The Production of High Velocity Mercury Vapor Jets by Spark Discharge

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(Received November 20, 1947)

Photographs of sparks of microsecond duration in hydrogen to a mercury electrode show high velocity jets from the mercury surface regardless of polarity. Spectroscopic examination shows that these jets are largely composed of mercury atoms which sweep away the hydrogen. Measurements of jet velocity demonstrate that it is independent of current and gas pressure over a wide range. While it decreases with distance from the source, the original velocity of the cathode jet is 1.9 (10^6) cm/sec., and that of the anode jet is 1.5 (10^6) cm/sec. The energy of the anode jet can be accounted for by positive mercury ions crossing the anode drop. It appears that the energy of the cathode jet cannot be accounted for unless it is assumed that, in addition to positive ions striking the cathode and rebounding as neutral atoms, many mercury atoms leave the cathode as negative ions.

SECTION I

Introduction

THE experiments of many investigators show that a migration of atoms of the metal composing the electrodes of a gap occurs in either an arc or spark discharge. There is, however, considerable disagreement in the results published by various investigators on the velocity of migration found with spark discharge, and the results obtained with arc discharge are approximately an order of magnitude higher than those obtained with sparks.

Spark Discharge

The earliest work appears to be that of Schuster and Hemsalech.[1] Using a spectrometer and moving film arrangement they showed that, during an oscillatory spark in air, radiation characteristic of the electrode material is found at increasingly greater distances from the electrodes with increasing time. Although the accuracy of their experiments was not very great, they were able to conclude that the atoms of the electrode materials migrated with a velocity of from 1.9 (10^5) to 0.4 (10^5) cm/sec. The velocity determined in this way was dependent on the electrode material and was found to decrease with distance from the electrodes. Mohler[2] photographed a spectrum of an electric spark along the line of discharge, then reversed the electrodes and took another spectrogram. From the observed Doppler shift he calculated an average velocity of the "particles" from the electrodes of 0.37 (10^5) cm/sec. Lawrence and Dunnington[3] through the use of a Kerr cell and spectrograph showed that the luminosity of the metallic vapors of the electrodes spread from the electrodes with a velocity which, using zinc, had an average value of 2.1 (10^5) cm/sec. On the assumption that the broadening of the zinc spectrum was due to the Stark effect of interatomic fields, they calculated that a third of the molecules in the discharge paths were ionized. Assuming that their measured velocity was that of the migration of positive ions from the anode, they estimated that approximately $\frac{1}{2}$ of the current was carried by positive ions.

Arc Discharge

Using a "vacuum" arc with copper electrodes Tanberg[4] found that a pressure is exerted on both the cathode and a vane suspended in front of it. On the assumption that the measured force was that caused by the force of reaction of evaporating copper atoms, he calculated a velocity of 16 (10^5) cm/sec. for these copper atoms. Tanberg supposed this high velocity to be caused by a high temperature of the cathode spot and calculated that the required temperature was 500,000°. Compton,[5] objecting to the postulation of such a high temperature, thought it much more likely that the force was due to the rebounding of

[1] A. Schuster and G. Hemsalech, Proc. Roy. Soc. **64**, 331 (1899).
[2] J. F. Mohler, Astrophys. J. **15**, 125 (1902).
[3] E. O. Lawrence and F. G. Dunnington, Phys. Rev. **35**, 396 (1930).
[4] R. Tanberg, Phys. Rev. **35**, 1080 (1930).
[5] K. T. Compton, Phys. Rev. **36**, 706 (1930).

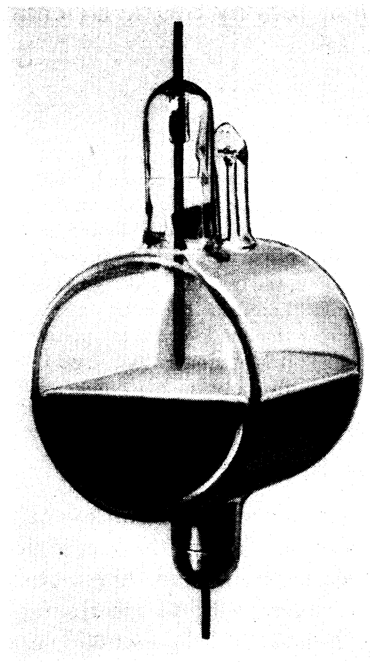

FIG. 1. Spark gap tube provided with flat glass window.

neutralized positive ions from the cathode with energy $(1-\alpha)$, where α is the accommodation coefficient of the positive ions. The force on the cathode of a mercury vacuum arc was measured by Kobel.[6] Again assuming that this force was due to the reaction of mercury vapor on the cathode he calculated a velocity very similar to that obtained by Tanberg.

That this high velocity could not be accounted for by an accommodation coefficient theory was shown by Slepian and Mason,[7] since the velocity calculated by Tanberg required energies of 70 volts, or considerably more than the cathode drop.

Tanberg and Berkley[8] then showed that the temperature of the cathode was only of the order of 3000°K and concluded that the high speed of the vapor stream cannot be caused by the temperature of the cathode itself. They suggested that the vapor gained a high velocity thermal agitation in a region just outside the cathode. Ludi and Risch[9] have suggested that there may be a large number of atoms ionized five or six times. Easton, Lucas, and Creedy[10] measured a force on the anode of the vacuum arc and, using the assumption of Tanberg that this force was the force of reaction of particles leaving the anode, calculated a velocity of the anode vapor of 10^6 cm/sec. They concluded that a thermal origin was indicated. Robertson[11] showed that the force on the cathode of a vacuum arc is a function of gas pressure, and Risch[12] showed that this was the expected result of a jet caused by the unbalanced gas pressure on the back of the cathode.

Considerable doubt that the calculations of vapor velocities in the vacuum arc are valid was expressed by Tonks[13] who calculated that a force of the order measured could be accounted for by the partial pressure of the electron gas. Loeb[14] showed that even if this correction were small the calculated velocity would be too high because of a difference between the number of atoms "evaporated" and those which actually escape into the vapor stream, since a large number of atoms is returned to the cathode as positive ions.

During the war an extensive study of fixed spark gaps has been made because of their importance in radar circuits. These studies have shown that, at least in air and hydrogen and using a wide variety of cathode material, a spark having a peak current greater than 100 amperes has a cathode and anode drop of the same order

FIG. 2. Photograph of five individual sparks showing cathode mercury vapor jets. The mercury pool (cathode) is at the bottom. (Retouched.)

[6] E. Kobel, Phys. Rev. **36**, 1636 (1930).
[7] J. Slepian and R. C. Mason, Phys. Rev. **37**, 779 (1931).
[8] R. Tanberg and W. E. Berkley, Phys. Rev. **38**, 297 (1931).
[9] F. Ludi and R. Risch, Zeits. f. Physik **75**, 812 (1932).
[10] E. C. Easton, F. C. Lucas, and F. Creedy, Elect. Eng. **53**, 1454 (1934).
[11] R. M. Robertson, Phys. Rev. **53**, 578 (1938).
[12] R. Risch, Phys. Rev. **57**, 1181 (1940).
[13] L. Tonks, Phys. Rev. **46**, 278 (1934).
[14] L. B. Loeb, *Fundamental Processes in Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), p. 633.

as that measured in a steady arc.[15] It is therefore reasonable to suppose that the mechanism of vapor jet production is the same in both arc and spark discharge. Indeed, it would be amazing if even much difference of a quantitative nature exists, and it appears likely that most of the large reported difference between the vapor velocities in sparks and those in arcs is due to the lack of proper corrections of vacuum arc velocities, making these calculated values too high, and to the use of average velocities in spark measurements, making these values comparatively low, especially in the Doppler measurements of Mohler.

SECTION II

Spark Photographs

At one stage in the investigation of fixed spark gaps used as switches in radar modulator circuits a mercury pool was used as one electrode in a tube, such as that shown in Fig. 1. These spark gaps consist simply of a rod of molybdenum, 1.5 millimeters in diameter, brought within 7 millimeters of a mercury pool in a tube containing purified hydrogen at a pressure of 90 centimeters of mercury. When tubes of this type were operated as a switch, sparks occurred between the tip of the molybdenum rod and the mercury pool at the rate of several hundred per second. The current in these sparks is a unidirectional pulse having a constant value of a few hundred amperes and a duration adjustable from 0.25 to 5 microseconds. It was found that regardless of polarity a jet of mercury vapor was

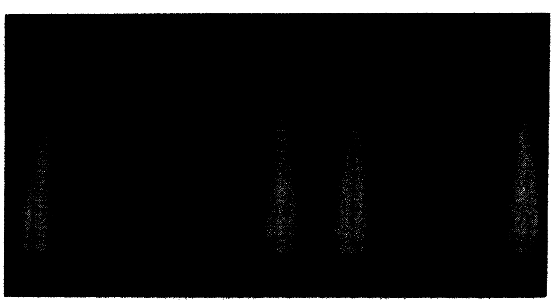

FIG. 3. Photograph of four individual sparks showing anode mercury vapor jets. The mercury pool (anode) is at the bottom. (Retouched.)

[15] F. S. Goucher, J. R. Haynes, W. A. Depp, and E. J. Ryder, "Spark gap switches for radar," Bell Sys. Tech. J. **25**, 594 (1946); Tech. Pap. Bur. Stand. **25** (October, 1946).

FIG. 4. Photograph of four individual sparks showing the effect of mercury on both electrodes. The mercury pool (anode) is at the bottom. (Retouched.)

ejected from the mercury pool during the time of discharge.

There is no doubt whatever about the physical reality of these jets. Perhaps the simplest and most striking proof is that obtained from Kodachrome photographs of individual sparks. These photographs were taken by reflecting the image of the spark from a rotating first surface mirror prior to focus on the Kodachrome film. A black and white reproduction* of a Kodachrome film obtained in this way, using the mercury pool as the cathode, is shown in Fig. 2. This is a photograph of five individual sparks, having a current of 230 amperes, which is constant for 5 microseconds. The images of the molybdenum anode are at the top and those of the mercury cathode at the bottom. A red positive column (dominated by the hydrogen α-line) reaches from the anode to the cathode. A well defined jet of the blue-green color characteristic of excited mercury vapor extends from the cathode spot into the positive column. It has been suggested that perhaps these are not real jets but are produced by the propagation of a high energy pulse along the spark column. That this is not the explanation is demonstrated in the first spark of Fig. 2 since the jet extends outside the spark column.

If the polarity of the mercury and molybdenum rod be reversed, so that the mercury is now the anode, similar jets are obtained as shown in Fig. 3. These sparks were obtained without changing the pulse conditions. In each spark photograph an anode jet may be seen extending from an anode spot on the mercury into the positive column. Again in the first spark photograph the anode jet is seen to pass outside the spark column.

If mercury is splashed on the molybdenum rod

* These photographs have been retouched to restore the contrast lost in black and white reproduction.

it wets the surface and, in this way, a drop of mercury may be hung from its tip. Under these conditions jets are obtained from both electrodes which are oppositely directed. The result is shown in Fig. 4. The gap between the rod and the mercury pool has been reduced so that the jets will collide while retaining a high energy. When the jets meet they splay out, giving the effect that one obtains by directing the streams of two garden hose against each other. This is a proof of the physical reality of these jets since they obviously have momentum.

The anode and cathode jets are always found to leave the mercury in a direction normal to the surface and proceed in a straight line regardless of the path of the spark. The apparent deviations from this rule, shown in Fig. 2, are produced by ripples on the mercury surface which are clearly visible during sparking. In extreme conditions with a mercury drop hung on the tip of the molybdenum rod the cathode spot may be produced on the side of the drop. When this occurs the jet still leaves in a direction normal to the mercury surface proceeding in a straight line which is nearly 90° from the average path of the spark.

A "dark" space of about 0.05 centimeter may be observed in Fig. 2 between the cathode spot and the jet. A similar dark space between the anode spot and jet, of much the same thickness, is clearly visible in the original Kodachrome of Fig. 3. The width of these dark spaces increases with current and time and is a function of the jet diameter. These spaces cannot be regions of cathode and anode drop since, from the space-charge law and existing voltage, the current density would be many orders of magnitude less on this assumption than that actually measured. It seems far more likely that these dark spaces constitute the distance required for the mercury atoms in the jet, all originally moving in the same direction with much the same velocity, to acquire a sufficiently random velocity by impact with hydrogen molecules to produce a high probability of excitation on mutual impact.

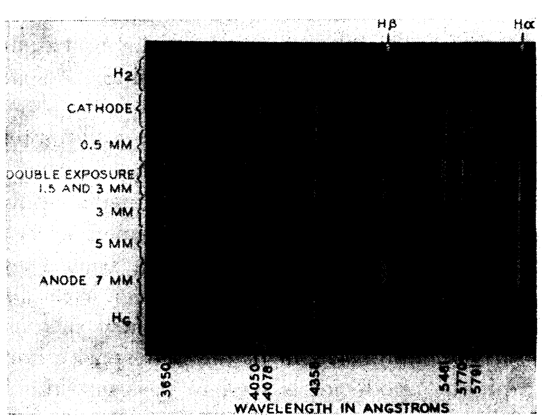

FIG. 5. Spark spectra of the light originating at progressively greater distances from a mercury cathode. At the extreme top and bottom are comparison spectra of hydrogen and mercury.

Analysis of spark photographs taken with peak currents ranging from 90 to 440 amperes and pulse durations of 0.25, 1, 2, and 5 microseconds, shows that the current density at the cathode is independent of current but decreases with time. An empirical formula for the current density ρ, at the cathode which fits the data closely, is given simply by

$$\rho = 10^6/(14.8 + 7.7 \log_e t), \tag{1}$$

where t is in microseconds. Thus, for a 0.25-microsecond pulse the enormous current density of 2.4 (10^5) amperes/cm² is obtained. Since in these experiments the current is maintained constant during the time of pulse, it follows that the cathode spot area is increasing rapidly.

SECTION III

Spectroscopic Examination

Examination with a spectrograph shows that the jets are composed largely of mercury atoms. A spectrogram obtained with the light from portions of sparks at progressively greater distances from a mercury cathode is shown in Fig. 5. These sparks were obtained with current pulses having a constant value of 300 amperes for two microseconds. At the top is a comparison spectrum of hydrogen and at the bottom a mercury spectrum obtained from a General Electric H4 lamp. Many of the lines which appear with increasing intensity near the cathode are easily identified with the mercury comparison spectrum. The additional strong lines are included in the spark spectrum of mercury. It is evident that the light near the cathode and hence from the jet is very largely that of excited mercury vapor.

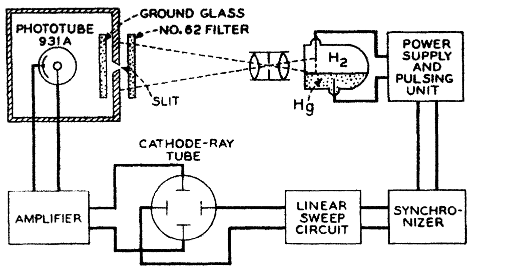

FIG. 6. Schematic diagram of apparatus used to measure mercury vapor jet velocities.

It will be observed that hydrogen β-line is quite broad. Its half-width at 6 millimeters from the mercury cathode was measured with a microphotometer and was found to be 99 angstroms. It has been shown by Finkelnburg[16] that this broadening is chiefly due to the Stark effect. Calculation shows that the implied interatomic field requires an ionization of the positive column of approximately 25 percent. This value agrees well with that of 33 percent, obtained by Lawrence and Dunnington, in the early stages of an electric spark in air. Since both of these calculations are based on the assumption that the density of the gas in the discharge path remains unaltered during the time of discharge, it follows that the gas density in the positive column cannot have decreased more than a factor of 3 or 4 at the most, since more than this would lead to greater than 100 percent ionization.

SECTION IV

Jet Front Velocity Measurements

It was obvious that an accurate measure of these mercury vapor jet velocities could be obtained by measuring the time required for the excited mercury atoms in the jet to go a known distance. Accordingly, an apparatus was constructed which enables these quantities to be determined. The light from sparks produced in the tube shown in Fig. 1 is focused on a horizontal slit by means of a lens provided with a diaphragm, as shown in Fig. 6. The horizontal slit allows only the light from a small segment of the spark column to reach a multiplier phototube. A No. 62 Wratten and Wainwright filter (mercury monochromat) is placed in the path of the light beam to limit the radiation reaching the photo-tube largely to that of the green line of mercury, and a ground glass screen is interposed to diffuse the light. The output of the photo-tube is amplified by a broad band amplifier and then placed on the vertical plates of a cathode-ray oscilloscope, the horizontal plates being connected to a fast linear time sweep synchronized to the pulsing circuit. It was shown that the deflection of the oscilloscope spot was closely proportional to the light reaching the photo-tube and that the time constant of the circuit was of the order of 0.02 microsecond. The dimensions of the slit and magnification of the image are such that the light reaching the phototube comes only from a section of the spark column a hundredth of a millimeter thick. It is therefore possible to place the slit across the spark image at any desired distance from the image of the mercury surface and to measure the time of arrival of the excited mercury atoms composing the jet by the consequent increase of light intensity passing through the mercury green line filter.

Typical characteristics obtained with this equipment are shown in Fig. 7. The current pulse shown in Fig. 7(b) was obtained on the oscillo-

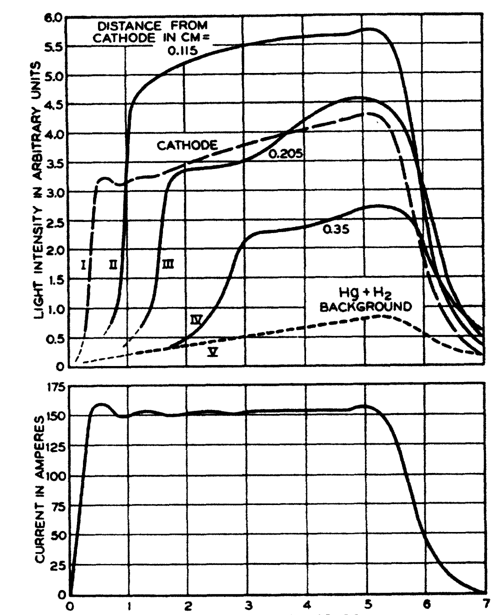

FIG. 7. (a) Typical characteristic of light intensity as a function of time, at various distances from a mercury cathode. (b) Current through the discharge on the same time scale.

[16] W. Finkelnburg, Zeits. f. Physik **70**, 375 (1931).

scope tube by well-known radar pulse technique using a nominal 5-microsecond pulse network. The current rises rapidly to a little over 150 amperes and remains substantially constant for approximately 5 microseconds. The consequent characteristics obtained from the light from the spark are shown in Fig. 7(a). When the slit is placed across the spark image so that the light reaching the photo-tube comes only from the cathode, or from its immediate vicinity, an approximate analog of the current pulse is obtained (curve I). However, close inspection of these curves shows that on the initial rise the light lags behind the current by about 0.15 microsecond. This delay time between the light and the current corresponds to the time required to initiate the arc discharge since it is known that currents in excess of 50 amperes are required to produce an arc (instead of a glow). A time delay of this same magnitude was found by Lawrence and Dunnington in the time required to produce zinc lines in a spark in air between zinc electrodes.

When a slit is placed so that the light reaching the photo-tube originates at a distance of 0.115 centimeter from the mercury cathode, an additional time delay of 0.6 microsecond is observed, as shown in curve II. At further distances longer time delays are observed, and the intensity rise due to the arrival of the jets diminishes so that a position of the slit is reached, near the anode,

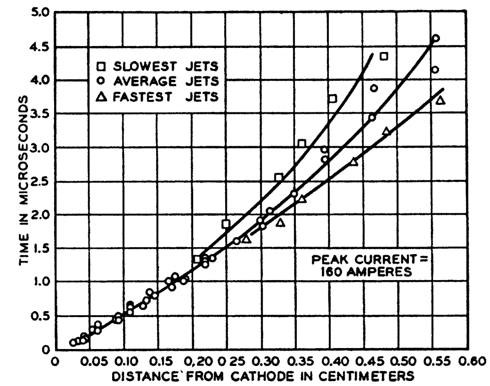

FIG. 9. Time of arrival of cathode jet front as a function of distance from mercury cathode.

where no effect of the jet can be found. In such a position the intensity varies with time, as shown in curve V. This is evidently the variation of the light from the positive column, already shown (Fig. 5) to be the mercury vapor and hydrogen spectrum characteristic of the gas in regions not reached by the jet.

The difference in time between any two analogous points on the light intensity rise characteristic, due to the arrival of the jet front and, at two different distances from the mercury cathode, might be used as a measure of the average velocity of the jet from between these distances.

The characteristics are seen to rise sharply to a fairly definite maximum after which the intensity increases much more slowly. Half of this maximum intensity was chosen as the comparison point.

The characteristics shown in Fig. 7 were made without changing the lens diaphragm and, therefore, show the relative maximum intensities. For the purpose of making measurements of the jet front velocity it is much more convenient to change the lens diaphragm aperture so that the change in intensity, due to the arrival of the jet, is maintained nearly constant. This also serves to reduce any errors due to non-linearity of the amplifier or cathode-ray tube. Typical characteristics obtained in this way are shown in Fig. 8. Only the initial rise of the characteristics are drawn and used as data. In the characteristics the upward shift with increasing time is produced by the mercury and hydrogen background light, shown in curve V, Fig. 7 (a).

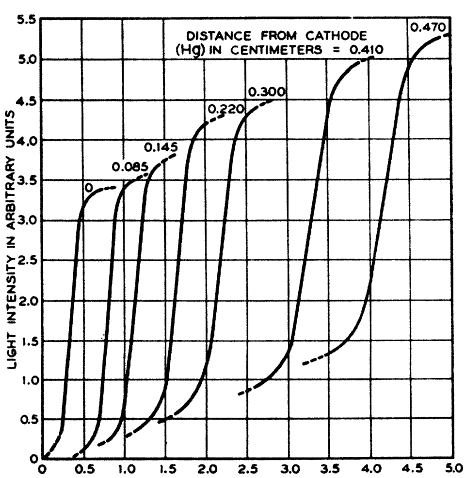

FIG. 8. Typical characteristics of light intensity as a function of time, used for obtaining jet velocity.

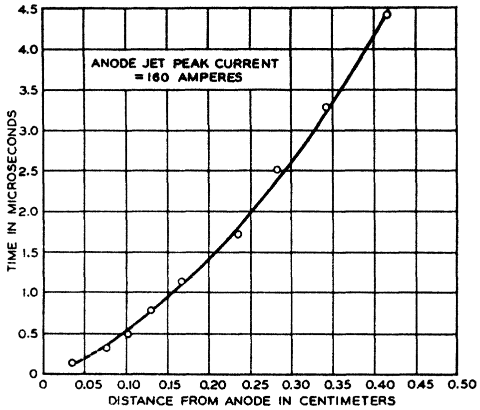

FIG. 10. Average time of arrival of anode jet front as a function of distance from mercury anode.

The characteristic obtained on the oscilloscope from light originating at the mercury cathode is a sharp line since the time uncertainty of synchronization of the apparatus is less than 0.02 microsecond. As the distance from the cathode is increased, the characteristics due to individual pulses (produced at the rate of 300 per second) have an increasing variation on the time axis of the oscilloscope tube and appear as a band of increasing width. This band width is evidently produced by variations in jet front velocity. It is possible, therefore, to trace the characteritics through points corresponding to jets having average velocities, as was done in obtaining data shown in Fig. 8, or through characteristics corresponding to the slowest or fastest jets. All three of these were done in order to obtain some idea of the distribution of jet velocities and the results are plotted in Fig. 9, where the time of arrival of a half-intensity point of a cathode jet front is plotted as a function of distance from the cathode. It will be observed that the maximum difference in jet front velocities is negligible at less than 0.2 centimeter from the mercury surface. Since all jets, therefore, have closely the same velocity near the cathode, it appears that the spread of jet velocities at greater distances is due to variations in density of the hydrogen produced by the previous spark.

Reversing the polarity of the mercury pool gives characteristics which are very similar to those shown in Fig. 8. Measurement of the delay in time of arrival of the half-intensity point at two known distances from the mercury anode, therefore, also gives a measure of the velocity of the front of the anode jet between these distances. Data obtained in this way are shown in Fig. 10 for average anode jets.

The velocity of the jet fronts may be derived from the curves of Figs. 9 and 10. This velocity for both cathode and anode jet fronts is plotted in Fig. 11. The velocities of each decrease linearly with distance from the mercury surface. This decrease is due to the mass reaction of the hydrogen with the jets. Extrapolation to the anode gives an initial velocity of the anode jet of 1.55 (10^5) cm/sec., and an initial velocity of the cathode jet of 1.9 (10^5) cm/sec. This initial velocity is found to be independent of hydrogen pressure from 5 to 153 centimeters of mercury and independent of current from 70 to 400 amperes. It has been shown[15] that both the electrode voltage drops and column voltage gradient are independent of current. As shown in Section III, the current density at the cathode is independent of current. Therefore, the initial jet velocity should be independent of current on any theory of the mechanism of jet production. The independence of jet velocity of gas pressure can be explained if the mechanism of jet production lies close to the electrode surface. Here the hydrogen is quickly swept away by the jet and the gas density is determined solely by the rate of mercury vaporization, so that the pressure of the surrounding hydrogen is of no consequence.

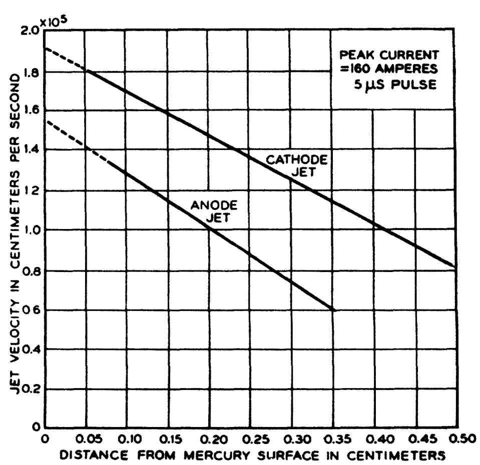

FIG. 11. Velocity of cathode and anode jet fronts as a function of distance from mercury surface.

SECTION V

Intensity Variation of Individual Lines

The variation of intensity of individual spectrum lines with time was investigated by a combination of the photo-tube and the spectrograph. The No. 62 filter was removed and the slit of the spectrograph substituted for the horizontal slit shown in Fig. 6. The usual photographic plate of a spectrograph was replaced by a metal plate provided with a narrow slot at the appropriate position to allow only the light from the chosen spectrum line to pass to the photo-tube. In this way the intensity of any chosen line could be investigated as a function of time and distance from the mercury surface.

When the spectrograph slit is so placed that the light reaching the photo-tube comes from a distance of 0.6 centimeter from the mercury cathode, the solid line characteristic, shown in Fig. 12, is obtained. This same solid line was obtained with three different wave-lengths. The diaphragm of the lens system was adjusted so that the peak intensities of the three lines examined (mercury 5461A, hydrogen 6563 and 3861A) had the same value. When this was done, it was found that the entire characteristic fell on the single solid line that was drawn. Since this radiation originates in a region not influenced by the jet, it is concluded that under these conditions all radiation in this band of energies rises and falls simultaneously. This conclusion was checked by showing that this same curve is obtained with the total light emitted from the spark in this region (to which the photo-tube is sensitive).

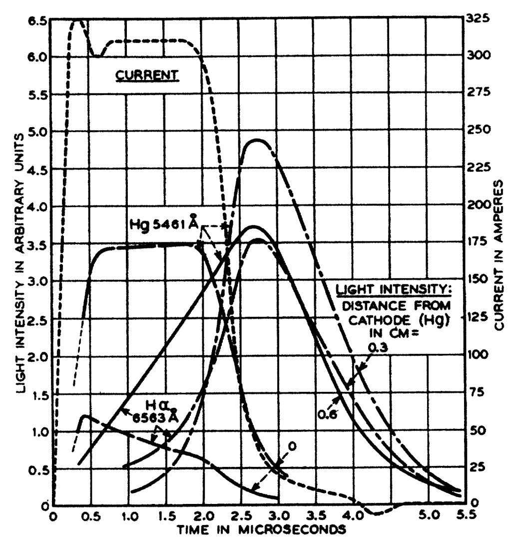

FIG. 12. Light intensity of individual spectrum lines as a function of time, at various distances from the mercury cathode surface. The solid line is the characteristic obtained by light originating at 0.6 cm from the mercury surface, the dash-dot lines at 0.3 cm, and the dash lines at 0.0 cm. The current through the discharge as a function of time is also dotted in for reference.

It will be observed that the light persists a considerable time after the pulse. This afterglow in hydrogen first recorded by Lord Rayleigh has been studied by numerous investigators including Meek and Craggs[17] with a photo-tube and cathode-ray oscilloscope arrangement. All of these investigations show that the afterglow in hydrogen persists for a much longer time than the calculated 10^{-8} second average life of an excited hydrogen atom. It has been suggested[17] that this long afterglow is caused by the persistence of a high temperature in a spark column, so that excitation persists as long as temperature permits. The fact that the variation of intensity of the green line of mercury is the same as hydrogen α and hydrogen β tends to confirm this "temperature" theory.**

It will also be observed that the peak of the light intensity time characteristic occurs after the current pulse, or more precisely some $\frac{3}{4}$ of the microsecond after the current has started to drop. On the "temperature" theory of excitation this implies that the "temperature" of the positive column continues to increase after the current drops. A time delay in sparks of this order of magnitude was predicted by Loeb[18] for conversion of the energy from ions and electrons to a velocity distribution of the gas molecules to something approaching a temperature. Thus, although the average kinetic energy of ions and electrons is decreasing, more impacts have energies above that required for ionization.

[17] J. M. Meek and J. D. Craggs, Nature **152**, 538 (1943); Proc. Roy. Soc. **A186**, 241 (1946).

** As Loeb points out, it is very loose to use the word "temperature" to describe the condition of the gas in a spark column, since equilibrium is not established. "Temperature" is used here in quotation marks to represent the kinetic energy of agitation of the gas molecules since it is clear that the molecules do not attain a Maxwellian distribution of velocities in these times.

[18] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), p. 539.

If the spectrograph slit is placed to receive the light originating nearer the mercury cathode (0.05 centimeter from the mercury surface) the jet appears after a time and alters the characteristics so that the time variation of mercury and hydrogen lines is no longer the same. The characteristics obtained under these conditions are shown in Fig. 12 (dash-dot lines). The intensity of the mercury radiation at 5461A is relatively low before the appearance of the jet. When the jet arrives the intensity of this mercury line increases about 25-fold. The hydrogen radiation at 6563A also increases on jet arrival but only 3.5-fold. No significance is to be placed on the relative magnitudes of intensities of these two radiations since they depend on the optical system and relative response of the photo-tube. The difference in increase of intensity found on appearance of the jet is significant, however, since it shows that the composition of the gas in the discharge path changes on jet arrival and, therefore, constitutes still another proof of jet reality. The increase in intensity of the hydrogen α-line on jet arrival may be partly due to excitation of hydrogen by excited mercury atoms through collisions of the second kind, but it probably is largely due to high collision excitation probability of the hydrogen near the jet surface.

When the spectrograph slit is placed so that the light reaching the photo-cell originates at the cathode, the characteristics are again quite different, as shown in Fig. 12 (dashed lines). The radiation of the mercury green line (5461A) is a fair analog of the current, but the radiation from the hydrogen α-line is not. It quickly reaches a peak and then decreases, in accord with the postulate that the hydrogen in front of the cathode is completely swept away by the jet. Some radiation from hydrogen remains, probably because the cathode spot is expanding so that hydrogen atoms are being constantly excited on the cathode spot periphery.

SECTION VI

Velocity of Jet Front, Center, and Back

In Section IV the velocity of the jet front was determined by measuring the increase in intensity of the light passing through a No. 62 filter as a function of time. By using short pulses, it is

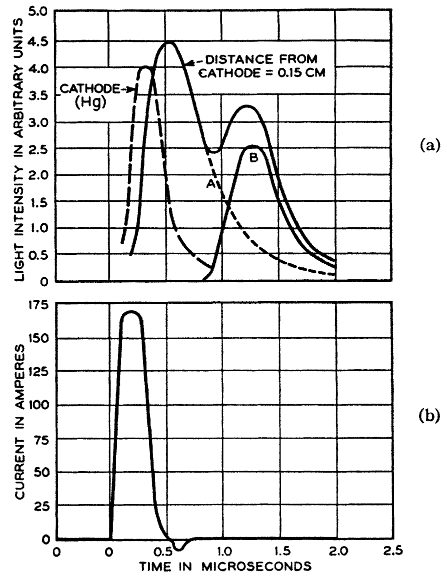

FIG. 13. (a) Light intensity as a function of time, I at the mercury cathode surface, and II at 0.15 cm from mercury surface. (b) Current through the discharge as a function of time.

possible, in this way, not only to measure this velocity of the front of the jet but also, by measuring the decrease in intensity, the velocity of the back of the jet. This has been done with both 0.25- and 1-microsecond pulses. The results obtained with both pulse durations are quite consistent but the data obtained with the 0.25-microsecond pulse are more easily interpreted since, because of the circuit used, the current cuts off much more rapidly. The apparatus was used as shown in Fig. 6. The current pulse obtained with the nominal 0.25-microsecond pulse network is shown in Fig. 13(b). The consequent intensity time variation obtained with light originating at the mercury cathode is shown in Fig. 13(a), (CATHODE). This curve is a fair analog of the current pulse with a time delay of about 0.15 microsecond as before.*** When the hori-

*** The pulse of light obtained from the cathode with these sparks is of considerable interest since it has a far faster decay than can be obtained with the usual gas discharge tubes used in high speed photography. The reason for the sharp decay in the light from the cathode is doubtless that the excited atoms are swept out of the cathode region by the jet.

Another interesting fact is the enormous intrinsic brightness of the cathode spot. Calculations based on either the effect of Kodachrome film or the average current through a photo-tube agree that the cathode spot of these 0.25-microsecond sparks has an intrinsic brilliancy of 10^6 candles/cm^2. This is some 500 times the intrinsic brilliancy

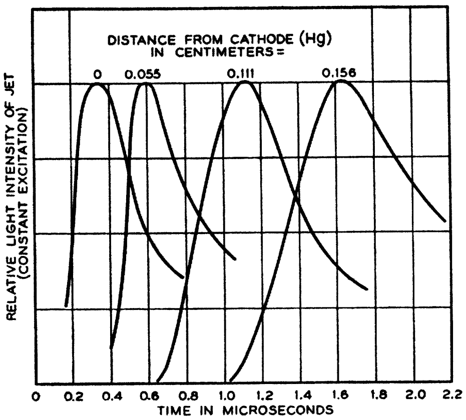

FIG. 14. Corrected relative light intensity of jet as a function of time.

zontal slit is placed so that the light reaching the photo-tube originates 0.15 centimeter from the mercury surface, the light intensity is greatly decreased. In order to compensate for this decrease the diaphragm of the lens was opened, giving the characteristics shown in Fig. 13(a). This curve has two peaks. If the slit is moved so that the light reaching the photo-tube comes from a region of the positive column not reached by the jet, the first part of this curve is retraced but the second peak disappears, the characteristic dropping along the dotted line A. Evidently, then, the first peak is due to the background light of the positive column and only the second peak is due to the jet. In order to obtain the intensity variation of the jet alone, the radiation due to the positive column (first peak and the dotted line) is subtracted from the original characteristic. The result is curve B.†

Curve B is evidently due to the jet but its profile is not due entirely to a change in number of mercury atoms in the spark since the intensity of radiation of the excited atoms is decreasing rapidly with time because the current has been cut off. This decrease in intensity with time was measured and it was found that for 0.25-microsecond pulse, the intensity of light from the jet, after the current is cut off, follows closely an exponential decay which can be written $i=i_0e^{-2.8t}$, where t is the time in microseconds. Therefore, by multiplying curve B by $e^{+2.8t}$ a profile is obtained that is due to a change in the number of mercury atoms only.

Such profiles, obtained at four different distances from the mercury cathode, are shown in Fig. 14. For comparison, the peaks of the four curves were arbitrarily made to have the same relative light intensity. The time of arrival of any analogous points on these curves might be plotted as a function of distance. In this manner one obtains the velocity of the corresponding part of the jet. The points chosen are those corresponding to the time required for the number of mercury atoms to reach the half-maximum, the maximum, and the half-minimum. These points, therefore, give a measure of the velocity of the mercury atoms at the front of the jet, at the maximum jet density, and at the back of the jet. The times of arrival of these parts of the jet are plotted in Fig. 15. It is obvious that near the mercury cathode all parts of the jet move with much the same velocity. The jet spreads out as the distance from the mercury surface increases, since the jet front apparently travels faster than the back. The dotted line is that of Fig. 9 obtained for the cathode jet front and a constant supply of energy, since a current of 160 amperes was maintained during the time recorded. The

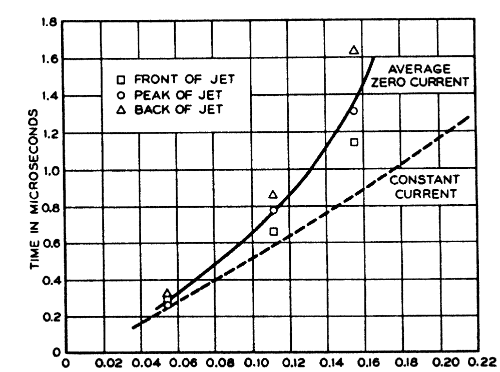

FIG. 15. Time of arrival of specified parts of the cathode jet as a function of distance from the mercury surface.

of tungsten at 3100°K and more than 10 times the brilliancy of the crater of a high intensity d.c. arc. These cathode spots are, in fact, apparently brighter than the surface of the sun (1.6×10^5 candles/cm²) or even the star Algol (0.8×10^6 candles/cm²). This enormous intrinsic brilliancy decreases with pulse duration since the cathode spot area increases with time, in accordance with Eq. (1), while the total light from the cathode remains substantially constant, as shown in Fig. 7.

† This is a permissible procedure since it has been shown in Section V that, even though some change in gas composition may exist as a function of distance from the mercury surface, the intensity variation of all the radiation to which the photo-cell is sensitive follows the same relative time variation in the absence of a jet.

solid curve is that for the average of the points. This curve, which closely approaches the dotted line near the mercury cathode, bends away at greater distances, in accordance with expectation, since no energy is being supplied to the jet column from external sources.

SECTION VII

Summary of Experimental Facts

The following experimental facts concerning the discharge in these tubes appear to have been established:

1. A jet of mercury vapor is ejected from a mercury cathode or anode during the time of discharge.
2. Jets are ejected in a direction always normal to the electrode surface (Section II).
3. Spectrograms show the jets are largely composed of mercury vapor (Section III).
4. At the cathode all parts of the jet move with approximately the same velocity (Section VI).
5. Jet velocity decreases with distance but the initial velocity of the cathode jet is $1.9\times(10^5)$ cm/sec., and that of the anode jet is $1.5\times(10^5)$ cm/sec. (Section IV).
6. These initial velocities are independent of current and gas pressure (Section IV).
7. The gas density in the positive column decreases little during the short discharge time (Section III).
8. The anode and cathode drop is 40 ± 5 volts[15] (the column drop is 300 volts/cm at 0.2 microsecond and 100 volts/cm at 1 microsecond).

Mechanism of Jet Production

These facts are incompatible with a "temperature" theory of jet production not only from a consideration of magnitudes but also because the jets are highly directive. If a "temperature" of the electrode were responsible one would expect a cosine distribution of direction of particles, leading to a diffused appearance of the jet at the electrodes. If the jets gain their energy thermally in a high energy region just in front of the electrodes, the velocity of the particles would be distributed at random and the jet would not be directive at all.

The facts are equally incompatible with a high ionic mobility for the following reasons:

1. From the known order of gas density in the positive column (Section III), mobility would have to increase a hundredfold under these conditions to reach the velocities recorded.
2. It is found experimentally that substitution of an oscillatory current pulse for the unidirectional pulse produces a little change in jet front velocity and, therefore, reversal of the field has a negligible effect.
3. Positive ions in the cathode jet would be moving the wrong way.
4. The jet direction may be removed nearly 90° away from the direction of the column gradient.

Since the jets always leave the electrodes in a direction normal to the surface, it is reasonable to attribute their energy to the anode and cathode drop. The energy of the anode jet can be accounted for by positive ions crossing the anode drop. From the laws of conservation of energy and momentum, the velocity of the anode jet, v_+ can be written

$$v_+=(M_+/M)[M/(M+M_h)][(2eV_a)/m]^{\frac{1}{2}}, \quad (2)$$

where M_+ is the effective mass of mercury atoms per unit time that cross the entire anode drop as positive ions, M is the mass of mercury atoms in the jet, M_h is the effective mass of hydrogen involved per unit time, e is the electronic charge, V_a the anode drop, and m the mass of the mercury atom. For the ratio of M_+/M to be as large as unity, every mercury atom vaporized from the anode would necessarily be immediately ionized by collision with an incoming electron. Since a large fraction of atoms may not be ionized until an appreciable portion of the anode drop has been crossed and others never ionized at all, M_+/M is considerably less than one. It is also evident that $M/(M+M_h)$, the fraction due to the mass reaction of the hydrogen, is less than unity. In any event, since V_a must be at least as great as the ionization potential of mercury (10.4 volts), substitution of this value in Eq. (2) shows that $(M_+/M)[M/(M+M_h)]=f$ must be less than 0.5, for otherwise v_+ becomes larger than the measured value ($1.5\times(10^5)$ cm/sec.).

Using the accommodation coefficient theory of cathode jet production,[5] the velocity of the cathode jet, v_-, may be written similarly:

$$v_-=(M_+/M)[M/(M+M_h)]\times[2eV_c(1-\alpha)/m]^{\frac{1}{2}}, \quad (3)$$

where M_+ is the effective mass of mercury atoms which are positively ionized at the anode end of the cathode drop and escape into the jet by rebounding from the cathode, retaining an average fraction of their incident energy of $(1-\alpha)$. V_c is the cathode drop and M, M_h, and e are the same as before. Both fractions M_+/M and $M/(M+M_h)$ are again certainly less than unity for exactly the same reasons. In fact, there is no reason to expect either to be greatly different numerically from those obtained with the anode jet. Since $V_a+V_c=40$ volts[15], V_c cannot be greater than 30 volts. The value of α for positive mercury ions on mercury has not been determined. From consideration of known values of α it is, however, unreasonable to assume that the average value of α in this case is less than 0.90 and it is probably very close to the 0.93 measured by Arnot and Milligan[19] for 50-volt positive mercury ions on tungsten. Substituting the value of $V_c=30$ volts and $\alpha=0.90$ in Eq. (3), one obtains

$$v_-=1.7\ (10^5)\,f\ \text{cm/sec.}$$

Using the maximum value of f obtained from the anode jet (0.5) as an approximation, $v_-=0.85\times 10^5$ cm/sec. which is approximately half the velocity or a quarter of the energy required.

The same conclusion is reached by use of the measured loss of mercury from the cathode with a current pulse of 200 amperes for 2 microseconds and a repetition rate of 300 per second. This value, corrected for the gross evaporation at the mercury surface, is 17 (10^{-4}) grams/coulomb. This measurement was made by simply weighing a small cup of mercury before and after sparking in hydrogen for one hour and making a small correction for vaporization at the mercury surface. This correction was made by measuring the temperature of the mercury surface with a thermocouple during sparking and repeating the experiment with the mercury surface held at the same temperature by means of a heating coil instead of sparks. Since it has been shown in Section VI that ejection of mercury from the cathode spot stops abruptly with the current, the result obtained in this way should be reasonably accurate. Tonks[20] obtained 2.5 (10^{-4}) grams/coulomb by extrapolating to zero current using an anchored spot and small currents. The value of 17 (10^{-4}) grams/coulomb may be too high, but it is believed nearer the true value for unanchored spots and these high current densities than the value obtained by Tonks.

Using a mercury loss of 17 (10^{-4}) grams/coulomb of the cathode, it is easily shown that $M_+/M=1.2(i_+/i)$ where i_+/i is the ratio of the positive ion currents to the total currents at the cathode. Also, using this value of mercury loss $M/(M+M_h)\approx 0.90$. Therefore, if $i_+/i=\frac{1}{2}$, which is certainly as large as one would like to assume, $f=0.54$ and $v_-=0.9\ (10^5)$ cm/sec. (maximum).

It therefore appears necessary to assume that in addition to positive ions striking the cathode and rebounding as neutral atoms, a large number of other particles leave the cathode drop with higher energies. This could be true if many mercury atoms leave the cathode as negative ions. That negative mercury ions can be produced at the cathode has been shown by Arnot and Milligan.[19] Under the conditions of their experiments, however, the probability of production of negative mercury atoms was extremely small. If it is assumed that this small probability is greatly increased due to the high field at the cathode surface under these conditions, the cathode jet velocity can easily be accounted for, since Eq. (3) can be rewritten as

$$\begin{aligned} v_- = {} & (M_+/M)[M/(M+M_h)] \\ & \times[2eV_c(1-\alpha)/m]^{\frac{1}{2}}+(M_-/M)[M/(M+M_h)] \\ & \times[2e(V_c-\phi)/m]^{\frac{1}{2}}, \quad (4) \end{aligned}$$

where M_- is the mass of negative ions and ϕ is the effective work function for a negative ion. Using the more probable values of $V_c=22$ volts, $\alpha-0.93$, and $\phi\approx 4$ volts, when $M_+/M=0.40$ and $M_-/M=0.36$, $V=1.9\ (10^5)$ cm/sec., and the measured velocity of the cathode jet is attained.

If this theory is correct one would expect the cathode jet to carry a high net negative charge and the anode jet to carry a high net positive charge. This condition appears to exist as shown by the direction in which the jets are bent in a magnetic field. In a field of approximately 2000 gauss it is observed that both the anode and cathode jets are bent out of the positive column in opposite directions corresponding to the net charges expected. The anode jet is bent some-

[19] F. L. Arnot and J. C. Milligan, Proc. Roy. Soc. **A156**, 538 (1936).
[20] L. Tonks, Phys. Rev. **54**, 634 (1938).

what less than the cathode jet, which has a mean radius of curvature of about a centimeter, implying a net charge to mass ratio for the jet of about -40 e.m.u./gram.

SECTION IX

Conclusion

The existence of mercury vapor jets in sparks, which have been shown to be arcs of short duration, appears to have been proven beyond reasonable doubt. The velocity of the jets is such that neither the positive nor negative ions in the arc column can reach the electrodes since the ion velocities are considerably less than the jet velocities and are oppositely directed. From consideration of the literature of vapor velocities in both sparks and arcs, it is certain that these vapor jets occur with a wide variety of electrode materials. An obvious test of the theory of the mechanism of jet production would be to obtain the initial jet velocity as a function of the atomic weight of the electrode material. Since the program of peacetime research prevents further work in this organization, it is hoped that the study will be carried on elsewhere.

It is a pleasure to acknowledge the advice and the assistance of many persons in this organization, particularly Drs. F. S. Goucher, W. Shockley, J. Bardeen, and C. H. Townes.

PHYSICAL REVIEW VOLUME 73, NUMBER 8 APRIL 15, 1948

On the Calculation of Self-Energies in Quantum Theory by Analytic Continuation

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(Received July 23, 1947)

Riesz's method of solving hyperbolic differential equations by analytic continuation has been used by Gustafson to eliminate infinities in quantum theory. Treating the one-electron case, he found finite values of the self-energy integrals in the second approximation, also for those integrals for which the λ-limiting process fails (without the further assumption of negative-energy photons). In the present paper it is shown that the general result of Gustafson's procedure is to remove all divergences normally appearing in self-energy expressions, except logarithmic divergences. Thus the total self-energy of the electron, to the second approximation, is found to be zero on the one-electron theory if calculated by this method, whereas in the hole theory the logarithmically divergent expression of Weisskopf is retained. A proposal by Pauli to alter the commutation rules in a certain way gives essentially the same results.

I. INTRODUCTION

We are going to investigate the general effect of evaluating self-energies in quantum theory by the method used by Gustafson in the case of the second approximation.[4–8*] Before entering on the problem we set down a few notations to be employed in the following.

A point in space-time is denoted by $x=x^\nu$ ($\nu=0, 1, 2, 3$; $x^0=ct$), and a metrical tensor with $-g_{00}=g_{11}=g_{22}=g_{33}=1$ is used. The length $r(x)$ of a vector x is defined by

$$-r(\mathbf{x})^2=g_{\mu\nu}x^\mu x^\nu=x_\nu x^\nu=(\mathbf{x},\mathbf{x}),$$

so that r^2 is positive for a time-like vector. Further, we write $\square$ for the wave operator $-\partial^2/\partial x^\nu \partial x_\nu$, and $\vec{x}$ for a vector (x^1, x^2, x^3) in ordinary space. Then $(x,y)=x_\nu y^\nu=\vec{x}\vec{y}-x^0y^0$.

Now, to form an expression for the self-energy of an electron or of a nucleon, one may start with the equations giving the interaction with the electromagnetic field and the meson field, respectively. Here we use the formulation of the theory in which the dynamic variables are operators satisfying field equations analogous to those of the classical theory. The specific state of the system is then characterized by a normalized and time-independent state vector C. A variable represented by an operator F has the expectation value $\langle F\rangle=C^*FC$.

* Numbered references will be found at the end of the text.

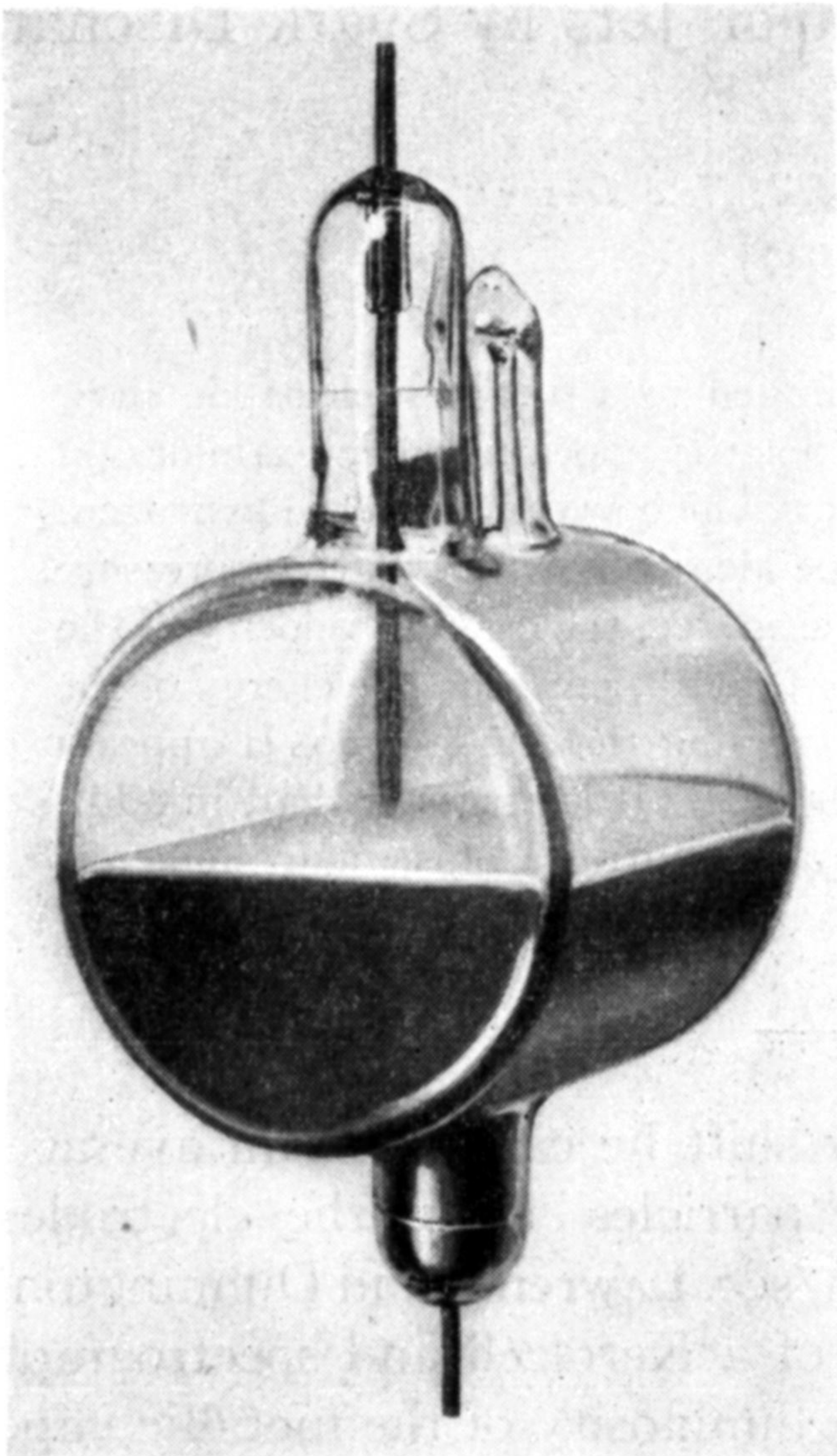

FIG. 1. Spark gap tube provided with flat glass window.

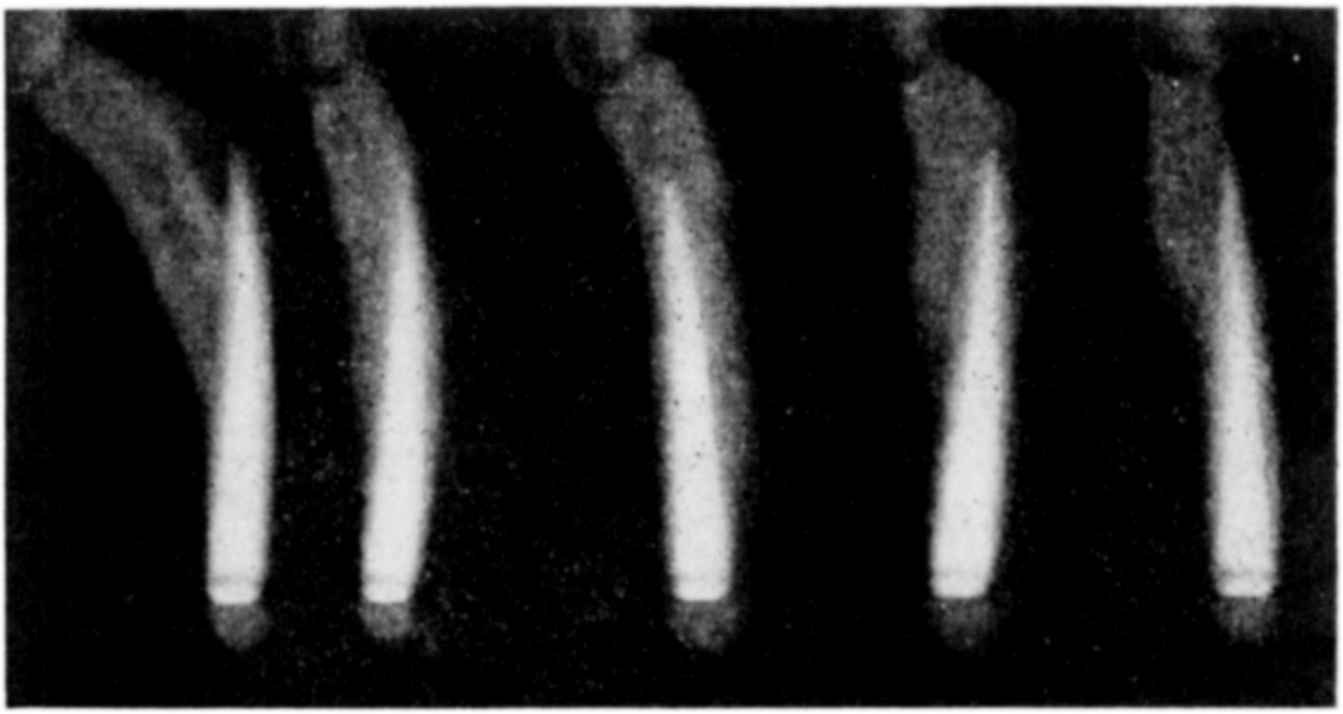

FIG. 2. Photograph of five individual sparks showing cathode mercury vapor jets. The mercury pool (cathode) is at the bottom. (Retouched.)

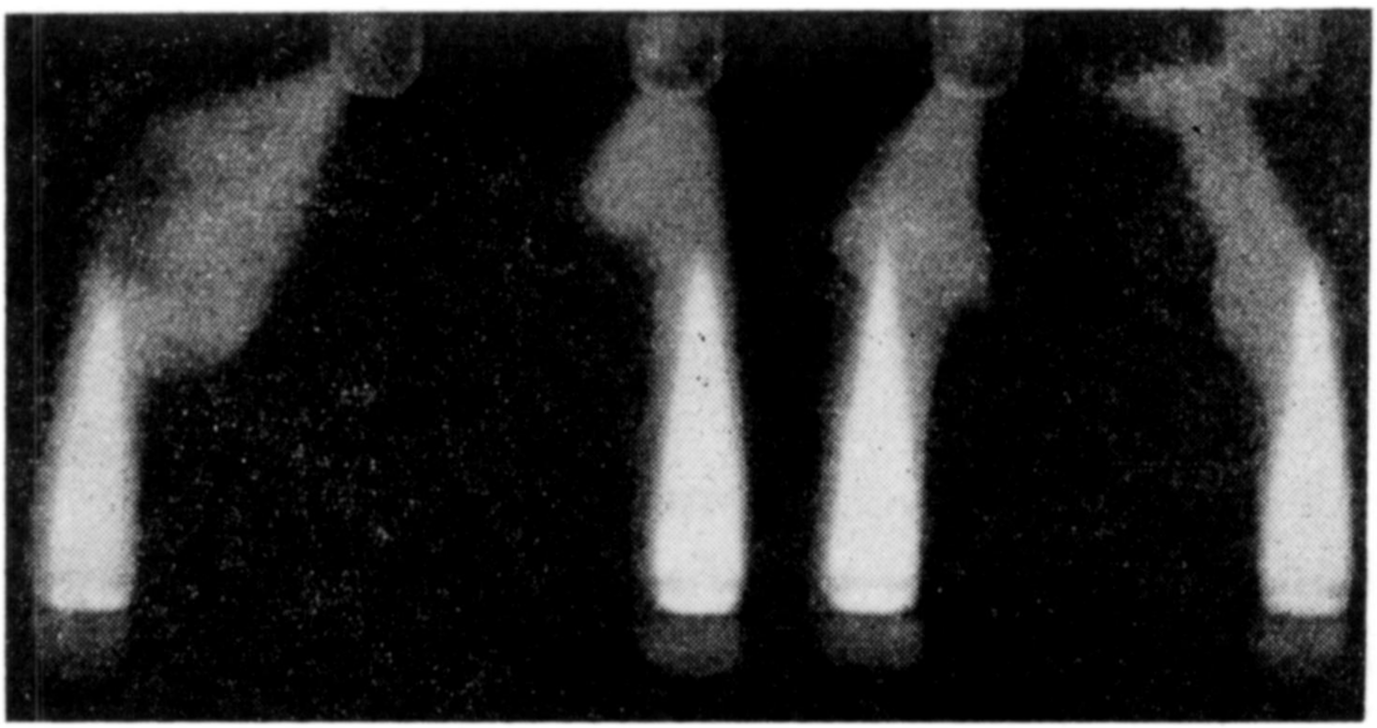

FIG. 3. Photograph of four individual sparks showing anode mercury vapor jets. The mercury pool (anode) is at the bottom. (Retouched.)

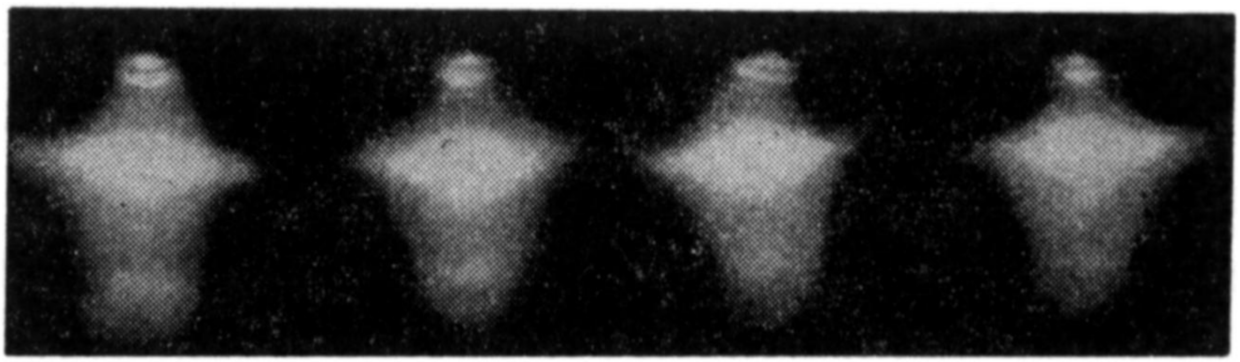

FIG. 4. Photograph of four individual sparks showing the effect of mercury on both electrodes. The mercury pool (anode) is at the bottom. (Retouched.)

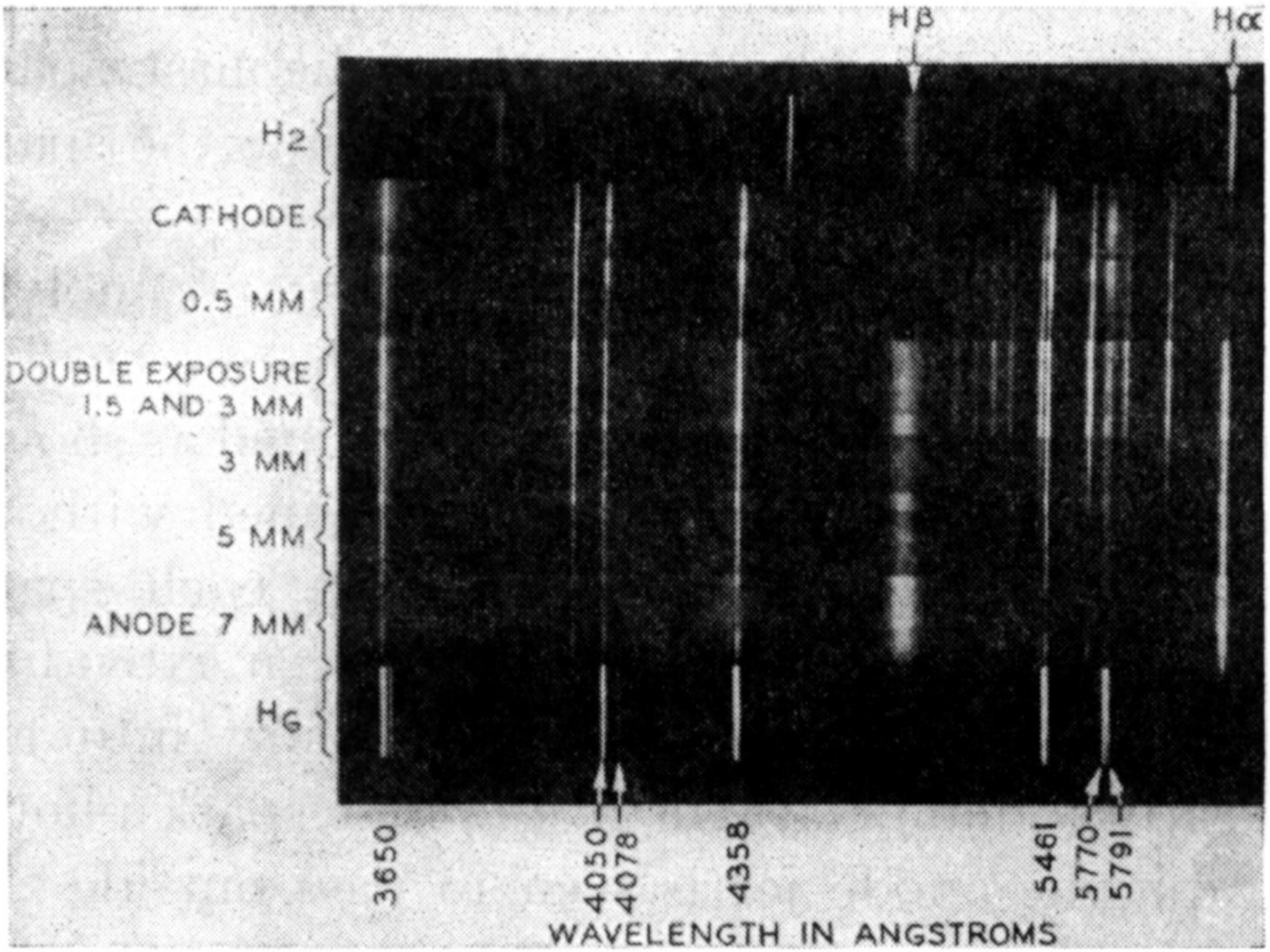

FIG. 5. Spark spectra of the light originating at progressively greater distances from a mercury cathode. At the extreme top and bottom are comparison spectra of hydrogen and mercury.