The existence of a solution $R(r)$ of the type in question will be concluded from the following theorem:[6] Suppose that $f(x)$ is any function possessing at every positive x derivatives of arbitrarily high order, and that the latter satisfy the conditions

$$(-1)^{n+1}d^nf(x)/dx^n \geqq 0 \quad \text{for} \quad n=0,1,2,\cdots; \quad 0<x<\infty. \quad (7)$$

Then the differential equation,

$$y''+f(x)y=0, \quad (8)$$

has a positive solution which is representable in the form

$$y(x)=\int_0^\infty e^{-xq}d\phi(q), \quad 0<x<\infty, \quad (9)$$

where $\phi(q)$ is a certain non-decreasing function which (since only $d\phi$ occurs) can be normalized by $\phi(0)=0$.

In order to apply this general theorem to the wave equation, put

$$f(r)=W_0-l(l+1)r^{-2}-V(r) \quad (10)$$

and $y=rR$. Then an easy calculation transforms Eq. (1) into an Eq. (8), where $x=r$. Since Eq. (9) becomes identical with Eq. (4), where $R=y/r$, it follows that

$$R(r)=\left\{\int_0^\infty e^{-rq}d\phi(q)\right\}/r. \quad (11)$$

But if the identity

$$1/r=\int_0^\infty e^{-rp}dp$$

is substituted into Eq. (11), a straightforward contraction reduces Eq. (11) to Eq. (4), since $\phi(0)=0$.

What remains to be verified is that the assumptions (7) for the existence of a solution of the form (9) are satisfied in the present case. But Eq. (10) shows that the case $n=0$ of the conditions (7), where $d^0f/dx^0=f$, is contained in the assumption (2) for a bound electron, and that the other conditions (7) reduce to

$$(-1)^n d^n V(r)/dr^n \geqq 0, \quad \text{where} \quad n=1,2,\cdots; \quad 0<r<\infty. \quad (12)$$

Clearly, the conditions (12) are satisfied by $V(r)=r^{-a}$ if $a>0$. Hence, they are satisfied by any potential of the form (3). They are also satisfied if $V(r)=e^{-r}/r^a$, since they are satisfied if $V(r)=e^{-r}$ (they are satisfied by $V(r)=V_1(r)V_2(r)$ if they are satisfied by both $V=V_1$ and $V=V_2$).

If $r=x$, Eq. (1) and Eq. (10) represent the particular case $g(x)=x^2$, $h(x)=f(x)$ of the general self-adjoint differential equation $(gz')'+hz=0$. It is not hard to verify that the above deduction can be extended to the case of the latter differential equation as soon as the conditions (7) are satisfied by both $f=-g^{-\frac{1}{2}}$ and $f=h$.

[1] E. Schrödinger, *Abhandlungen zur Wellenmechanik* (J. A. Barth, Leipzig, 1928), p. 3.
[2] E. Schrödinger (see the above), pp. 4–10 and pp. 131–134.
[3] E. G. C. Poole, *Introduction to the Theory of Linear Differential Equations* (Clarendon Press, Oxford, 1936), Chap. V.
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Stark Spectrum of H_2O*

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THE Stark spectrum of the rotation line of H_2O[1] at 22,235.22±0.05-mc/sec. has been observed, using higher electric fields than previously reported.[2] The apparatus used is similar to types described by others,[3,4] utilizing 1.25 cm silver wave guide with a 6 kc/sec. square wave electric field oriented parallel to the electric vector of the microwave radiation. At fields of 7500 volts/cm, four of the six components predicted by theory were completely resolved.

The unperturbed line had a minimum breadth of approximately 250 kc/sec.; because of the irregularities in the perturbing electric field, the components had widths of approximately 500 kc/sec.

Figure 1 shows the experimental data and fitted curves for the Stark splitting. It appears that the unresolved $M=0$, 1 line has a "center of gravity" approximating the splitting of the $M=1$ line. This is to be expected since the latter is doubly degenerate and the former a single line; the resulting computed intensity ratio is 36/70. The relative intensities of the components qualitatively follow a

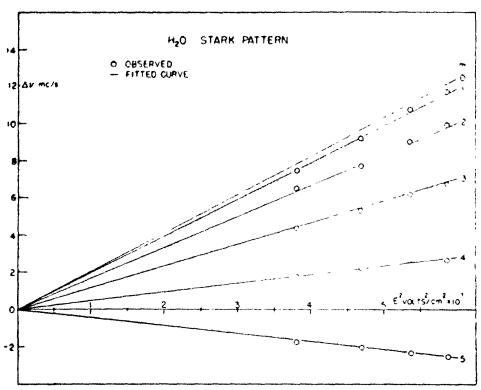

FIG. 1. H_2O Stark pattern.

theoretical (J^2-M^2) law, $(J=6)$, but this has not been substantiated by integration over the components.

The experimental data can be represented by the formula

$$\Delta\nu=(20.65-1.003M^2)E^2\times 10^{-8} \text{ mc/sec.},$$

where $M=0, 1, 2, 3, 4, 5$; E=electric field strength in volts/cm. The probable error of fit is approximately 0.3 percent. This is to be compared with the theoretical formula calculated by means of conventional perturbation theory:

$$\Delta\nu=(18.42-0.9165M^2)E^2\times 10^{-8} \text{ mc/sec.},$$

with the dipole moment equal to 1.84×10^{-18} esu-cm.[5]

Certain of the direction cosine matrix elements involved in the calculation were determined from the line strengths for H_2O calculated by King, Hainer, and Cross[6] while others were determined by interpolation from tables of line strengths by these same authors.[7] The term values involved in the second-order perturbation theory were taken from the assignments by Randall, Dennison, Ginsburg, and Weber[8] with the exception that the observed microwave absorption frequency was used in place of that computed from the assigned term values.

Despite favorable agreement between the experimentally and theoretically derived formulas, the existing discrepancy requires some comment. From the theoretical point of view there are three possible sources of error. Induced polarization effects due to the Stark field have been examined and for the present case may be safely neglected. There is some uncertainty in the dipole moment, particularly since the value usually obtained involves an implicit averaging over the various vibrational states of the molecule. The most serious error is probably in the use of the rigid rotor wave functions in the theoretical calculations.

Experimentally, the greatest source of error is in the determination of the field strength, since the field is not homogeneous over the cross section of the wave guide. Calculations show that corrections for fringing of the field near the side walls of the wave guide are negligible. However, the error in the measured field strength can be as much as ± 2 percent because of lack of centering of the Stark electrode and the wave shape of the applied Stark voltage.

The ratio of the constant and M-dependent coefficients in the formulas is 20.59 for the observed data and 20.10 for the theoretical formula, giving a 2.5 percent agreement. This ratio is independent of both field strength and dipole moment.

To apply the theory described to our measurements, a value for the dipole moment of 1.94 ± 0.06 Debye units would be required.

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