Then the equations given above become

$$x=\frac{\lambda H_0}{(2\pi f)\omega}[1-\cos\omega s],$$

$$y=0,$$

$$z=\left(\frac{v_0/c}{\omega(1-v_0^2/c^2)^{\frac{1}{2}}}+\frac{\lambda^2H_0^2}{16\pi f\omega^2}\right)\omega s-\frac{\lambda^2H_0^2}{32\pi f\omega^2}\sin 2\omega s,$$

$$ct=\left(\frac{1}{\omega(1-v_0^2/c^2)^{\frac{1}{2}}}+\frac{\lambda^2H_0^2}{16\pi f\omega^2}\right)\omega s-\frac{\lambda^2H_0^2}{32\pi f\omega^2}\sin 2\omega s.$$

It is evident from these equations that the projection of the orbit in the x, z plane is a distortion of $x=\sin^2 z$ such that the particle stays one side of the z axis, returning to the z axis when z changes by

$$2\pi\left(\frac{v_0/c}{\omega(1-v_0^2/c^2)^{\frac{1}{2}}}+\frac{\lambda^2H_0^2}{16\pi f\omega^2}\right).$$

The maximum displacement from the z axis occurs when

$$s=\pi/\omega,$$

and is given by

$$x_{\max}=\frac{2\lambda H_0}{2\pi f\omega}.$$

Here the particle has maximum energy given by

$$m_0c^2V^4=\frac{m_0c^2}{(1-v_0^2/c^2)^{\frac{1}{2}}}+\frac{\lambda^2H_0^2}{8\pi f\omega}.$$

In case the function H has a phase angle α, that is,

$$H=H_0\cos[2\pi f(ct-z)+\alpha],$$

the orbit of the particle no longer returns to the z axis when $V_0^1=V_0^2=0$. The projection of the motion on the x axis has in addition to the motion described above a constant velocity proportional to $\sin\alpha$.

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The Threshold for the Positive Pre-Onset Burst Pulse Corona and the Production of Ionizing Photons in Air at Atmospheric Pressure

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AN analysis of the mechanism of the pre-onset burst pulse corona indicated that use might be made of the theory to throw some light on the photoelectric ionization processes responsible for various other breakdown mechanisms in gases. In view of the work currently being done on this subject, it is of importance to report the conclusions arrived at. The discussion will be limited to the positive point to plane corona in air at atmospheric pressure, fields X being expressed in volts per cm and pressure p in mm of Hg.

Currents from a corona point of radius r below the onset potential V_g of the intermittent Geiger regime consist of a field intensification of the negative ion currents produced in the volume of the corona gap by an external source. Field intensification can only begin when the potential of the point reaches a value V_f such that the ratio X_f/p at the point surfaces exceeds 90. At this field the negative O_2^- ions produced in lower field regions by external agencies begin to shed

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their electrons in impacts with neutral molecules. The current increases with potential above V_f as ionization by collision from further out in the gap becomes possible. The multiplication of the incoming electrons into an *electron avalanche* by the field may be calculated by the well-known Townsend function $\exp\int_r^{a+r} \alpha dx$. Here α is the first Townsend coefficient given by the Townsend relation $\alpha/p=f(X/p)$, Sanders values of $f(X/p)$ being here used. In the equation r is the point radius and a is the distance from the point surface at which the electron either becomes free or at which α in the rapidly declining field reaches negligible values. Experience has set the limit for a in the latter event at fields where $\alpha\sim 1$. Since the distance at which $X/p=90$ is less than that at which $\alpha=1$, the former condition will usually fix a. Where photo-ionization produces electrons in the gas near the point at which $\alpha\sim 1$, at $X/p=40$, the value of a set by $\alpha=1$ can be used as there then is little chance for electron attachment.

The $\exp\int_r^{a+r}\alpha dx$ electrons created in an avalanche are rapidly drawn to the point, and the equal number of positive ions drift away from the point relatively slowly. Accompanying the production of the electrons in an avalanche, there are produced $f\exp\int_r^{a+r}\alpha dx$ photons of all sorts capable of ionizing constituents of the gas photoelectrically. The quantity f is a fraction of great interest in this analysis. When the point potential reaches a value V_g such that the photons produced can, on the average, yield one new electron in the gas beyond a, then a new electron avalanche is created for each initial ion imposed by external sources. Thus a successive chain of avalanches can continue independently of the external source for each initial ion. This condition should mark the threshold of a self-sustaining corona. Because of the statistical character of the quantity $f\exp\int_r^{a+r} \alpha dx$ and the chance β that a photon will be produced beyond a, the succession is likely to be interrupted. It can further be interrupted by distortion of the field X_g by positive ion space charges left behind by the avalanches when these become great enough. In consequence, V_g marks the threshold of the intermittent *pre-onset burst pulse corona* initiating the Geiger counter regime. This regime was first investigated as a corona (as distinct from a counter) phenomenon by G. W. Trichel[1] and A. F. Kip[2] and has later been extensively investigated by G. L. Weissler[3] and recently by W. N. English.[4] While its statistically conditioned threshold is slightly uncertain, it can clearly be recognized by characteristic burst pulses of from 50–2000 μsec. on the oscillograph screen, an abrupt increase in current, and at very slightly higher potentials by the film of glow adhering closely to the point surface. At very slightly higher potentials (50–150 volts depending on r) and gap length the appearance of *pre-onset streamer pulses* on the oscillograph and the brush-like glow of a few mm length marks the onset of the streamer regime. At a potential V_0 some 100–500 volts higher, the Geiger counter regime ends in the onset of *steady self-sustaining burst pulse corona*.

It is the threshold V_g which is of current interest. Unfortunately, most onset studies have been made with hemispherically capped cylindrical points of around 0.05-cm diameter and plane cathodes with gaps from 1 to 5 cm in length. In these, that field X is a function of distance x from the point center is not accurately known. The one careful study, made by K. E. Fitzsimmons,[5] using a confocal paraboloid system where fields can be computed, was done under considerable difficulties in wartime when discussion and consultation were impossible. It appears that the threshold V_f for field intensified currents was mistaken by Fitzsimmons for the threshold V_g. Repetition of these studies for an exact evaluation of V_g lies well in the future. Accordingly, fairly reliable estimates of ionization were made using a hemispherically capped cylindrical point of $r=0.019$ cm and a plate distance of 3 cm having $V_g=5050$ volts. This estimate yields $\exp\int_r^{a+r} \alpha dx$ as being about 6×10^4 ions, with $a=0.04$ cm.

One can now set the threshold for the burst pulse onset as determined by the relation $\beta f\exp\int_r^{a+r} \alpha dx=1$ (1). Here β, the chance that a photon-electron is produced at a distance from the point surface in excess of a, may roughly be

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set as $\beta = 0.5 \exp(-\bar{a}\mu)$ (2). In this relation the factor 0.5 takes account of the half of the photons produced at the point surface which are absorbed by it. The quantity μ is the absorption coefficient of the gas for such photons and $\bar{a}$ is an average path of the photons in the hemispherical shell between r and $r+a$. It can be roughly computed as $\bar{a} = (a/\cos\bar{\theta})$, where $\bar{\theta}$ is the average angle of photon-emission with the normal to the point surface given by $\bar{\theta} = (\int_0^{\pi/2} \theta \sin\theta d\theta)/(\int_0^{\pi/2} \sin\theta d\theta) = 1$ radian. The rigorous calculation for β has been carried out by R. A. Wijsman and differs numerically little from the approximate value. Thus $\beta = 0.5 \exp(-1.86\mu a)$. With a set as 0.04 cm the value of μ is required. The wave-length corresponding to photo-ionization of N_2, whose ionization potential is the highest of the gases present, is about 785A. E. G. Schneider[6] gives values of μ in cm^{-1} at 760 mm for wave-lengths from about this value down to 1000A at which ionization is unlikely. The values range irregularly from 640 at 791.5A, 60 at 848A, 230 at 966A, 14 at 1000 to 12 at 1050.5A. A. M. Cravath[7] and Dechene[8] give μ as 10 and 6 cm^{-1}, respectively, for photons from corona that ionize air within 2–6 cm from the corona. H. Raether,[9] using more filtering, observes values of 2 cm^{-1} from sparks in air. Dechene observes a decrease in μ as the radiations are filtered by increasing thicknesses of air. For purposes of discussion one may use values of μ of 10 and 100 cm^{-1}. This yields values of β of 0.238 and 3.05×10^{-4} and of f of 7×10^{-5} and 5.5×10^{-2} from Eqs. (2) and (1) for a μ of 10 and 100 cm^{-1}, respectively. Cravath estimated the number of active photons per ion as 10^{-4} from his studies. Both values of f given above are well within reason. It must be concluded that photons with μ much greater than 100 cannot be active in *burst pulse onset* as f becomes too great.

It follows that if a gas can be ionized by photons of longer wave-lengths whose value of μ is of the order of 10 cm^{-1}, the value of f as required for the burst pulse corona threshold can be as low as 10^{-4}. In burst pulse corona almost all photoelectrons created beyond a have a chance of reaching some section of the point surface. In the case of the photon production for *streamer advance*, where the effective r about the positive space charge is much less and photoelectrons produced beyond an $a = 0.04$ cm may not reach the point, it follows that μ must be greater, requiring a larger f or else enhanced fields and a larger $\exp\int_r^{a+r} \alpha dx$ than for burst pulse onset. For a streamer progress in a plane parallel gap $\exp\int_r^{a+r} \alpha dx$ is of the order of 10^7. If μ be set as 100, then $f = 3 \times 10^{-4}$, which is reasonable. The appearance potential of *pre-onset streamers from the point in question* is 5175 volts. This increases the electron avalanche by at best a factor of the order of 2. In this case the field about the point, in contrast to a uniform field, is such that successive avalanches as well as spreading laterally over the point as in the burst pulse, can feed into the same spot. Local ion space charge fields can then enhance the field at the point surface so as to lead to a rapid increase of the $\exp\int_r^{a+r} \alpha dx$ at that spot. Thus the streamer propagating ionization and photon density with a larger μ and a reasonable f can cause streamer advance. From these considerations it is believed that one can narrow down the range of values of the absorption coefficients for secondary photo-ionizing mechanisms leading to air discharges to between 10 and 100 cm^{-1} and that the efficiency of active photon production f by the electrons must lie in the order of 10^{-4} to 10^{-3}.

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