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The Penetration of Gamma-Radiation through Thick Layers

II. Plane Geometry, Iron, and Lead

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The method for considering the multiple scattering of gamma-rays given in Part I is extended to include the photoelectric effect and pair formation. Numerical calculations are made for the penetration of a monochromatic beam of gamma-rays through targets of either iron or lead. The thicknesses required to reduce the energy flux of factors of 2, 10, 100, 1000, and 10^6 are tabulated for gamma-rays of 1, 3, and 5 Mev passing through air, water, concrete, iron, or lead; and the corresponding absorbtion coefficients are given. The distinction between roentgen per unit time and energy flux is considered and numerical tables are given for the decrease in dosage with increasing target thickness. Rough experimental agreement is obtained with the rough experimental data now available. The need for careful measurements of the absorbtion of monochromatic gamma-rays passing through very thick targets is pointed out.

IN the paper[1] which is Part I in this series, a method was developed for considering the multiple scattering of gamma-rays in passing through a thick target. For simplicity, only Klein-Nishina scattering was considered. In the present paper we extend this method to take into account the absorbtion of gamma-radiation by the photoelectric effect for low energies and pair formation for large energies. Actual numerical calculations of both the energy flux and the roentgens are made for the penetration of 1-, 3-, or 5-Mev gammas through either iron or lead. For convenience, a table is given (see Table II) for the thicknesses required to attenuate the original beam by 0.5, 0.1, 0.01, 0.001, or 10^{-6} for not only these metals but for air, water, and concrete in which the pair formation and photoelectric effects may be ignored.

I. PAIR FORMATION

The cross section per electron for pair formation for energies less than $\alpha = 10m_0c^2$ can be estimated from the relation

$$\sigma_{pf}/r_0^2 = 0.2545(Z/137)[\alpha - 2.332]. \tag{1}$$

Here $r_0 = e^2/m_0c^2$, the classical radius for the electron, Z is the atomic number, and α is the gamma-ray energy in units of m_0c^2. This expression for the cross section gives good agreement with the numerical values cited by Heitler.[2]

II. PHOTOELECTRIC EFFECT

The cross section per electron for the photoelectric effect, σ_{pe}, can be simply represented provided that the gamma-ray energy is considerably larger than α_k, the K x-ray limit. According to Gray[3]

$$\frac{\sigma_{pe}}{r_0^2} = 3.45\times 10^{-6}\frac{Z^3}{\alpha^3}(1+0.008Z)\times\left(1-\frac{\alpha_k}{4\alpha}-\frac{\alpha_k^2}{1.21\alpha}\right). \tag{2}$$

For iron, $\alpha_k = 0.0140$; for lead, $\alpha_k = 0.173$. Thus for

$$\text{iron:}\quad \sigma_{pe}/r_0^2 = (0.0733/\alpha^3)[1-(0.00365/\alpha)]; \tag{3}$$

$$\text{lead:}\quad \sigma_{pe}/r_0^2 = (3.151/\alpha^3)[1-(0.0678/\alpha)]. \tag{4}$$

III. INCLUSION OF PAIR FORMATION AND THE PHOTOELECTRIC EFFECT IN MULTIPLE SCATTERING EQUATIONS

The result of both pair formation and the photoelectric effect is to absorb gamma-rays but

[1] J. O. Hirschfelder, J. L. Magee, and M. H. Hull, "Penetration of gamma-radiation through thick layers. I. Plane geometry, Klein-Nishina scattering," Phys. Rev. **73**, 851 (1948).

[2] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), 2nd edition, Appendix II, p. 259.

[3] Hulme, McDowgall, Buckingham, and Fowler, Proc. Roy. Soc. London **139**, 150 (1935). Formula for photoelectric cross section is given and reference to J. A. Gray, Proc. Roy. Soc. Canada **21**, 179 (1927).

not to scatter or to degrade those which are transmitted. The formulation of the equations for multiple scattering proceeds exactly as in the case where only the Klein-Nishina cross sections are considered. However, the intensity of each beam is further diminished by virtue of the absorbtion cross section.

Thus for a wide beam of gamma-rays impinging at right angles to a thick target we get the following set of equations:

$$dI_0/dX=-[\sigma_0+(\sigma_{pe})_0+(\sigma_{pf})_0]NI_0, \quad (5)$$

$$\frac{dI_1}{dX}=\frac{\sigma_0 NI_0}{1+\alpha_0(1-\bar{\mu}_0)}-\frac{[\sigma_1+(\sigma_{pe})_1+(\sigma_{pf})_1]NI_1}{\bar{\mu}_0}, \quad (6)$$

$$\frac{dI_n}{dX}=\left(\frac{\sigma_{n-1}NI_{n-1}}{\bar{\mu}_0\bar{\mu}_1\cdots\bar{\mu}_{n-2}[1+\alpha_{n-1}(1-\bar{\mu}_{n-1})]}\right)-\left(\frac{[\sigma_n+(\sigma_{pe})_n+(\sigma_{pf})_n]NI_n}{\bar{\mu}_0\bar{\mu}_1\cdots\bar{\mu}_{n-1}}\right). \quad (7)$$

Here, as in Part I, the subscript n refers to gamma-rays which have been scattered n times; N is the number of electrons per unit volume; σ_n are the Klein-Nishina cross sections corresponding to α_n, as before; $\bar{\mu}_n$ is the cosine of the effective angle of Klein-Nishina scattering, as given in Part I, and for the same values of $D=\sigma_0 NX$. The solution to these equations can be written

$$\frac{I_n}{A}=\sum_{j=0}^{n}K_{j,n}\times\exp(-[\sigma_j'+(\sigma_{pe})_j'+(\sigma_{pf})_j']D/\sigma_0), \quad (8)$$

Where the prime again has the meaning

$$\sigma_j'=\sigma_j/(\bar{\mu}_0\bar{\mu}_1\cdots\bar{\mu}_{j-1}), \quad (9)$$

$$(\sigma_{pe})_j'=(\sigma_{pe})_j/(\bar{\mu}_0\bar{\mu}_1\cdots\bar{\mu}_{j-1}), \quad (10)$$

$$(\sigma_{pf})_j'=(\sigma_{pf})_j/(\bar{\mu}_0\bar{\mu}_1\cdots\bar{\mu}_{j-1}). \quad (11)$$

The values of $K_{j,n}$ are determined as before from the boundary condition that when $X=0$

$$I_0=A, \quad (12)$$

$$I_1=I_2=\cdots=I_n=0, \quad (13)$$

or

$$K_{0,0}=1, \quad (14)$$

and for all other values of n,

$$K_{n,n}=-\sum_{j=0}^{n-1}K_{j,n-1}. \quad (15)$$

The non-diagonal $K_{j,n}$ are determined by the recursion formula:

$$K_{j,n}=\frac{\sigma_{n-1}'K_{j,n-1}}{[\sigma_n'+(\sigma_{pe})_n'+(\sigma_{pf})_n'-\sigma_j'-(\sigma_{pe})_j'-(\sigma_{pf})_j'][1+\alpha_{n-1}(1-\bar{\mu}_{n-1})]}. \quad (16)$$

The method and the approximations are then very much the same as for the Klein-Nishina scattering alone.

IV. NUMERICAL CALCULATIONS

Numerical calculations were made for both iron and lead for values of $D=\sigma_0 NX$ equal to 1, 2, 5, 10, and 20. Notice that, because of the dependence of $\bar{\mu}_n$ on D, it is still desirable to express the unit of distance in Klein-Nishina mean free paths for the initial gamma-rays. The results are shown in Table I. The energies, α_n, are the same as for the Klein-Nishina scattering and are given in Table V of the previous paper, Part I. The total intensity of gamma-rays which have passed through either iron or lead targets may be summarized by the following equations which were obtained by curve fitting the information given in Table I.

Iron:

$$\alpha=2,\quad I/A=(1+0.2206X+0.00435X^2)e^{-0.462X}, \quad (17)$$

$$\alpha=6,\quad I/A=(1+0.1000X+0.00123X^2)e^{-0.281X}, \quad (18)$$

$$\alpha=10,\quad I/A=(1+0.0635X+0.00123X^2)e^{-0.245X}. \quad (19)$$

Lead:

$$\alpha=2,\quad I/A=[1.3222+0.0786X-(0.4027/\{1.25+0.565X\})]e^{-0.647X}, \quad (20)$$

$$\alpha=6,\quad I/A=(1+0.1159X+0.00264X^2)e^{-0.430X}, \quad (21)$$

TABLE I.

(A) Iron

	1 Mev	$\alpha=2$		$(X=2.17D$ cm)	
X(cm)	2.17	4.34	10.85	21.70	43.40
D	1	2	5	10	20
R/R_0	0.5597	0.2807	0.02671	3.58×10^{-4}	3.75×10^{-8}
I/A	0.5498	0.2737	0.02577	3.44×10^{-4}	3.59×10^{-8}
$-d/dD(I/A)$	0.4270	0.2382	0.02479	3.42×10^{-4}	3.60×10^{-8}
I_0/I	0.6668	0.4910	0.2570	0.1275	0.0535
I_1/I	0.2388	0.2997	0.2886	0.2121	0.1232
I_2/I	0.0699	0.1334	0.2121	0.2172	0.1686
I_3/I	0.0188	0.0510	0.1257	0.1731	0.1751
I_4/I	0.0046	0.0179	0.0654	0.1182	0.1527
I_5/I	0.0010	0.0055	0.0309	0.0725	0.1178
I_6/I		0.0015	0.0134	0.0409	0.0830
I_7/I			0.0053	0.0214	0.0543
I_8/I			0.0019	0.0104	0.0332
I_9/I				0.0047	0.0192
I_{10}/I				0.0020	0.0104
I_{11}/I					0.0053
I_{12}/I					0.0025
I_{13}/I					0.0011

	3 Mev	$\alpha=6$		$(X=4.00D$ cm)	
X(cm)	4.00	8.00	20.00	40.00	80.00
D	1	2	5	10	20
R/R_0	0.5105	0.2309	0.01591	12.12×10^{-5}	3.87×10^{-9}
I/A	0.4618	0.1985	0.01270	9.23×10^{-5}	2.85×10^{-9}
$-d/dD(I/A)$	0.4129	0.1935	0.01365	1.026×10^{-4}	3.20×10^{-9}
I_0/I	0.7036	0.5319	0.2853	0.1422	0.0605
I_1/I	0.2177	0.2881	0.2963	0.2254	0.1349
I_2/I	0.0570	0.1129	0.1933	0.2093	0.1689
I_3/I	0.0161	0.0430	0.1118	0.1586	0.1646
I_4/I	0.0044	0.0159	0.0608	0.1083	0.1415
I_5/I	0.0011	0.0056	0.0309	0.0698	0.1101
I_6/I		0.0019	0.0148	0.0422	0.0801
I_7/I			0.0067	0.0242	0.0558
I_8/I				0.0132	0.0370
I_9/I				0.0069	0.0236
I_{10}/I					0.0144
I_{11}/I					0.0034

	5 Mev	$\alpha=10$		$(X=5.56D$ cm)	
X(cm)	5.56	11.12	27.80	55.60	111.20
D	1	2	5	10	20
R/R_0	0.4165	0.1543	0.00585	1.592×10^{-5}	5.77×10^{-11}
I/A	0.3587	0.1223	0.00411	1.030×10^{-5}	3.52×10^{-11}
$-d/dD(I/A)$	0.4045	0.1465	0.00533	1.379×10^{-5}	4.77×10^{-11}
I_0/I	0.7153	0.5383	0.2709	0.1201	0.0435
I_1/I	0.2110	0.2870	0.2972	0.2130	0.1140
I_2/I	0.0533	0.1116	0.2027	0.2167	0.1640
I_3/I	0.0149	0.0416	0.1132	0.1660	0.1678
I_4/I	0.0042	0.0158	0.0612	0.1134	0.1455
I_5/I	0.0011	0.0057	0.0318	0.0741	0.1155
I_6/I			0.0157	0.0459	0.0873
I_7/I			0.0074	0.0271	0.0626
I_8/I				0.0153	0.0428
I_9/I				0.0083	0.0282
I_{10}/I					0.0179
I_{11}/I					0.0109

(B) Lead

	1 Mev	$\alpha=2$		$(X=1.77D$ cm)	
X(cm)	1.77	3.54	8.85	17.70	35.4
D	1	2	5	10	20
R/R_0	0.4127	0.1522	0.00659	3.00×10^{-5}	4.87×10^{-10}
I/A	0.4076	0.1495	0.00642	2.90×10^{-5}	4.69×10^{-10}
$-d/dD(I/A)$	0.4381	0.1671	0.00733	3.32×10^{-5}	5.37×10^{-10}
I_0/I	0.7810	0.6779	0.5092	0.3686	0.2436
I_1/I	0.1937	0.2649	0.3444	0.3677	0.3419
I_2/I	0.0236	0.0511	0.1187	0.1882	0.2486
I_3/I	0.0015	0.0055	0.0244	0.0604	0.1167
I_4/I	0.0001	0.0004	0.0033	0.0130	0.0383
I_5/I				0.0020	0.0091
I_6/I				0.0002	0.0016
I_7/I					0.0002

	3 Mev	$\alpha=6$		$(X=3.26D$ cm)	
X(cm)	3.26	6.52	16.30	32.60	65.20
D	1	2	5	10	20
R/R_0	0.3806	0.1298	0.00397	8.13×10^{-6}	1.792×10^{-11}
I/A	0.3464	0.1126	0.00321	$6{:}30\times10^{-6}$	1.343×10^{-11}
$-d/dD(I/A)$	0.4104	0.1417	0.00431	8.67×10^{-6}	1.874×10^{-11}
I_0/I	0.7114	0.5393	0.2827	0.1312	0.0508
I_1/I	0.2312	0.3226	0.3610	0.2832	0.1694
I_2/I	0.0494	0.1088	0.2254	0.2825	0.2504
I_3/I	0.0073	0.0251	0.0958	0.1816	0.2360
I_4/I	0.0006	0.0036	0.0286	0.0842	0.1630
I_5/I		0.0003	0.0058	0.0288	0.0844
I_6/I			0.0008	0.0071	0.0334
I_7/I			0.0001	0.0012	0.0101
I_8/I				0.0002	0.0024
I_9/I					0.0004

	5 Mev	$\alpha=10$		$(X=4.53D$ cm)	
X(cm)	4.53	9.06	22.65	45.30	90.60
D	1	2	5	10	20
R/R_0	0.2073	0.04208	2.771×10^{-4}	4.123×10^{-8}	3.164×10^{-16}
I/A	0.1753	0.03197	1.831×10^{-4}	2.535×10^{-8}	1.867×10^{-16}
$-d/dD(I/A)$	0.3170	0.05897	3.519×10^{-4}	5.023×10^{-8}	3.776×10^{-16}
I_0/I	0.6721	0.4342	0.1240	0.0203	0.0014
I_1/I	0.2510	0.3541	0.3217	0.1556	0.0435
I_2/I	0.0634	0.1555	0.3023	0.2816	0.1535
I_3/I	0.0121	0.0459	0.1653	0.2647	0.2392
I_4/I	0.0015	0.0092	0.0648	0.1657	0.2378
I_5/I	0.0001	0.0011	0.0179	0.0776	0.1709
I_6/I		0.0001	0.0033	0.0265	0.0953
I_7/I			0.0004	0.0065	0.0408
I_8/I				0.0011	0.0133
I_9/I				0.0002	0.0033
I_{10}/I					0.0006
I_{11}/I					0.0001

$$\alpha=10,\quad I/A=(1+0.0405X+0.00461X^2)e^{-0.442X}. \tag{22}$$

For many practical purposes it is desirable to know the thickness of various types of targets which will attenuate gamma-rays to a given fraction of their initial intensity. As a result of the calculations given in this paper and its predecessor, we can estimate these thicknesses either for substances which have only Klein-Nishina cross sections such as air, water, or concrete, or iron or lead for which we have made separate calculations. In any case,

$$D=0.02392\rho(2Z/M)(\sigma_0/r_0^2)X. \tag{23}$$

For mixtures of elements, the weight average of $2Z/M$ is to be taken over all the elements. For air at N.T.P. (0°C, 1 atmos.), $\rho=0.001293$ g/cm^3 and $(2Z/M)=1$, so that

$$D=3.093\times10^{-5}(\sigma_0/r_0^2)X. \tag{24}$$

For water at 2°C, $\rho=1$ g/cm^3 and $2Z/M=(16+4)/18.016$, so that

$$D=0.02655(\sigma_0/r_0^2)X. \tag{25}$$

For concrete,[4] 1–2–4 mix, $\rho=2.29$ g/cm^3 and

[4] O. W. Eshbach, *Handbook of Engineering Fundamentals* (John Wiley and Sons, Inc., New York, 1936).

$(2Z/M)=1$, so that

$$D=0.0548(\sigma_0/r_0^2)X. \tag{26}$$

For iron, $\rho=7.86$ g/cm^3 and $(2Z/M)=0.9312$, so that

$$D=0.1750(\sigma_0/r_0^2)X. \tag{27}$$

For lead, $\rho=11.34$ g/cm^3 and $(2Z/M)=0.7915$, so that

$$D=0.2147(\sigma_0/r_0^2)X. \tag{28}$$

Thus we obtain the results shown in Table II. Here by "Klein-Nishina" is meant any substance for which pair formation and photoelectric effect are negligible.

In most treatments of gamma-ray scattering and absorbtion, the decrease in energy flux as a function of target thickness is expressed in terms of "absorbtion coefficients," a, which are defined by the relation:

$$I/A=\exp(-aX). \tag{29}$$

In Table III, the mass absorption coefficients, a/ρ, are tabulated. The column labeled "concrete" was calculated for pure Klein-Nishina scattering with $(2Z/M)=1$. For any other material for which Klein-Nishina scattering is valid

$$a/\rho=(2Z/M)(a/\rho)_{\text{concrete}} \tag{30}$$

The meaning of our absorption coefficients should be emphasized. They correspond to the total energy which has been absorbed from the beam of gamma-rays and should not be confused with the coefficients for absorption plus scattering. Our coefficients could be experimentally determined by allowing the rays to enter the counting chamber from all directions; whereas the coefficients for absorbtion plus scattering only consider the flux from the unscattered beam and require a fine slit system to insure that only the undeflected rays can enter the counter.

V. ROENTGEN

The physiological effects of gamma-rays are determined, not by the energy flux, I, but by the ionization which they produce in the body tissues. This is proportional to the dosage in roentgen units. A roentgen is defined as that amount of radiation which will produce 2.08 $\times 10^9$ ion pairs per cm^3 of air under N.T.P. Since both air and the body tissues are composed of elements of small atomic number, in the energy range considered here only Klein-Nishina scattering is important and the ionization produced in the body tissues differs from the ionization produced in air only by the ratio of

TABLE II. Thickness in cm required to reduce intensity of incident beam by stated factors.

	Klein-Nishina alone					
	I/A	0.5	0.1	0.01	0.001	10^{-6}
	D	1.17	3.40	6.26	8.96	16.68
$\alpha=2$	Air (N.T.P.)	1.44×10^4	4.18×10^4	7.70×10^4	1.10×10^5	2.05×10^5
(1 Mev)	Water (2°C)	16.7	48.6	89.5	128.	239.
	Concrete (1–2–4 mix)	8.1	23.6	43.4	62.2	116.
	D	1.05	3.15	5.90	8.51	16.07
$\alpha=6$	Air (N.T.P.)	2.37×10^4	7.12×10^4	1.33×10^5	1.92×10^5	3.63×10^5
(3 Mev)	Water (2°C)	27.6	82.8	155.	224.	423.
	Concrete (1–2–4 mix)	13.4	40.3	75.5	109.	206.
	D	0.98	3.01	5.71	8.30	15.79
$\alpha=10$	Air (N.T.P.)	3.08×10^4	9.45×10^4	1.79×10^5	2.61×10^5	4.96×10^5
(5 Mev)	Water (2°C)	35.9	110.	209.	304.	578.
	Concrete (1–2–4 mix)	17.3	53.3	101.	147.	279.
	Klein-Nishina plus pair production and photoelectric effect					
	I/A	0.5	0.1	0.01	0.001	10^{-6}
$\alpha=2$	Iron	2.49	7.22	13.31	19.08	35.63
(1 Mev)	Lead	1.40	4.23	8.11	11.91	23.12
$\alpha=6$	Iron	3.61	11.11	21.00	30.46	57.68
(3 Mev)	Lead	2.15	6.88	13.27	19.45	37.26
$\alpha=10)$	Iron	3.77	12.14	23.54	34.52	66.05
(5 Mev)	Lead	1.76	5.98	12.19	18.35	36.13

TABLE III. Mass absorbtion coefficients.

		a/ρ cm^2/g				
	I/A	0.5	0.1	0.01	0.001	10^{-6}
	Water	0.0415	0.0474	0.0514	0.0540	0.0578
$\alpha=2$	Concrete	0.0374	0.0426	0.0463	0.0485	0.0520
(1 Mev)	Iron	0.0354	0.0406	0.0440	0.0460	0.0494
	Lead	0.0437	0.0480	0.0501	0.0512	0.0527
	Water	0.0251	0.0278	0.0297	0.0308	0.0327
$\alpha=6$	Concrete	0.0226	0.0249	0.0266	0.0277	0.0293
(3 Mev)	Iron	0.0244	0.0263	0.0279	0.0289	0.0305
	Lead	0.0284	0.0295	0.0306	0.0313	0.0327
	Water	0.0193	0.0209	0.0220	0.0227	0.0239
$\alpha=10$	Concrete	0.0175	0.0189	0.0199	0.0205	0.0216
(5 Mev)	Iron	0.0234	0.0242	0.0249	0.0254	0.0266
	Lead	0.0348	0.0340	0.0333	0.0332	0.0337

the electron densities in the two cases. This is why the roentgen is a satisfactory unit for physiological dosage. Unfortunately, Geiger counters are not constructed to measure roentgens, but rather the ionization produced in metal foils. However, they are calibrated in roentgens with different calibration curves corresponding to different energies of incident gamma-rays.

The energy absorbed in 1 cm^3 of air N.T.P. is equal to $3.69\times10^{-5}I(\sigma_a/r_0^2)$. Here 3.69×10^{-5} is the product of the number of electrons per cm^3 in air times the square of the classical electron radius r_0, in cm; I is the energy flux in Mev per cm^2; and (σ_a/r_0^2) is the Klein-Nishina cross section for energy absorption per electron in units of r_0^2. A formula for σ_a is given in Rutherford, Chadwick, and Ellis,[5] and its numerical value is given in Table IV. The number of roentgens is simply proportional to the energy absorbed in 1 cm^3 of air. Since 1 Mev of energy absorbed produces $10^6/33.2$ ion pairs, and 1 roentgen unit corresponds to 2.08×10^9 ion pairs, the number of roentgens per unit time, R, is given by

$$R=5.34\times10^{-10}I(\sigma_a/r_0^2), \tag{31}$$

and for a beam of gamma-rays having many energy components

$$R=5.34\times10^{-10}\sum_n I_n\times(\sigma_a/r_0^2)_n. \tag{32}$$

Since (σ_a/r_0^2) is not independent of energy, the ratio of R/R_0, where R_0 is the roentgens per unit time of the initial gamma rays before passing through the target, is not equal to I/A.

[5] Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (The Macmillan Company, New York, 1930), p. 464.

TABLE IV. σ_a/r_0^2.

α	σ_a/r_0^2	α	σ_a/r_0^2
0.25	0.9548	1.5	1.2150
0.30	1.0217	2.0	1.1659
0.35	1.0729	2.5	1.1134
0.40	1.1132	3.0	1.0625
0.50	1.1695	4.0	0.9711
0.60	1.2035	5.0	0.8941
0.75	1.2313	6.0	0.8289
1.00	1.2431	8.0	0.7258
		10.0	0.6478

Numerical values are given for R/R_0 for iron and lead in Table I. Similar calculations were made for substances in which only the Klein-Nishina scattering occurred. The ratio

$$(R/R_0)/(I/A)$$

is almost unity for 1-Mev gammas, and becomes large of the order of 1.52 for 5-Mev gammas which have plowed through a large thickness of target. This ratio can be expressed in the form:

$$R/R_0=(I/A)\exp(bX)=\exp[(b-a)X]. \tag{33}$$

Here the calculated values of b have been curve-fitted in the form:

$$b=c/(d+X). \tag{34}$$

The values of c and d are given in Table V for water, concrete, iron, and lead. For any substances for which only the Klein-Nishina scattering occurs

$$c=(c)_{\text{concrete}},\quad d=\frac{d_{\text{concrete}}}{(\rho/2.29)(2Z/M)}. \tag{35}$$

VI. CONCLUSIONS

There are no adequate experimental data with which to make a quantitative comparison with the theoretical results presented here. Most of the experimental work on the scattering and absorption of gamma-rays has been designed to study single rather than multiple scatterings, and when a target of many mean free paths has been used, a slit system has been incorporated to insure "perfect geometry" and eliminate the scattered rays. The review articles such as Gentner's[6] and Hall's[7] do not even mention the

[6] W. Gentner, Physik. Zeits. **38**, 836 (1937).
[7] Harvey Hall, Rev. Mod. Phys. **8**, 358 (1936).

problems of multiple scattering. Kohlrausch[8] and the earlier workers were troubled by scattered radiation but their targets were not thick enough to furnish useful data for our purposes and their radiation was not monochromatic. Sizoo and Willemsen[9] studied the absorption and single scattering of radium gamma-rays through thin targets of lead and iron. They were able to make theoretical calculations agreeing perfectly with their experimental data by taking into account the exact spectral distribution of the primary beam and using the same cross sections for Klein-Nishina scattering, the photoelectric effect, and pair production which we use in the present report. Our theoretical treatment should be accurate for both the unscattered and the singly scattered beams since no approximations are made once the cross sections have been chosen. Thus the work of Sizoo and Willemsen guarantees that our absorption coefficients are valid for thin targets of less than two mean free paths.

The authors know of an unpublished experiment[10] in which the mass absorbtion coefficient for Ra$(B+C)$ gamma-rays passing through 7 cm of lead is 0.047. For 1-Mev gamma-rays we calculate 0.048; for 3-Mev gamma-rays we calculate 0.030. According to Roberts,[11] the spectral distribution of gamma-rays from a Ra$(B+C)$ source is very complex with the energies ranging from 0.5 Mev to 3.0 Mev so that it is difficult to make any sort of comparison. Furthermore, 7 cm of lead correspond to a reduction of the energy flux by a factor of only ten; i.e., $I/A=0.1$, and for this attenuation, 90 percent of the flux is in the form of either the unscattered or the singly scattered rays. Thus, if an accurate comparison were possible it would only check the approximations which we make in estimating the remaining 10 percent of the rays which are multiple scattered. Thus it is clear that very thick targets, such as 15 or more cm of lead, are required to provide an adequate check of our results.

TABLE V. Ratio of roentgens per unit time to energy flux.

	$(R/R_0)/(I/A)=\exp(cX/d+X)$	c	d
$\alpha=2$	Water	0.0345	11.20
	Concrete	0.0345	5.43
	Iron	0.0467	3.64
	Lead	0.0437	5.06
$\alpha=6$	Water	0.318	61.5
	Concrete	0.318	29.80
	Iron	0.340	9.99
	Lead	0.318	8.07
$\alpha=10$	Water	0.468	93.2
	Concrete	0.468	45.1
	Iron	0.556	15.50
	Lead	0.602	10.78

There is more than academic interest in both experimental and theoretical considerations of multiple scattering, since it is the total radiation which is physiologically important.

The authors regret that John L. Magee and M. H. Hull were unable to participate in this present report, although they had a great deal to do with the methods used here. The basic idea for the sequence of differential equations was suggested by John von Neumann. The calculations were carried out by Jean Patterson, Ruth Shoemaker, and Wallace Spaulding.

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[8] K. W. F. Kohlrausch, Akad. Wiss. Wien Ber. **126** (4), 441 (1917); 683 (1917); (2a) 705 (1917); (2a), 887 (1917).
[9] G. J. Sizoo and H. Willemsen, Physica **5**, 100 (1938).
[10] H. M. Parker, Clinton Laboratory Lecture Series (unclassified), Lecture V (May 1944).
[11] J. E. Roberts, Phil. Mag. **36**, 264 (1945).