(n, p) and a (n, α) disintegration insofar as the energy of the α-particles was measurable. The energy difference $E(n, p) - E(n, \alpha)$ of corresponding transitions must be equal to the difference of the Q values: $Q(n, p) - Q(n, \alpha)$ ~ 0.9 Mev.

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On the Resolving Time and Genuine Coincidence Loss for Geiger-Mueller Counters*

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THE coincidence method is one which has been widely used in experiments relating to nuclear disintegrations and cosmic radiation. In investigations of nuclear reactions and radioactive nuclei by coincidence techniques, the use of a very short resolving time ($\tau \leq 0.1$ microsecond) would have the obvious advantage of minimizing accidental coincidences arising from the presence of a strong background of uncorrelated phenomena. Circuit design has not been the limiting factor in achieving such short resolving times; the difficulties lie in the Geiger-Mueller counter mechanisms. A number of workers[1-4] have reported substantial losses in the counting of genuine coincidences when employing G-M counters in coincidence circuits having resolving times less than one microsecond. These losses have been attributed to fluctuating time delays (as much as one microsecond) between the initial ionizing event and the change in the potential of the counter wire of sufficient magnitude to register.

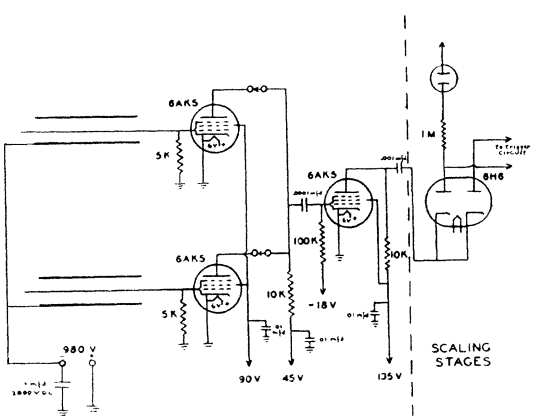

FIG. 1. Fast coincidence circuit—resolving time 0.08 microsecond.

Coincidence studies of radioactive nuclei have been in progress for some time at this laboratory and recent attempts to use resolving times under one microsecond have yielded preliminary results of a promising nature.

Thin-walled Geiger-Mueller counters were constructed to the following specifications: sensitive counting volume—5 cm in length, 1.5 cm in diameter; thickness of glass walls —0.013 cm; anode wire diameter—0.0075 cm; aquadag cathode thickness—0.0013 cm.

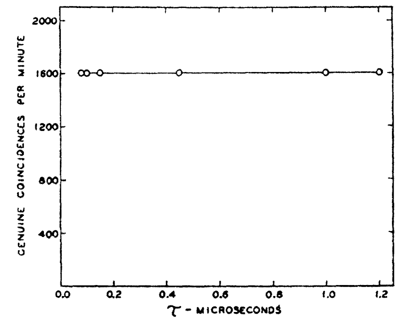

FIG. 2. Genuine coincidences as a function of the resolving time.

The counters were washed in a sulphuric acid solution and rinsed in distilled water before the cathode coating was applied. After baking in vacuum for 2 hours at 420°C to drive off occluded vapors, the anode wires were flashed and the counters filled with an argon-ether mixture to a pressure of 12 cm Hg. The operating potential for these counters was 980 volts.

For operating resistors of 1000 to 5000 ohms, the counter pulse shapes were examined on a fast sweep oscilloscope. The pulse rise time as viewed on the oscilloscope was 0.15 microsecond, the rise time of the oscilloscope amplifier. The actual rise time of the counters may be less than this amount. The pulses had a uniform amplitude of several volts.

A matched set of 2 of these Geiger-Mueller counters was operated in coincidence with the circuit shown in Fig. 1. The anode wires were connected directly to the grids of the Rossi valves as indicated. For an operating resistance of 5000 ohms, the counters delivered a negative pulse of approximately 4.5 volts to the grids of the 6AK5's. The resolving time of the circuit was varied by altering the values of the input grid resistors and the bias of the output tube. The coincidence circuit was checked for delays in one channel with respect to the other and for inter-channel coupling which might give rise to spurious coincidences.

Genuine coincidences were produced by a collimated source of hard beta-rays which were allowed to traverse both counters. As the resolving time, τ, was reduced from 1.2 microseconds to 0.08 microsecond, no loss in genuine coincidences was observed. More than 1600 genuine coincidences were recorded at each point to insure a high statistical accuracy. The genuine coincidence rate is plotted as a function of the resolving time in Fig. 2. As a check on the results obtained, radioactive sources of Sc^{46}, Ti^{51}, and Ta^{182} were placed between the counters, and the beta-gamma coincidence rate was determined as a function of the resolving time. No losses in the genuine coincidence rate were observed down to the shortest resolving time used.

No variations in these results were observed when four pairs of counters of the type described above were tested.

The results obtained suggest that for these particular counters, the delay times are a fraction of one-tenth of a microsecond, and that long delays reported by other workers are not inherent in all Geiger-Mueller counters. It has been suggested that long time delays are due to the presence of electronegative vapors.[5,6] Rossi and Nereson[7] have reported delays of a few tenths of a microsecond in brass-walled argon-alcohol counters used in cosmic ray experiments. Counter delays of a tenth microsecond or less have also been reported.[8–10]

For argon-alcohol counters of diameter 1.2 cm, an extrapolated delay time of 0.04 microsecond has been calculated.[9] This result is in agreement with the fundamental experiments of Ramsey[11] and the recent analysis of Korff.[12]

Circuit limitations prevented the investigation of coincidence losses at resolving times less than 0.08 microsecond. A fast rise time amplifier is now in development for extending observations to 10^{-8} second, since no counter delays have as yet been detected.

Geiger-Mueller counters of this type should widen the use of the coincidence method and in particular should aid in the search for short-lived metastable states.[10,13]

Stimulating discussions with W. E. Ramsey and with Professor W. C. Elmore of Swarthmore College are gratefully acknowledged. We are indebted to Dr. Elmore for having measured the rise time and other counter pulse characteristics with his apparatus. Austin G. Nester of the Bartol Research Foundation constructed the Geiger-Mueller counters.

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On the Solutions of Radial Wave Equations

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FOR a fixed value, $W=W_0$, of the eigen parameter, the separation of the spherical harmonics in a Schrödinger equation, belonging to a potential of the type $V=V(r)$, is known to reduce the three-dimensional wave equation to the ordinary differential equation[1]

$$(r^2R')'/(r^2R)+W_0-l(l+1)/r^2-V(r)=0, \qquad (1)$$

for $R=R(r)$, where $R'=dR/dr$ (and $8\pi^2\mu/h^2=1$). Let the constant W_0 be in the range of "bound electrons," so that

$$W_0\geqq 0, \quad \text{and} \quad V(r)\geqq 0 \quad \text{for} \quad 0<r<\infty, \qquad (2)$$

while the potential is a sum of the form

$$V(r)=Ar^{-a}+Br^{-b}+\cdots, \qquad (3)$$

in which $a, b, \cdots$ and $A, B, \cdots$ are positive constants.

In the case of the hydrogen atom, the sum (3) reduces to a single term, with $a=1$ and $A=Ze^2>0$. In this case, the solution of Eq. (1) leads[2] to Laguerre functions, and the preponderant factor behaves, roughly, as $e^{-\frac{1}{2}r}$ if $r\rightarrow\infty$. The success of this substantially explicit method depends on the accidental applicability of the classical theory (Riemann-Fuchs) of "regular" singular points of linear differential equations,[3] a circumstance due to the particular choice, Ze^2/r, of $V(r)$.

Because of the same circumstance, a closer inspection of the characteristic equation and of the recursion formulae is possible. It reveals that the coefficients of the Laguerre functions[4] obey a certain law of positive regularity. It will not be necessary to give a description of this law, since it turns out to have nothing to do with the specific nature of a Coulomb force or, for that matter, with the possibility of *some* "explicit" integration of Eq. (1). In fact, it will be proved below that the situation is as follows:

If the potential *graph*, $V=V(r)$, over the half-line $0<r<\infty$ of the (r, V) plane, besides satisfying condition (2), satisfies a certain *qualitative* condition (that specified by the inequalities (12) below), then Eq. (1) possesses for $0<r<\infty$ a positive solution which not only is *a Laplace transform in the real domain*, say

$$R(r)=\int_0^\infty e^{-rq}\phi(q)dq, \quad \text{where} \quad 0\leqq q<\infty, \qquad (4)$$

but belongs to a Laplace transform of a *non-negative* density, which, in addition, is *non-increasing* when q increases. In other words, the function $\phi(q)$ occurring in Eq. (4) is subject to

$$\phi(q)\geqq 0 \quad \text{and} \quad d\phi(q)\geqq 0 \quad \text{for} \quad 0\leqq q<\infty. \qquad (5)$$

Needless to say, Eq. (4) is claimed to represent an *exact* solution of Eq. (1), rather than just an approximation, asymptotic to some solution as $r\rightarrow\infty$. If the distribution determined by $\phi(q)$ is known,[5] a positive solution of Eq. (1) follows from Eq. (4), whereupon the "other" (linearly independent) solution of Eq. (1) is supplied by the product

$$R(r)\cdot\int_r^\infty \{pR(p)\}^{-2}dp, \qquad (6)$$

in which R denotes the first solution. This can, of course, be verified by substituting the function (6) into Eq. (1) and assuming that the latter is satisfied by the first factor of the product (6).