rarita. It has been pointed out by several authors[24] that relativistic effects, resulting from the motion of the constituents in the deuteron, could likewise cause deviations from additivity in the same direction and of comparable magnitude as those introduced through the mere presence of the $D$ state. The estimates vary widely, dependent on the different assumptions about the type of interaction between the proton and the neutron. While no definite answer can thus be given at the present state of the theory, it is not justifiable, either, to overlook these relativistic corrections.

Finally, deviations from additivity for the magnetic moments can appear for a third independent reason. If one assumes that the excess moment of the proton and the moment of the neutron are due to the meson field, one is led to the possibility that the modification of this field, which causes their interaction, affects also the resultant magnetic moment of proton and neutron in the deuteron. The existing theories indeed contain features which lead in general to a modification of the intrinsic moments through that of the meson field.[25] In the special case of the deuteron, however, this mechanism contributes no correction by virtue of the particular symmetry properties with respect to an interchange of proton and neutron.

It is evident that the present knowledge of nuclear forces is insufficient to give a satisfactory explanation of the observed value (45). With its considerable accuracy this experimental number may serve as an important and rather severe test for future theories.

## VII. ACKNOWLEDGMENT

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