

required to account for the observed quadrupole moment of the deuteron.

The large moments which are observed for H^3 and He^3 could be accounted for *without* exchange provided the extreme assumption is made that the admixture for the ground state is ${}^2P^2=0.383$ and ${}^4P^2=0.617$, while ${}^2S^2={}^4D^2=0$. These values were calculated using the formula for the moment of H^3 given by Sachs.⁸ This would mean that the ground state of H^3 and He^3 is essentially a P state rather than an S state. This seems extremely improbable on almost any assumption as to form of the nuclear forces, as has already been pointed out by Mayer and Sachs.⁹

It appears much more reasonable to accept Villars¹⁰ explanation that the excess magnetic moment is caused by exchange currents and to accept the Gerjuoy-Schwinger admixture. If this is done, one finds for the part of the magnetic moment due to the exchange currents approximately 0.27 nuclear magneton, positive for H^3 and negative for He^3 .

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The Conductivity of Metals at Microwave Frequencies

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February 9, 1948

A NOTE by B. Serin under the above title has recently appeared in The Physical Review.¹ The formulation given by him is, however, not in accordance with the physical nature of the problem and, under certain conditions, the solution which he obtains is mathematically unsound. The problem of metallic conductivity at microwave frequencies is much more complicated than is suggested by Serin's account; a more exact formulation has already been given by one of us,² and the full solution is in course of publication.^{3,4}

Serin assumes at the outset that the electric field in the metal must take the form of a damped simple harmonic wave. This is clearly correct for an unbounded metal provided that wave propagation is possible at all; we shall show, however, that no wave propagation in the ordinary sense is, in fact, possible when the mean free path of the electrons exceeds a certain critical value.

Following Serin (and using his notation), we write the Boltzmann equation for the distribution function of the

electrons:

$$\frac{\partial f}{\partial t} - \frac{e}{\hbar} E(z) e^{2\pi i v t} \frac{\partial f}{\partial k_x} + v_x \frac{\partial f}{\partial z} = -\frac{f-f_0}{\tau}. \quad (1)$$

In order to simplify the discussion, we shall assume that $v\tau \ll 1$, so that the term $\partial f/\partial t$ may be neglected. Equation (1) is to be solved for an arbitrary field distribution $E(z)$, taking into account the boundary conditions appropriate to the problem under consideration. For an unbounded metal, the correct solution is easily shown to be

$$f = f_0 - \frac{e}{\hbar v_x} \frac{\partial f_0}{\partial k_x} \exp\left(-\frac{z}{\tau v_x}\right) \int_z^\infty E(z') \exp\left(\frac{z'}{\tau v_x}\right) dz', \quad (v_x < 0), \quad (2)$$

$$f = f_0 + \frac{e}{\hbar v_x} \frac{\partial f_0}{\partial k_x} \exp\left(-\frac{z}{\tau v_x}\right) \int_{-\infty}^z E(z') \exp\left(\frac{z'}{\tau v_x}\right) dz', \quad (v_x > 0).$$

$E(z)$, of course, must be such that the integrals converge; the physical interpretation of this obvious mathematical requirement is discussed below. The expressions (2) may now be used to calculate the current density as a function of $E(z)$, and, on combining the resulting expression with Maxwell's equations, the following equation for $E(z)$ is obtained:

$$\frac{d^2 E(z)}{dz^2} = \frac{3i}{2l\delta^2} \int_{-\infty}^\infty k\left(\frac{z-z'}{l}\right) E(z') dz', \quad (3)$$

where $l = \tau v =$ mean free path, $\delta =$ classical skin depth, and

$$k(u) = \int_1^\infty \left(\frac{1}{y} - \frac{1}{y^3}\right) e^{-y|u|} dy.$$

Equation (3) belongs to a well-known type of integral equation, and it may be shown⁵ that the only solutions are of the form

$$E(z) = \sum_j A_j \exp\left(-\frac{s_j z}{l}\right), \quad (4)$$

where the s_j 's are the solutions of the equation

$$s^2 = \frac{3il^2}{2\delta^2} \int_{-\infty}^\infty k(u) e^{-su} du = i\alpha K(s) \quad (\text{say}) \quad (5)$$

in the region in which the integral converges.

Examination⁴ of the function $K(s)$ reveals that it is regular in the strip $|\Re s| < 1$, and that Eq. (5) has two roots $\pm s_1$ in this strip if $\alpha (= \frac{3}{2} l^2 / \delta^2)$ is less than $\alpha_0 (\approx 2.63)$, i.e., if l is less than 1.32δ , but none otherwise. For $\alpha < \alpha_0$, then, an exponential solution exists, and on substituting

$$E(z) = E_0 \exp\left(-\frac{s_1 z}{l}\right) = E_0 \exp\left(-\frac{2\pi i v M z}{c}\right) \quad (\text{say})$$

in Eq. (2), Serin's result is obtained.

The absence of solutions for $\alpha > \alpha_0$ may be interpreted physically as follows. The exponential wave given by Serin's solution under these conditions is attenuated by a factor e in a distance smaller than the mean free path of the electrons. In this case the current at any point may be contributed by electrons which, though few in number, have come without collision from a region of the wave where the electric field is very great; indeed, the further the electrons have traveled in the wave direction the greater will be their total contribution to the current,

although their number will be diminishing. This physical picture is confirmed by substituting Serin's solution in (2) when it is found that the integrals diverge for $\alpha > \alpha_0$. This point was not noticed by Serin, since, in assuming at the outset that the field was exponential, he, in effect, wrote down a finite particular solution of the Boltzmann equation and neglected infinite terms in the general solution (2).

It is therefore clear that if any solution is to be found for $\alpha > \alpha_0$, the surface of the metal must be specifically taken into account in the calculation. If this is done, solutions may be found for all values of α . There is thus a fundamental physical difference in the nature of the electric field for values of α less or greater than α_0 . In the former case the field in the metal, at sufficient distances from the surface, is unaffected by the presence of the surface, which simply acts as a means for exciting a damped traveling wave. In the latter case, however, the form of the excitation is conditioned at all points by the presence of the surface and never approximates to an exponential form, being, in fact, a true surface excitation. For further details we refer to the work already quoted.²⁻⁴

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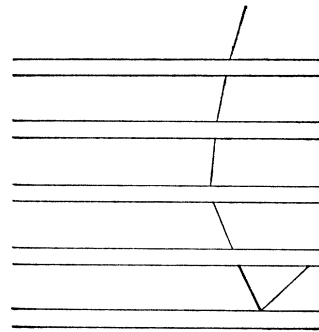
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Mesotron Decay*

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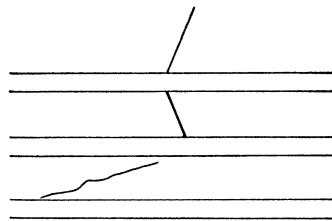
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March 2, 1948

THE cloud-chamber pictures obtained in the experimental study of the mass of the mesotron¹ have been examined for evidence of charged decay particles. The radius of curvature of the track of the mesotron was measured in one cloud chamber placed in a magnetic field of 4750 gauss. The range in lead for the same mesotron was measured in a cloud chamber placed immediately below the first. It contained 15 horizontal lead plates 0.27 inch thick, both surfaces of which were covered with 0.001-inch aluminum foil. Sixteen positive mesotrons appear to have stopped in the lower chamber. In five of these cases a lightly ionized track is seen to leave the plate in which the mesotron stopped. Of the 27 negative mesotrons which appear to stop in the lower chamber, only one is accompanied by a possible decay track. In two cases, two particles are seen leaving the plate in which the mesotron stopped. It is possible that the decay electron initiated a cascade of which two particles are able to get out of the lead plate. Two of these events are illustrated by drawings made from measurements on the pictures. (See Figs. 1 and 2.) The width of the line represents roughly the density of ionization of the track. In each case the increase in the density of ionization of the mesotron track can be seen and the track of the decay particle is near minimum ionization. In the stereoscopic view the mesotron track and the track of the decay particle seem to be associated



PICTURE NUMBER 7049

FIG. 1.



PICTURE NUMBER 7811

FIG. 2.

in space and time for all cases. In several pictures a delay in the reduction of the clearing field voltage has resulted in a separation of the track and the time association was checked by measurement. For the negative mesotron the decay particle comes out of the bottom of the lead plate in which the mesotron stops, and the space association of the tracks may be doubtful. The two tracks are of the same age. It seems very unlikely that the mesotron could have stopped and decayed in the thin aluminum foil. The scattering and density of the track do not suggest that the mesotron stopped in the top foil. If it stopped in the lower foil, one would expect the decay electron to have an energy of approximately 50 Mev. The scattering of the electron suggests that its energy is very much less than 5 Mev.

The data on these events are summarized in Table I.

Using curves given by Rossi and Greisen² one can make a rough estimate of the range of electrons in lead. Using this range with the assumption that the mesotron might have stopped anywhere in the lead plate and that the decay electrons are emitted with spherical symmetry, one can calculate the probability that a decay electron will get out of the lead plate and produce a track in the cloud chamber. For 50-Mev electrons the probability calculated in this way for the experimental arrangement used here is 0.52; for 25-Mev electrons the probability is 0.16. The experimental value found here for those cases which are identified as positive mesotrons is 0.31. Decay electrons which barely get out of the lead plate or which stay close to the plate, especially in a backward or forward direction,