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Deuteron-Induced Reactions

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A simple semiclassical model is described for the computation of *d-p* and *d-n* cross sections. It is found that the stripping process is responsible for practically the entire observed *d-p* cross section at any bombarding energy E_d. The few available quantitative *d-p* measurements—for Na^{23}, Co^{59}, Cu^{63}, Br^{81}, and Bi^{209}—agree with curves calculated by assuming that only the stripping process is effective. As E_d increases above the barrier height of the target nucleus, the measured *d-p* cross sections diminish; this is interpreted as due to effective *d-n* competition, in which the excited nucleus re-emits a neutron after acquiring one in the *d-p* stripping process. It is expected that the *d-n* excitation curves are also predominantly due to the stripping process for incident energies $E_d \gtrsim 10$ Mev.

To compare observed magnitudes with the calculated values, it is necessary to specify as a nuclear parameter the average "sticking probability" ξ of an elementary particle. Comparison of the *d-p* and *d-n* stripping processes indicates that for a given target the ratio σ_{dp}/σ_{dn} will exceed unity at all energies E_d comparable with the Coulomb barrier and may approach the limit $\sigma_{dp}/\sigma_{dn} \rightarrow \xi_n/\xi_p \approx 1$ as $E_d \rightarrow \infty$.

I. INTRODUCTION

THIS study discusses nuclear reactions induced by deuterons at moderate incident energies, 2–15 Mev. The models used for calculation are of the simplest type, so that the structural details of individual nuclei are neglected, and the computed results should be regarded as of a semiquantitative nature. Nevertheless, fair agreement is obtained with some absolute excitation curves, and it appears possible to draw certain general conclusions that have not previously been emphasized about the mechanism of these reactions and to give a very simple formula that accounts, to first order, for the observed excitation curves. It is found for all but perhaps the lightest elements (i) that the observed *d-p* excitation curve is due almost entirely to the Oppenheimer-Phillips or stripping reaction throughout the entire range of deuteron bombarding energies, and not just at energies well below the barrier height of the nucleus, and (ii) that the major portion of the observed *d-n* cross section is due to an analogous stripping effect with the roles of neutron and proton interchanged. Only a secondary contribution is made by the direct reaction, in which the deuteron is absorbed as a whole and a single particle subsequently emitted; the predominance of the stripping process is greater for *d-p* than for *d-n* reactions, and for both types this predominance increases with the atomic number of the target.

Excitation curves for the *d-p* reaction are computed for those elements of which thorough quantitative measurements have been made: Na^{24}, Co^{59}, Cu^{63}, Br^{81}, and Bi^{209}. These cross sections are obtained experimentally by measuring the specific activity of the residual nucleus, which has experienced a net gain of one neutron in the reaction. If the deuteron is absorbed as a whole with high incident energy, however, the most probable result is the emission of two or more particles, and the simple *d-p* reaction becomes very unlikely. Available statistical theories of nuclear level densities[1] suggest that the two-particle competition to the direct reaction usually becomes important in the region around $E_d > 10$ Mev. But in this energy region the stripping process still allows an appreciable probability that the proton, in escaping, will carry away all of the incident energy or even more, leaving the neutron in a negative energy (bound) state in the final nucleus. The energy of the neutron will be measured from a zero corresponding to the potential energy of infinite separation from the nucleus; therefore, a neutron bound in the nucleus will be called a "negative energy" neutron. The preponderance of the *d-p* stripping process at these energies is estimated more quantitatively in the examples below. Of course, even the stripping reaction yields relatively fewer negative energy neutrons as the incident deuteron energy increases: the neutrons tend to be more often absorbed with positive energy, causing the immediate re-emission of a

[1] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1939).

neutron (an uncertainty of a few kev in the energy limit for re-emission is unimportant for these considerations). Therefore, as the deuteron energy is increased well above the nuclear barrier, the *d-p* cross section will decline because of effective *d-pn* competition in the stripping reaction, which has not been observed.

At low incident energies, the charge asymmetry of the deuteron greatly favors the *d-p* process by stripping, because the proton does not, on the average, have to penetrate so far into the Coulomb barrier as in the direct reaction. This is what is ordinarily known as the Oppenheimer-Phillips reaction.

The *d-n* reactions at high incident energies are also due to stripping in accord with the argument above: only the stripping process, in which the statistical distribution of emergent energies is largely governed by the internal wave function of the deuteron, permits the appreciable absorption of protons in energy states low enough to avoid emission of another particle by the residual nucleus. At low energies, the proton must penetrate the same Coulomb barrier to reach the nuclear surface, regardless of whether the entire deuteron is absorbed or not. Therefore, the stripping process is not tremendously favored, as in the corresponding *d-p* case, but the low binding energy and wide average separation of the deuteron components make it possible even in this instance to ascribe roughly half the observed cross section to a stripping type reaction in which the neutron escapes without encountering the nucleus.

These considerations indicate that for a given target nucleus, the observed *d-p* cross section will generally exceed the *d-n* cross section. For when both reactions are predominantly due to stripping, the Coulomb repulsion of the nucleus will always favor proton stripping relative to neutron stripping; and as a first approximation we may assume other (nuclear) factors to be the same. At high incident energies, the Coulomb field is less effective, so the ratio σ_{dp}/σ_{dn} decreases toward unity as E_d decreases. Observations of this sort have been made on Bi^{209}.[2,3]

In this simplified treatment, the nucleus is introduced merely as a geometrical surface characterized by the phenomenological constant ξ, the "sticking probability" for an elementary particle. Curves computed on this basis are compared with quantitative cross section measurements, and it is found possible to achieve a moderate fit by assigning a reasonable constant value to ξ. Unfortunately, quantitative measurements are now available for only a few elements, but the values of the parameter ξ obtained here lie generally within the range encompassed by other measurements of this quantity. The values of ξ_n derived here for heavy elements seem to be definitely smaller than those for light and medium elements. It might be of interest to extend these investigations of the sticking probability to a more complete list of nuclei than has previously been studied.

II. METHOD OF TREATMENT

The outstanding peculiarity of the deuteron is its relatively low binding energy of about $I=2.18$ Mev and the concomitant wide average separation of its constituents, about

$$1/2\alpha=\hbar/2(MI)^{\frac{1}{2}}=2.2\times10^{-13}\text{ cm}.$$

This wide separation is the dominant feature in deuteron reactions and accounts for the preponderance of the stripping process. For it is statistically rather unlikely that the two particles of the deuteron will arrive simultaneously at the surface of the nucleus; the first particle to arrive may be immediately absorbed by the nucleus, abandoning the second particle which will usually escape without encountering the nucleus, especially if it is the proton under the action of the repulsive Coulomb field.

Previous treatments of the Oppenheimer-Phillips process[4-6] take as a point of departure the usual perturbation formula for the cross section. Unfortunately, the corresponding matrix element includes nuclear wave functions and a nuclear potential for which no explicit expressions are known. The unknown factors may be evaluated approximately in terms of the neutron sticking probability ξ_n, derived from considering the absorption cross section when the target

[2] H. E. Tatel and J. M. Cork, Phys. Rev. **71**, 159 (1947).
[3] J. M. Cork, Phys. Rev. **70**, 563 (1946).
[4] J. R. Oppenheimer and M. Phillips, Phys. Rev. **48**, 500 (1935).
[5] H. A. Bethe, Phys. Rev. **53**, 39 (1938).
[6] G. M. Volkoff, Phys. Rev. **57**, 866 (1940).

nucleus is bombarded with a beam of fast neutrons. Of course, the sticking probability is a quasiclassical notion depending for its validity on the statistical behavior of the nucleus.

In the present study, quasiclassical concepts like ξ_n are used directly in setting up the problem, instead of entering in the evaluation of formal expressions which cannot be computed analytically. The statistical methods for dealing with nuclear behavior have been discussed by several authors;[1, 7, 8] we shall follow the outline of reference 1. Because of the short range and great intensity of nuclear forces, it is possible to consider the nucleus as having a fairly well-defined boundary, i.e., as a classical sphere of radius $R=r_0A^{\frac{1}{3}}$ if A is the mass number of the nucleus. The most popular value at present is about $r_0=1.5\times10^{-13}$ cm, although sizable fluctuations from this mean may be apparent in particular nuclei. With this classical notion of the nucleus as a solid sphere, the absorption process for an incident particle naturally decomposes into two stages: (i) first, the penetration of whatever potentials are effective in the region outside the nuclear surface, and (ii) the probability that after reaching the surface the particle will be bound into the nucleus by energy exchange with the other nuclear components. These two stages are not independent, as the probability of absorption at the surface depends on the slope or phase of the incoming wave function, giving rise to resonance phenomena. However, if the reaction involves an average over many resonance levels—as it certainly will in the case of deuteron bombardment—we may, as a first approximation, consider the phase relations at the surface to be purely random. Then one can simply compute factor (i) from well-known wave equations and assign to factor (ii) a parameter ξ which represents the average sticking probability of the particle at the nuclear surface. The sticking probability and the definite radius R are the two classical attributes used to characterize the nucleus.

The same considerations may be applied if the incident beam consists of deuterons instead of elementary particles. Since we neglect the effect on ξ of the phase of the incident wave function, the absorption of a neutron or proton depends simply on its sticking probability and the probability that, as a member of the incident deuteron, it will reach the nuclear surface; if this latter probability is computed with the restriction that the second particle of the deuteron remain *outside* the nucleus, we can obtain the cross section for a one-particle absorption process like the Oppenheimer-Phillips reaction. Furthermore, because of the random phase relations assumed at the nuclear surface, the absorption process is independent of the condition of the abandoned particle, and *vice versa*. In qualitative terms, the nucleus is simply assumed to "snatch up" the first particle of the deuteron which it encounters, while the second particle is abandoned in whatever state it happens to find itself at the time and generally escapes without striking the nucleus. An advantage of this approach to deuteron-induced reactions is that it seems to be free of embarrassment in application to deuteron energies at or above the Coulomb barrier. If we can find satisfactory wave functions for the region external to the nucleus, the validity of the method proposed should not alter greatly with increased energy—at least until the point where the nucleus begins to break up under the impact, which requires on the order of hundreds of Mev. The argument is equally applicable to *d-p* or *d-n* reactions, so the stripping process is expected to be of importance in both.

Suppose a beam of elementary particles like protons, for example, is incident on the nucleus. At very high energies the effect of the Coulomb barrier is negligible, and the geometrical cross section of the nucleus in the beam will be πR^2. The absorption cross section is then $\xi_p\pi R^2$ where ξ_p is the sticking probability of the protons. We assume that at lower energies the cross section decreases in a manner proportional to the average density $\rho_p(R)$ of the protons at the nuclear surface, so that

$$\sigma_{\text{abs}}=\xi_p\pi R^2\rho_p(R). \qquad (1)$$

If the wave function $\varphi_i(r,\theta)$ represents a plane wave of unit amplitude of protons in the Coulomb field, we average over all angles to get

[7] E. J. Konopinski and H. A. Bethe, Phys. Rev. **54**, 130 (1938).
[8] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).

the mean density at the nuclear surface:

$$\rho_p(R)=1/4\pi\int|\varphi_i(R,\theta)|^2d\omega,$$

where $d\omega$ is the element of solid angle. As the incident energy $E_d\rightarrow\infty$, $\varphi_i\rightarrow\exp[i\mathbf{k}\cdot\mathbf{r}]$ and $\sigma_{\text{abs}}\rightarrow\xi_p\pi R^2$ as mentioned before; and, of course, for a plane wave of neutrons, $\varphi_i=\exp[i\mathbf{k}\cdot\mathbf{r}]$ for all energies and $\rho_n(R)$ is always unity. Notice that no angular momentum restrictions are imposed: at these high energies the nucleus is assumed able to absorb particles of all angular momenta with equal ease.

This form for σ_{abs} at once invites extension to the case where deuterons are the incident particles. For instance, we may wish to compute the absorption cross section for neutrons, while the proton is allowed to escape unscathed: this is the *d-p* stripping cross section. Then we have

$$\sigma_{dp}=\xi_n\pi R^2\rho_n(R), \tag{2}$$

where $\rho_n(R)$ is the average density of neutrons at the surface of the nucleus, supplied by deuterons whose protons are outside the nucleus. Thus, if the wave function for an incident plane wave of deuterons, normalized to unit amplitude at infinity, is $\varphi_i(\mathbf{r}_n,\mathbf{r}_p)$, we have

$$\rho_n(R)=\int_{p>R}dV_p\int(d\omega_n/4\pi)|\varphi_i(\mathbf{R},\mathbf{r}_p)|^2. \tag{3}$$

As in the case of bombardment with elementary particles, we do not analyze φ_i into terms corresponding to angular momentum $l\hbar$, because the nucleus is assumed to assimilate all angular momenta. For this reason it is also not necessary to consider the intrinsic spin moments of the neutron and proton. The tensor forces in the deuteron have been omitted as a refinement incommensurate with the approximations used here.

Although the preceding discussion supposes that the neutron is captured and the proton freed, the reverse case is exactly analogous, in which the proton is absorbed and the neutron escapes. Then we should have

$$\sigma_{\text{reverse}}=\pi R^2\xi_p\rho_p(R),$$

where the density of proton at the nucleus is

$$\rho_p(R)=\int_{r_n>R}dV_n\int(d\omega_p/4\pi)|\varphi_i(\mathbf{r}_n,\mathbf{R})|^2. \tag{4}$$

The density $\rho_p(R)$ is expected to be always less than the corresponding $\rho_n(R)$, as the proton has to penetrate less deeply into the Coulomb field in the latter case.

To evaluate the factor $\rho_n(R)$, we must find an expression for the wave function $\varphi_i(\mathbf{r}_n,\mathbf{r}_p)$. It is usual to introduce the relative and center of gravity coordinates, $\mathbf{s}=\mathbf{r}_p-\mathbf{r}_n$, $\mathbf{S}=\frac{1}{2}(\mathbf{r}_p+\mathbf{r}_n)$; then the wave function $\varphi_i(\mathbf{s},\mathbf{S})$ satisfies the equation

$$[(\hbar^2/4M)\nabla_S{}^2+(\hbar^2/M)\nabla_s{}^2+E-I-V_0(s) -Ze^2/|\mathbf{S}+\tfrac{1}{2}\mathbf{s}|]\varphi_i(\mathbf{s},\mathbf{S})=0, \tag{5}$$

where M is the proton or neutron mass, E is the kinetic energy of the deuteron, $I=2.18$ Mev is its binding energy, and $Ze^2/|\mathbf{S}+\frac{1}{2}\mathbf{s}|=Ze^2/r_p$ is the Coulomb repulsive potential of the nucleus, acting solely on the proton. The potential $V_0(s)$ represents the specifically nuclear forces between the neutron and proton and is taken to be a simple scalar potential dependent on the separation distance alone. We look for approximate solutions of this equation, among which the simplest are the following. The first and most obvious choice is approximation (a): the deuteron center of gravity motion is assumed to be given by the wave function $\varphi_c(\mathbf{S})$ for a plane wave in a Coulomb field, while the internal coordinate is described by the wave function $\chi_0(\mathbf{s})$, the solution for the bound state unperturbed by the Coulomb field,

$$[\hbar^2/M\nabla_s{}^2-I-V_0(s)]\chi_0(\mathbf{s})=0. \tag{6}$$

This implies that the most important factor determining the reaction is the "natural spread" of the deuteron due to its low binding energy, for it completely neglects the effect of the charge asymmetry of the deuteron in the Coulomb field, according to which the proton is less likely to reach the nuclear surface than the neutron. This neglect entails a serious objection, since it means that the *d-p* and *d-n* stripping cross sections would be the same for a given nucleus at all energies. Actually, however, the *d-p* cross section markedly exceeds the *d-n* cross section, especially at low energies.

An attempt to improve the zero-order approximation (a) leads to the so-called adiabatic approximation (b). The wave function corresponding to approximation (a) does not satisfy Eq. (5) but leaves a remainder term proportional to $(Ze^2/S)-(Ze^2/|\mathbf{S}+\frac{1}{2}\mathbf{s}|)$, which represents the Coulomb energy difference between the positions of the proton and center of gravity. This remainder term is introduced as an additional potential energy into Eq. (6) for the internal motion of the deuteron, and the center of gravity wave function remains $\varphi_c(S)$ as before; this amounts to assuming that all the Coulomb energy gained by the proton in moving out from the center of gravity position goes into distorting the internal motion of the duteron. As this would be true only if the center of gravity remained fixed in position, the accuracy of the adiabatic approximation is somewhat problematical. The adiabatic internal function cannot be found in closed form, although the W.K.B. approximation is available for the case in which the neutron and proton are collinear with the nucleus. This solution cannot be handled analytically, however.

In both solutions (a) and (b) the appearance of $\varphi_c(S)$ implies that the internal motion of the deuteron is much more rapid than its passage through the Coulomb field, so that the average center of charge coincides with the center of gravity. This in turn implies a high binding energy and narrow intrinsic spread of the deuteron, which is almost the reverse of the truth. Approximation (b) is further inconsistent by distorting the internal motion so that the centers of charge and of gravity do not coincide, while continuing to treat the external motion as if they did.

A third approximation (c) suggests itself as the antithesis of the previous two: assume that the internal motion of the deuteron is much *slower* than its passage through the Coulomb field, or at least through that part of the field where the neutron has an appreciable probability of striking the nucleus. This is reasonable at high incident energies, and in the extreme limit it describes the deuteron behavior as that of a rigid framework which maintains a fixed magnitude and orientation as it traverses the Coulomb field. The distribution of this magnitude and orientation is specified by χ_0^2. Thus, the Coulomb wave function applies to the proton coordinate, and the incident wave function is $\varphi_i=\varphi_c(\mathbf{r}_p)\chi_0(\mathbf{s})$. The wave function φ_c, however, still refers to a particle of mass $2M$ as long as the deuteron is not dissociated by photoelectric action of the Coulomb field. Such a photoelectric process is of second order relative to the reaction considered here, so that it may be neglected. With this neglect, we have approximation (c) for the incident wave function: $\varphi_i=\varphi_c(\mathbf{r}_p)\chi_0(\mathbf{s})=\varphi_c(\mathbf{r}_p)\chi_0(\mathbf{r}_n-\mathbf{r}_p)$. This approximation has the cardinal advantage of being very tractable analytically.

We may now write an explicit form for $\rho_n(R)$. Substituting approximation (c) into Eq. (3), we have

$$\rho_n(R)=\int_{r_p>R}|\varphi_c(\mathbf{r}_p)|^2(dV_p/4\pi)\times\int d\omega_n|\chi_0(\mathbf{R}-\mathbf{r}_p)|^2. \quad (7)$$

The second integral, over the nuclear surface, can be performed at once by assuming the usual square-well model for the potential $V(s)$. We obtain

$$\int|\chi_0|^2d\omega_n=(\alpha/r_pR)(e^{2\alpha a}/(1+\alpha a))\times\{F[2\alpha(r_p-R)]-E[2\alpha(r_p+R)]\}, \quad (8a)$$

where the assumed radius of the square-well potential is $a=2.8\times10^{-13}$ cm, and

$$\begin{aligned}F(x)=E(x)&=\int_x^\infty\frac{dt}{t}e^{-t} \qquad \text{for } x\geq 2\alpha a,\\ &=E(2\alpha a)+\tfrac{1}{2}e^{-2\alpha a}[\ln(2\alpha a/x)\\ &\quad+Ci(2\alpha a\pi/x)-Ci(\pi)] \quad \text{for } x\leq 2\alpha a.\end{aligned} \quad (9)$$

Here

$$Ci(x)=-\int_x^\infty(\cos t/t)dt,$$

and the numerical factor $e^{2\alpha a}/(1+\alpha a)$ has the value 1.59. For nuclei with $2\alpha R>2$,

$$F[2\alpha(r_p-R)]\gg E[2\alpha(r_p+R)],$$

so we may take

$$\int|\chi_0|^2d\omega_n=(\alpha/r_pR)(e^{2\alpha a}/(1+\alpha a))\times F[2\alpha(r_p-R)]. \quad (8b)$$

Substituting in Eq. (7), with $dV_p/4\pi=r_p^2dr_p$, we

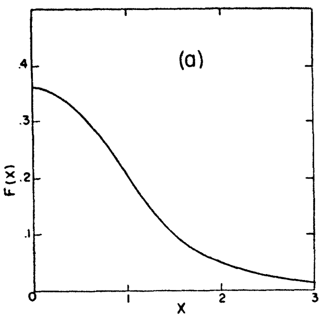

FIG. 1a. The average of the internal function over the nuclear surface,
$$\int |\chi_0|^2(d\omega_n/4\pi) \approx F[2\alpha(r_p-R)].$$

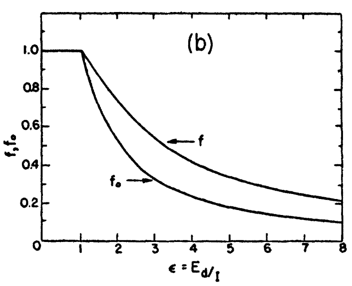

FIG. 1b. The reduction factors due to energy distribution of the stripped particle. The upper curve is $f(E_d)$ for *d-p* stripping; the lower curve f_0 is just $f'(E_d)$ for *d-n* stripping, computed for the case that $E_d'=E_d-Ze^2/R\leq 0$. In case $E_d'>0$, a rough approximation if
$$f'(E_d)=f(E_d)-Ze^2/E_dR[f(E_d)-f_0(E_d)].$$

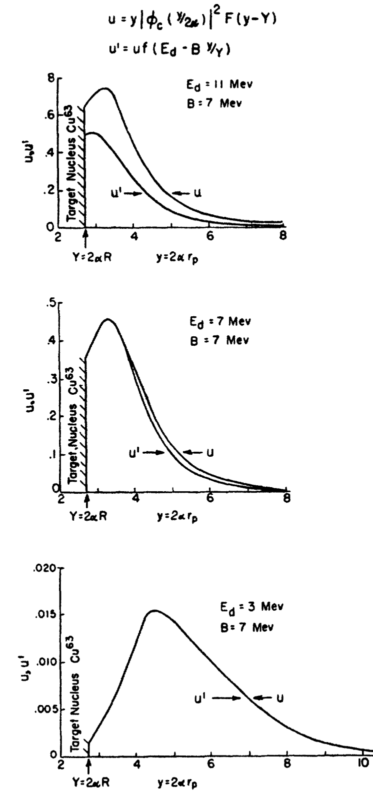

FIG. 1c. Comparison of integrand functions for $\rho_n(R)$ for Cu^{63}, at deuteron energies $E_d=3$, 7, and 11 Mev; the nuclear barrier height is $B=7$ Mev. Function u is without correction for *d-pn* competition as calculated from energy distribution of stripped protons; function u' is reduced by this correction.

obtain finally

$$\rho_n(R)=0.8\int_Y^\infty (ydy/Y)|\varphi_c(y/2\alpha)|^2F(y-Y), \quad (10)$$

where $y=2\alpha r_p$, $Y=2\alpha R$. For the purpose of computation, a curve of $F(x)$ is presented in Fig. 1a. For the factor $|\varphi_c|^2$ representing the deuteron plane wave in the Coulomb field, the expressions in Konopinski and Bethe[7] were used. No angular momentum restrictions were imposed, so that partial waves of all l values were included. In the notation of reference (7),

$$|\varphi_c(r)|^2=l_c^2/l_0^2=\sum_{l=0}^{\infty}(2l+1)P_l(r)/(kr)^2,$$

representing the intensity at a position r of a beam of unit intensity at infinity. The variation of ρ_n with E_d for a particular nucleus is to be obtained by integrating graphically Eq. (10). The expressions used tend to overestimate $|\varphi_c|^2$ at high energies, so that our computed $\rho_n(R)$ will also be somewhat too large for high E_d.

Equation (10) is not complete, for it is necessary to consider the effect of the energy distribution of the liberated protons. Although we have assumed no coherence between the neutron absorption process and the final proton state, conservation of energy requires that if the neutron is to be captured in a negative energy state (and essentially only such neutrons will contribute to the observed *d-p* activity, the rest causing re-emission in a *d-pn* reaction), the proton must escape with an energy E_p for which $E_d-I\leq E_p\leq E_d-I+E_0$. Here E_0 is the maximum binding energy of the neutron in the residual nucleus, taken in these calculations to be $E_0=3.5I=7.6$ Mev. This range represents only a fraction of the total proton spectrum, so that the effective $\rho_n(R)$ must be somewhat reduced from the expression calculated above.

By virtue of the assumption that the absorption process is very rapid and of random phase relative to the deuteron's internal motion, we can, in first approximation, take the proton energy to be specified simply by its membership in the deuteron. For a deuteron in the absence of a Coulomb field, moving with kinetic energy E_d and wave vector $\mathbf{K}$, the usual Fourier inversion shows that the proton has a momentum distribution

$$P(\mathbf{k})d\mathbf{k} \sim k^2 dk d\Omega / [\alpha^2 + |\tfrac{1}{2}\mathbf{K} - \mathbf{k}|^2]^2, \quad (11)$$

where $K^2 = 4ME_d/\hbar^2$, $k^2 = 2ME_p/\hbar^2$, $\alpha^2 = MI/\hbar^2$. Integrating over the solid angle $d\Omega$ and expressing the results in terms of energy,

$$P(E_p)dE_p \sim E_p^{\frac{1}{2}} dE_p / [I + 2E_p - E_d]^2 + 4IE_d. \quad (12)$$

To carry this over to the deuteron in a classically accessible region of the Coulomb field, we assume that an equation of the form (12) applies, except that the zero point of the energy scale is shifted by an amount Ze^2/r_p for E_p and E_d, so that the effective distribution is

$$P(E_p')dE_p' \sim E_p'^{\frac{1}{2}} dE_p' / [I + 2E_p' + E_d']^2 + 4IE_d', \quad (13)$$

with

$$E_p' - E_p = E_d' - E_d = Ze^2/r_p. \quad (14)$$

This is based on the classical idea that the energy Ze^2/r_p is stored by the incident deuteron in the Coulomb field and is regained by the proton as it escapes to an infinite distance, but is therefore not available when the deuteron breaks up at r_p. Then for a given proton position r_p at breakup, the fraction of escaping protons corresponding to neutrons captured in negative energy states is

$$f(E_d') = \frac{\int_{E_d'-I}^{E_d'-I+E_0} P(E_p')dE_p'}{\int_0^{E_d'-I+E_0} P(E_p')dE_p'}. \quad (15)$$

For a given incident energy, E_d' decreases and $f(E_d')$ increases as r_p moves in from infinity; and $f(E_d')$ reaches the value unity at $r_p = r_0$, where $E_d - Ze^2/r_0 = I$. From this point on we take $f \equiv 1$, $r_p \leq r_0$. This means that the proton is assumed always to seize enough energy to escape without having to penetrate through a Coulomb barrier, and this condition requires that all neutrons be left bound in the nucleus if the reaction occurs at $r_p \leq r_0$.

This method of estimating the energy distribution of the released protons is admittedly crude but should impart at least the right direction to our results. The estimate is somewhat biased toward the high energy end of the proton spectrum, because no allowance is made for the possibility that the proton may penetrate out through the Coulomb barrier. This means that we overestimate the percentage of low energy neutrons and hence the observed cross section. The reduction factor $f(E_d)$ is plotted in Fig. 1b.

We have also assumed that all the stripping protons will escape without further encounter, whereas actually a small fraction of them will subsequently reach the nucleus and be absorbed. This effect is equivalent to absorption of the deuteron as a whole, and its inclusion would therefore reduce the computed cross section for. stripping. At incident energies on the order of the target barrier height B, however, this is a second-order effect, although for $E_d \gg B$, it greatly reduces[9] the stripping cross section from about $\pi/2R^2$ to $\pi R/4\alpha$. Furthermore, at low E_d the reduction in d-p stripping appears to be roughly compensated by the d-p contribution from the direct reaction, so that to a first approximation the entire d-p cross section may be computed from Eq. (2) above.

Accordingly, we write

$$\begin{aligned} \sigma_{dp} &= \pi R^2 \xi_n \rho_n(R), \\ \rho_n(R) &= 0.8 \int_Y^\infty dy (y/Y) |\varphi_c(y/2\alpha)|^2 \\ &\quad \times F(y-Y) f(E_d - B(y/Y)), \end{aligned} \quad (16)$$

where $B = Ze^2/R$ is the Coulomb barrier of the target nucleus. The effect of the factor $f(E_d') = f(E_d - BY/y)$ is illustrated in Fig. 1c, where the integrand of ρ_n for Cu^{63} is plotted for several values of E_d. The reduction is negligible for low E_d, and of predominant importance for energies above the barrier height of 7 Mev, where it

[9] R. Serber, Phys. Rev. **72**, 1114 (1947).

accounts for the decline of σ_{dp} with increasing energy. Each curve displays a fairly sharp peak, which persists for E_d well above the barrier height, because the increase of $|\varphi_c(r_p)|^2$ with r_p in the neighborhood of R is still strong for these energies if all values of l are included in $|\varphi_c|^2$. The variation of these peak values with E_d is not a good approximation to the variation of ρ_n because of the variable width of the curves, as illustrated in Fig. 1c.

The present method of treatment has been discussed in terms of the *d-p* reaction, because it has been more extensively studied than the *d-n* reaction, which has generally been tacitly assumed to proceed entirely by direct absorption of the deuteron. Application of the same approach to the *d-n* case, however, indicates that an appreciable or even major fraction of the observed cross section is again due to a stripping process. In an analogous fashion to that above, the *d-n* stripping cross section is

$$\sigma_{dn}=\pi R^2\xi_p\rho_p(R), \tag{17}$$

where ξ_p is the proton-sticking probability. The proton density at the nuclear surface is

$$\begin{aligned}\rho_p(R)&=\int_{r_n>R} dV_n\int(d\omega_p/4\pi)|\varphi_i(\mathbf{r}_n,\mathbf{R})|^2\\&=\int(d\omega_p/4\pi)|\varphi_c(\mathbf{R})|^2\\&\quad\times\int_{r_n>R} dV_n|\chi_0(\mathbf{r}_n-\mathbf{R})|^2 \qquad (18)\\&\approx|\varphi_c(R)|^2[\tfrac{1}{2}+((1-e^{-4\alpha R})/8\alpha R)]\\&=|\varphi_c(R)|^2F_0(R),\end{aligned}$$

where $F_0(R)$ is simply the fraction of deuterons whose neutrons happen to lie outside the nucleus when the proton arrives at the surface.

Some of these neutrons will subsequently strike the nucleus and be absorbed, but at low E_d such absorption will most probably be followed by single neutron emission, so that the *d-n* stripping cross section will not be much reduced by this effect which is neglected in Eq. (18). At very high E_d, however, the *d-n* and *d-p* stripping cross sections are reduced in the same manner.[9] Therefore, although our absolute σ_{dp} and σ_{dn} may not be valid for $E_d \gg B$, the ratio σ_{dp}/σ_{dn} should remain correct to first order for all energies.

This also permits us a crude means of estimating the cross section for the absorption of the entire deuteron: without allowing for the detailed, compound nature of the process, these simple geometrical considerations would indicate that the cross section for formation of a compound nucleus with the addition of the entire deuteron is, for E_d comparable with B,

$$\sigma_d=\pi R^2\xi_p|\varphi_c(R)|^2[1-F_0(R)], \tag{19}$$

corresponding to the condition that the neutron is *inside* the nucleus when the proton arrives at the surface and is also absorbed. Since $F_0(R) > 1-F_0(R)$ for finite R, it is expected that even for the *d-n* reaction the stripping process will roughly equal the direct absorption, although the ratio of compound nucleus formation by the two methods is only

$$\begin{aligned}F_0(R)/1-F_0(R)&\approx(1+1/4\alpha R)/(1-1/4\alpha R)\\&\approx 1+1/2\alpha R=1.2\text{–}1.5,\end{aligned}$$

so that the disparity is of a much lower order than in the *d-p* reactions.

For high deuteron energies (above 10 Mev) the observed *d-n* activity is almost entirely due to stripping because the compound nucleus which has absorbed the whole deuteron is so highly excited that single particle emission is much less probable than the stripping process in which a proton is retained. In the *d-n* stripping reaction, we must also apply a reduction factor $f'(E_d)$ to account for the fact that the observed cross section is produced only by protons that are absorbed into the nucleus with energies less than E_0 above the ground state, for otherwise, a neutron is almost certain to be emitted, and a *d-2n* reaction will be observed instead. In this case, however, the escaping particle is not subject to the Coulomb field. Therefore, to specify the neutron energy distribution $P'(E_n)$, we replace E_p by E_n in Eq. (12) and E_d by $E_d'=E_d-Ze^2/r_p=E_d-Ze^2/R$ (or by zero if $E_d\leq Ze^2/R$, the nuclear Coulomb barrier). The reduction factor $f'(E_d)$ is then determined from $P'(E_n)$ in the same manner as $f(E_d)$ from $P(E_p)$. In Fig. 1b, the curve designated as $f_0(E_d)$ is the factor $f'(E_d)$ computed for $E_d'\leq 0$, the case most frequently encountered for heavy targets. If $E_d'>0$, one may take as a crude approximation

$$f'(E_d)\approx f_0(E_d)+(E_d'/E_d)[f(E_d)-f_0(E_d)]$$
$$=f(E_d)-Ze^2/E_dR[f(E_d)-f_0(E_d)].$$

The total *d-n* cross section to be observed from stripping is then

$$\sigma_{dn}=\pi R^2\xi_p|\varphi_c(R)|^2F_0(R)f'(E_d). \qquad (20)$$

For incident deuteron energies well below the barrier height, the dominant factor in expressions 19 and 20 is $|\varphi_c(R)|^2$, and hence the decrease in the observed σ_{dn} with decreasing E_d is sharper than for the corresponding σ_{dp}. This also means that a quantitative fit with experimental cross sections at $E_d\lesssim B$ is more sensitive to the choice of R for *d-n* than for *d-p* reactions, so that Eq. (20) should be used to fit experimental curves only over a wide energy range. As in the *d-p* reaction, the *d-n* curve will show a maximum in the neighborhood of the barrier height. As E_d increases beyond this point, the contribution of the direct *d-n* reaction is generally negligible; while the decreasing factor $f'(E_d)$ and the increasing factor $|\varphi_c(R)|^2$ of Eq. (20) are more equally balanced than in the corresponding *d-p* formula. Therefore, the observed *d-n* curves should decline more slowly than the *d-p* for energies above their peaks. It is also clear that the ratio σ_{dp}/σ_{dn} should, in general, exceed unity; for example, compare the expressions (16) and (20) for σ_{dp} and σ_{dn} from stripping at $E_d\approx B$:

$$\sigma_{dp}/\sigma_{dn}=\xi_n/\xi_p\cdot\rho_n/\rho_p. \qquad (21)$$

The terms ρ_n and ρ_P are computed in the same way, except that where $|\varphi_c(r)|^2f(E_d')$, $r\geq R$ occurs in the integrand for ρ_n, it is replaced by $|\varphi_c(R)|^2f'(E_d)$ in the integrand for ρ_p. Since

$$f'(E_d)<f(E_d)\leq f(E-Ze^2/r),$$

and $|\varphi_c(r)|^2>|\varphi_c(R)|^2$ for $r>R$, and ξ_p and ξ_n should generally be of the same order of magnitude, we have

$$\sigma_{dp}/\sigma_{dn}\gg\xi_n/\xi_p\approx 1 \qquad (22)$$

for incident deuteron energies comparable with the nuclear barrier height. At low deuteron energies, the direct *d-n* reaction will also contribute to σ_{dn}; but this will no more than double the σ_{dn} computed from stripping alone, so that the inequality (22) still should hold. At very high energies, $|\varphi_c(r)|^2\approx|\varphi_c(R)|^2\approx 1$, and $f'(E_d)\approx f(E_d-Ze^2/r)\to 0$, so that $\rho_p\to\rho_n\to 0$, and $\sigma_{dp}/\sigma_{dn}\to\xi_n/\xi_p$. This suggests the only case in which the observed σ_{dn} might exceed σ_{dp}: if for some nucleus ξ_p happens to exceed ξ_n, then for high energies (E_d several times the barrier height), we may find $\sigma_{dp}/\sigma_{dn}\lesssim 1$. Observations at these energies have not yet been reported even qualitatively, although a study of the σ_{dp}/σ_{dn} ratio in Bi^{209} up to $E_d=18$ Mev. (12) shows that $\sigma_{dp}/\sigma_{dn}>1$ in this energy range.

Of course many-particle reactions induced by deuteron bombardment, such as the *d*-2*n*, *d*-3*n*, *d*-(*p*,α), *d*-(*p*,2*n*), may also be separated into two components, one due to the formation of the original compound nucleus by a stripping process, the other due to the formation of a different compound nucleus by the absorption of the deuteron as a whole. In these complex reactions, however, the chief determining factor appears to be the level density of the compound nucleus and its behavior in emitting several particles in cascade. Therefore, the method of formation of the compound nucleus is of secondary importance, and an analysis to distinguish between the two modes of formation is scarcely possible with the present limited knowledge of level densities.

III. APPLICATION

Quantitative cross section measurements for deuteron-induced reactions have been made up to the present time only on a limited number of elements, and there mostly for the *d-p* reaction. We compare the formulas obtained above with these measurements.

Figure 2 shows the case of $_{29}Cu^{63}$, using an

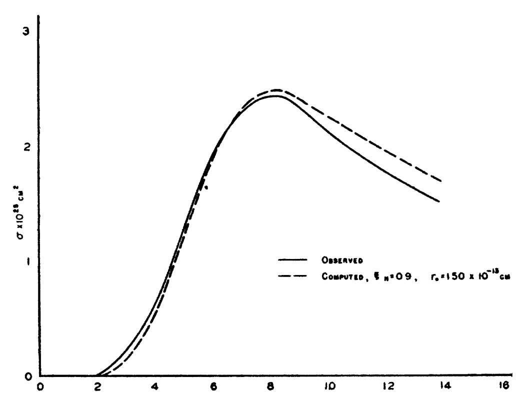

FIG. 2. *d-p* excitation curves for Cu^{63}.

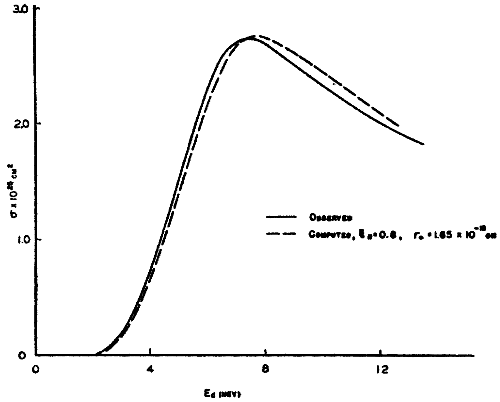

FIG. 3. *d-p* excitation curves for Co^{59}.

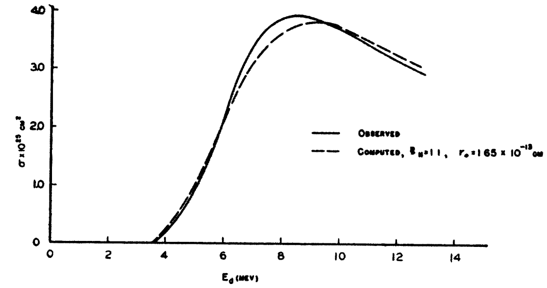

FIG. 4. *d-p* excitation curves for Br^{83}.

unpublished curve measured by Clarke and Irvine. The computed curve was obtained from Eq. (16), using $r_0 = 1.50 \times 10^{-13}$ cm, so that the nuclear radius was $R = 6.0 \times 10^{-13}$ cm, and the effective barrier height (corrected by a factor $A+2/A$ for the center-of-gravity motion of the system of deuteron plus nucleus) is (Ze^2/R) eff $= 7.0$ Mev. The general features of this curve are typical of all *d-p* excitation functions over wide enough energy ranges: at energies below the barrier, it rises toward a fairly well-defined peak, above which it declines again rather slowly. The rising portion of the curve is relatively flat compared with the excitation curve expected in case the entire deuteron were absorbed by the nucleus; this is due, of course, to the effect of the deuteron's wide natural spread. It is to be noted that the peak of the curve generally occurs at incident energies a Mev or two above the Coulomb barrier, and not at the barrier height itself. This arises from a combination of two factors. Since the expression for $|\varphi_c|^2$ includes the partial waves for all angular momenta, the value of $|\varphi_c|^2$ will continue to increase fairly rapidly with increasing deuteron energy, even above the barrier height. On the other hand, the reduction factor $f(E_d')$, determined by the energy distribution of the escaping protons, begins to become effective only for energies above the Coulomb barrier, and tends to reduce the cross section more and more as the deuteron energy increases. The opposing tendencies of $|\varphi_c|^2$ and $f(E_d')$ therefore result in a maximum that occurs somewhat above the barrier energy; beyond this point, the factor $f(E_d')$ is predominant, and the excitation curve declines. This decline may be thought of as due to competition with the unobservable *d-pn* reaction that follows when the stripping neutron is absorbed with positive energy. An effort has been made to explain this decline at high energies by assuming limitations on the angular momentum[10] so that only the first few partial waves contribute to $|\varphi_c|^2$, but this seems a rather artificial restriction in contrast to the present interpretation.

The computed curve in Fig. 2 is fitted to the observations by assuming a sticking probability of $\xi_n = 0.9$. It is at once evident that the agreement is close enough to suggest that practically the entire observed *d-p* cross section is due to this stripping process, and that the contribution of the direct *d-p* reaction may be neglected in first approximation. Indeed, upon examination this direct contribution proves to be very small for all energies of the incident deuteron. This is essentially because at low energies the deuteron has difficulty penetrating as a whole through the Coulomb barrier, and, more important, the proton is inhibited from escaping from the compound nucleus because of the same Coulomb barrier. This unfavorable ratio for *p* emission relative to *n* emission persists up to incident deuteron energies above the barrier height, beyond which appreciable competition begins to occur from the *d-pn* reaction. Thus, the direct *d-p* reaction hardly appears at all, because of strong *d-n* competition at low energies, and (*d*-2 particle) processes at higher energies.

The smallness of the *d-p* reaction may be estimated in a quantitative fashion by using the results of Weisskopf and Ewing[1] on the evaporation model of the nucleus. Equation (19) gives

[10] E. T. Clarke and J. W. Irvine, Jr., Phys. Rev. 66, 231 (1944).

the cross section σ_d for the formation of a compound nucleus by absorption of the entire deuteron; the corresponding *d-p* cross section is then

$$\sigma_{dp}=\sigma_d(\Gamma_p/\Gamma)(\epsilon), \tag{21}$$

where Γ_p and Γ are the proton and total emission widths and ϵ is the maximum kinetic energy with which the proton may be emitted. If the energy of the incident deuteron is E_d and the neutron binding energy is $E_0=8$ Mev, we have

$$\epsilon=E_d+E_0-I=E_d+6 \text{ Mev}.$$

For $E_d=9$ Mev, $\epsilon=15$ Mev, and from Fig. 2 of reference 1 we find $\Gamma_p/\Gamma(15 \text{ Mev})=0.05$; and at this energy $|\varphi_c(R)|^2=0.5$. Also, for Cu^{63}, $1-F_0(R)=0.4$, and we take $\xi_p=\xi_n=0.9$ as already determined. Then the direct *d-p* cross section for Cu^{63} at 9 Mev is, according to Eqs. (19) and (21),

$$\sigma_{dp}=\pi R^2\xi_p|\varphi_c(R)|^2(\Gamma_p/\Gamma)[1-F_0(R)] =0.1\times10^{-25} \text{ cm}^2.$$

This is less than five percent of the observed cross section at this energy, and the percentage will decrease at lower energies as Γ_p/Γ becomes less favorable and $|\varphi_c(R)|^2$ diminishes. At a high energy like $E_d=15$ Mev, on the other hand, we must be concerned with competition from the *d-pn* reaction. The relative probability of single proton emission is, according to the statistical formula, approximately

$$f_p=(1+(\Delta\epsilon'/T_B'))\exp[-\Delta\epsilon'/T_B'], \tag{22}$$

with

$$\Delta\epsilon'=\epsilon-V-E_0=(E_d+6)-V-8=E_d-9$$

with $V=7$ Mev, the Coulomb barrier of the nucleus. $T_B'=T_B(\epsilon-V)=2[5(E_d-1)/A_{\text{Cu}}]^{\frac{1}{2}}$. For Cu^{63} at $E_d=15$ Mev, we have $\Delta\epsilon'=6$, $T_B'=2.1$, so $f_p=0.2$. The other factors in σ_{dp} remain the same, except for $|\varphi_c(R)|^2$, which has increased to 0.8, and Γ_p/Γ (21 Mev), which may be estimated as about 0.1. Then the direct *d-p* cross section is

$$\sigma_{dp}=\pi R^2\xi_p|\varphi_c(R)|^2[1-F_0(R)](\Gamma_p/\Gamma)f_p =0.07\times10^{-25} \text{ cm}^2,$$

again about five percent of the observed cross section. This percentage will now decrease with increasing energy, as the direct reaction cross section will be dominated by the $\exp[-\Delta\epsilon'/T_B']$ factor for single proton emission.

In Figs. 3 and 4 are shown similar comparisons of observed and computed curves for $_{27}Co^{59}$ [11] and $_{35}Br^{81}$.[10] In each case the computed curves show only the cross section expected from stripping; the values of ξ_n used to fit the curves were $\xi_n=0.8$ for cobalt and $\xi_n=1.1$ for bromine. These cases show no distinctive features, except that for both elements it was necessary to assume $r_0=1.65\times10^{-13}$ cm to fit the experimental curves. The computed curves are not particularly sensitive to variations in r_0 because of the smoothing effect of the integrations involved but, as in these cases, it should be possible to assign a value for r_0 to within ten percent from comparison with experiment.

The *d-p* reaction has been studied quantitatively[11] for one light element, $_{11}Na^{23}$. Although the statistical attitude underlying our formulation may not be valid in this case, it is of interest to see how closely the curve computed on this basis may resemble the observations. In Fig. 5 are shown the excitation curves: the computed curve involves only the *d-p* stripping process with an assumed $r_0=1.50\times10^{-13}$ cm, and a value of $\xi_n=1.8$. The agreement in shape of the curves is quite good, except at very low energies, but agreement in magnitude is achieved only by taking $\xi_n=1.8$, whereas from its definition we should expect $\xi_n\leq1$. Of course, some irregularity in the effective ξ values is expected for light nuclei, where the conditions for definition of the sticking probability as an independent nuclear

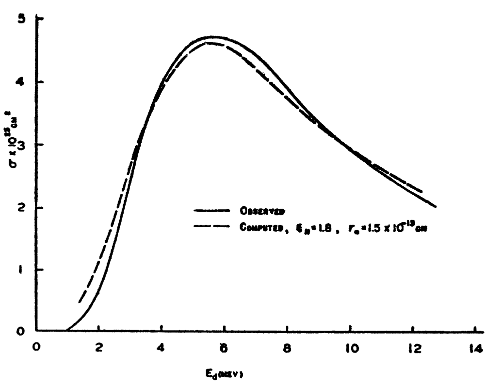

FIG. 5. *d-p* excitation curves for Na^{24}.

[11] E. T. Clarke and J. W. Irvine, Jr., unpublished.

parameter are least likely to be satisfied. A possible explanation might be that for such a light nucleus the *d-p* reaction by direct absorption would make a strong contribution. In the particular case of Na^{23}, however, it appears that the direct reaction is again negligible. It happens that the binding energy of a proton in the Mg^{24} nucleus is about 3.7 Mev greater than the binding energy of a neutron in the Na^{24} nucleus; in addition, the proton must overcome a 4-Mev Coulomb barrier in escaping. Thus, in an excited Mg^{25} nucleus there is effectively some 7.7 Mev more energy available for the emission of a neutron than of a proton, so that the direct *d-n* reaction is expected to overwhelm the *d-p* until incident deuteron energies are reached at which two-particle emission becomes prominent. For a very rough estimate of the σ_{dp}/σ_{dn} ratio we may use the statistical model as employed in reference 1; this shows that σ_{dp} (direct) $<0.05\times\sigma_{dp}$ (stripping) at $E_d=6$ Mev, the peak of the observed curve. Thus the stripping mechanism is expected to account for practically all the observed *d-p* cross section on Na^{23}. The sticking probability in this case might be reduced to approximately unity by taking $r_0\approx 1.8\times 10^{-13}$ cm; this would shift the peak of the computed curve to around 0.5 Mev lower energy.

The only heavy element for which a thorough quantitative study has been made on deuteron-induced reactions is Bi^{209}. Figure 6 shows the computed curves for both the *d-p* and *d-n* cross sections, as compared with a recent set of observations reported by E. Segrè.[12] For both

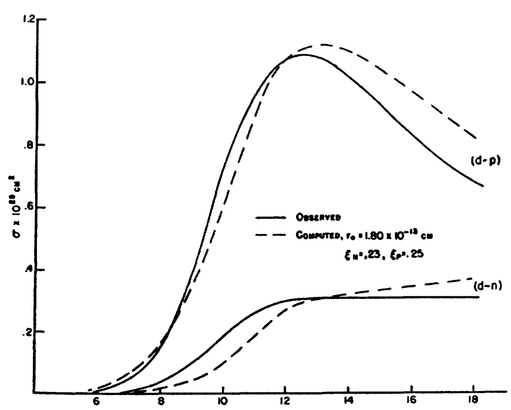

FIG. 6. *d-p* and *d-n* excitation curves for Bi^{209}.

[12] E. Segrè, unpublished.

TABLE I. Sticking probabilities.

Element	ξ_n	Element	ξ_n
Pt^{196}	0.2	Tl^{205}	0.5
Pt^{198}	0.4	Pb^{208}	0.3
Au^{197}	0.3	Bi^{209}	0.1
Hg^{196}	1.7	Th^{232}	0.2
Hg^{204}	0.3	U^{238}	0.2

computed curves a standard radius of $r_0=1.80\times 10^{-13}$ cm was assumed, leading to a nuclear radius of $R=10.7\times 10^{-13}$ cm. The values of the sticking probabilities were taken as $\xi_n=0.23$ and $\xi_p=0.25$ for the σ_{dp} and σ_{dn} curves, respectively. In computing the σ_{dn} curve with Eq. 20, the additional assumption was made that the threshold for a $Bi^{209}(p, n)Po^{209}$ reaction is some 2–3 Mev, in accordance with the presumed great instability of the Po^{209} nucleus. Only the component from the stripping process was considered in computing the σ_{dn} as well as the σ_{dp} curve; the direct *d-n* reaction is expected to encounter not only *d-2n* competition at energies around $E_d=10$ Mev, but strong *d-α* competition at all observable energies, as the compound Po^{211} is a very likely α-emitter.

The *d-p* and *d-n* reactions on Bi^{209} have previously been studied[3] at energies up to 14 Mev in an effort to determine the point at which the ratio σ_{dp}/σ_{dn} fell below unity, in accordance with the view that the Oppenheimer-Phillips supremacy of the *d-p* reaction would vanish as E_d approached the nuclear Coulomb barrier. This expectation is not fulfilled experimentally, and the present treatment indicates that since for Bi^{209} $\xi_p\approx\xi_n$ the observed ratio σ_{dp}/σ_{dn} will exceed or equal unity for all deuteron energies. This ratio approaches ξ_n/ξ_p as $E_d\rightarrow\infty$, so that the σ_{dp} and σ_{dn} curves will cross only at high E_d and only in the fortuitous (for this simple theory) circumstance that $\xi_n/\xi_p<1$.

This exhausts the list of elements for which quantitative excitation curves are available over a wide range of energies. Another set of measurements has been published[13,14] on quantitative cross sections for a series of heavy elements at the relatively low energies of $E_d=6$–9 Mev. The

[13] R. S. Krishnan and E. A. Nahum, Proc. Roy. Soc. **A180**, 321 (1942).

[14] R. S. Krishnan and E. A. Nahum, Proc. Roy. Soc. **A180**, 332 (1942).

shapes of the curves over this narrow region are not particularly informative, but in Fig. 7 are shown a couple of typical cases, the *d-p* excitation functions for Au^{197} and Th^{232}, representing examples from each end of the range of elements studied. For all calculations on these elements, a standard radius of $r_0 = 1.50 \times 10^{-13}$ cm was assumed, and the stripping reaction only was considered. The observed and computed curves for all intervening elements lie between these respective limiting cases, except for Bi^{209}, where these authors report an exceptionally steep curve, markedly steeper than that for any other element. Just for Bi^{209}, however, there exists an independent set of measurements over the same energy range,[2] which by contrast show a normal excitation curve with slope similar to that of the computed functions.

The quantitative values given in reference 14 may be used to calculate the corresponding sticking probabilities for the elements concerned; these are shown in Table I.

The corresponding ξ_p values were not computed from the few cross sections given for σ_{dn}, because at $E_d = 9$ Mev the results are extremely sensitive to the choice of nuclear radius, and it is not certain that the contribution from the direct reaction is negligible. A reasonable value of ξ_p could be obtained only from fitting a quantitative curve over an energy range exceeding the barrier height of the nucleus, as in the case of Bi^{209}.

The ξ_n values were all fitted to the experimental cross sections reported at 9 Mev, using Eq. (16) for the cross sections due to the stripping process alone. The direct *d-p* reaction can be neglected as usual. These values for ξ_n are not so reliable as those obtained from fitting the over-all curve, as in the case of the lighter elements, because the computed cross sections at 9 Mev—below the barrier height in all cases—are rather sensitive to the value selected for the nuclear radius. There remains, however, the suggestion that the sticking probabilities for the heavy elements are consistently lower than those of the light and medium nuclei, where $\xi_n \approx 1$. In this connection, it would be of interest to study the reactions of Table I quantitatively over a wide energy range in order to fix the values of ξ_n with more certainty, and to extend the measurements to elements in the range $83 < A < 196$.

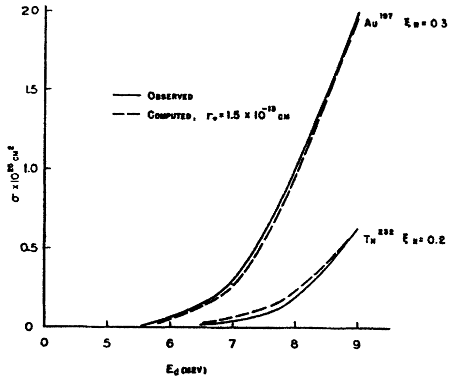

FIG. 7. *d-p* excitation curves for Au^{197} and Th^{232}.

The values of r_0 used to fit the over-all curves in Figs. 2–6 tend to be larger than the usually assumed value of 1.5×10^{-13} cm. This might be ameliorated by assuming that for *d-p* stripping to occur the proton must be outside the range of the *n-p* forces in the deuteron, as well as outside the nuclear surface. The actual radius of the nucleus would then be somewhat smaller than the effective radius appearing in the calculation.

ACKNOWLEDGMENTS

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