would be 14.5 volts, since the energy of the ^2P state is 3.56 volts. It is therefore clear that the reaction (1) is energetically possible.

Reaction (2) is an electron-atom collision in which a metastable atom collides with a low velocity electron and gives up its potential energy to the electron in a collision of the second kind. This is the reverse of a collision of the first kind, one in which the kinetic energy of the electron is transferred to potential energy of the atom. Available evidence[3] indicates that the collisional cross section is very large for collisions of the first kind when the kinetic energy of the electron is only slightly above that necessary to raise the atom to an excited state, and the electron is therefore able to give up practically all of its kinetic energy. For electron energies higher than this the collisional cross section falls off rapidly. By the principle of microscopic reversibility a large collisional cross section should be expected for the reverse process, and here also the collisional cross section should fall off rapidly as the velocity of the impinging electron increases.

[3] I. S. Bowen, Rev. Mod. Phys. 8, 55 (1936).

If it is assumed that these are the two principal reactions which occur in *all* afterglows, it is possible for the present, to account (at least qualitatively) for the principal features of the afterglow spectra. Collisions of the second kind postulated in reaction (2) will account for the delayed maximum in the intensity of forbidden radiations. They will also provide slow electrons which can excite the first-positive bands of nitrogen by collisions with metastable molecules ($A^3\Sigma$) and highly vibrating molecules in the $X'\Sigma$ state. The latter may be produced by three body collisions in which two normal nitrogen atoms are involved. On this hypothesis one would expect a modified discharge-like spectrum in the early part of the afterglow, and this is effectively what one observes in the auroral afterglow and, incidentally, also in the spectra of polar auroras. Superposed on this may be contributions from such reactions as those of Mitra, but these do not play the principal roles. I plan, in designing future experiments and also in applying the results of laboratory studies to the physics of the upper atmosphere, to test these hypotheses as carefully as possible.

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The Microwave Spark[1]

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Electrical breakdown of air was studied at 3 microwave frequencies: wave-lengths of 1.25 cm, 3 cm, and 10 cm. Breakdown field strength was determined as a function of pressure, gap height, intensity of initial ionization, width of a single microwave pulse which sufficed for breakdown, and the pulse repetition frequency.

It is found that the field strength for breakdown is strongly dependent upon the width of the microwave pulse, the intensity of initial ionization, and, of course, upon gap height and pressure of enclosed gas. A lesser dependence is found on the repetition rate of the pulses. The validity of Paschen's law at these high frequencies is investigated, and a semitheoretical formulation of breakdown is obtained based on electron ionization competing with the process of electron attachment to neutral molecules (relatively immobile in the gas). Prediction is made from this theory as to certain expected results. For experimental conditions these were not realizable at the time the experiments were performed but they may now be realized in certain laboratories.

1. INTRODUCTION

DURING the course of the radar work at the Radiation Laboratory, M.I.T., serious problems were posed by undesired sparking of radiofrequency components when high power emanated from magnetrons. Accordingly, investigations were started to study the nature of microwave sparking.[2]

[1] This article is based on work done for the Office of Scientific Research and Development under Contract OEMsr-262 with the Massachusetts Institute of Technology.

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[2] A part of the experimental material discussed in the present article appears in Radiation Laboratory Report 731, D. Q. Posin, Ina Mansur, H. Clarke, November 1, 1945.

After the completion of the spark breakdown experiments, a mathematical attempt was made to establish a sparking criterion utilizing as basic assumptions ionization by electrons which are activated by the microwave field, and disappearance of electrons by attachment to neutral molecules. For this purpose use was made of an equation giving the electron ionization as a function of pressure and ratio of field strength to pressure, similar to the functions well known in d.c.[3]

Other theories of microwave breakdown, though usually applicable only to the low pressure phenomena and for continuous waves (not pulsed microsecond bursts), have been recently developed by a number of writers[4–6] and it is hoped that more theoretical work will soon be forthcoming on electrical breakdown phenomena of centimeter waves.[7]

2. GENERAL REMARKS

To obtain reliable and repeatable breakdown data in the experiments discussed in this paper it was found necessary to provide sufficient initial ionization in the gap where breakdown was to take place. The initial ionization was supplied by placing a capsule of radioactive cobalt (of 3.2-millicurie intensity) on the outside of the wave guide above the breakdown gap. Otherwise, if the initial ionization is sporadic, like the normal ionization from cosmic rays, one might, in raising the field strength, pass the true breakdown value if no strategically placed electrons are available at the critical moment; later, electrons might appear appropriately in the gap and the spark may soon pass, but at too high a field.

It was found at microwave frequencies that with artificially provided initial ionization the breakdown field may be considerably lower than the field with only the normal natural ionization. In d.c. one does not expect the initial ionization to change the breakdown field unless the initial ionization is sufficient to cause space charge distortion[8] of the applied field. In pulsed microwave breakdown even a moderate amount of externally supplied initial ionization may change the breakdown field significantly; possibly this occurs for the following reason: If extra electrons are available when the pulse begins, then the pulse may be tall enough and wide enough to generate a sparking quantity of ions. That is, the field may be sufficiently intense and may be operative long enough to cause accumulation of electrons leading to a spark. If the initial ionization is low, the pulse may have to be made taller in order to generate the same sparking number of ions by the time the pulse has come to the end of its width. Hence, we should be guilty of overvolting.[9]

After the completion of the straightforward planned investigations, the writer observed an effect which appears to be of some interest: the lowering of the electrical breakdown field strength in certain cases for the 3-cm waves, caused by an externally applied magnetic field.[10]

Another point of interest found during the experiments is the following: when a spark or glow occurs within the narrowed wave-guide section constituting the gap, an increase in the magnitude of the standing wave occurs in the region between the discharge and the power source, and this change detected by a pick-up probe in a slotted section, may be used to advantage as an indication that breakdown is occurring, for the case of the low pressure, soft, inaudible glow discharge; or for any other, if desired. This is obviously one of the many

[3] The mathematical approach here indicated was first made by the writer in Radiation Laboratory Report 53.1, November 29, 1945, and dealt with both pulsed voltage applications and continuous wave applications.

[4] T. Holstein, Phys. Rev. **69**, 50A (1946).

[5] Ernest Mayer, Phys. Rev. **69**, 51A (1946).

[6] H. Margenau, Radiation Laboratory Report 967, January 10, 1946.

[7] See also T. Holstein, Phys. Rev. **70**, 367 (1946), and Margenau, McMillan, Dearnley, Pearsall, and Montgomery, Phys. Rev. **70**, 349 (1946).

[8] Varney, White, Loeb, Posin, Phys. Rev. **48**, 818 (1935).

[9] The fact that an external supply of electrons, due ultimately to the radioactive capsule, lowers the apparent breakdown field may be used to locate weak spots in a large r-f component assembly, such as wave guides, chokes, and flanges, stub supports, rotary joints, etc., as follows: Without using the radioactive cobalt the assembly is connected on the high power bench and the power is raised until something sparks; the field strength is then lowered just out of reach of breakdown and the radioactive capsule is then passed (by a pair of long safety tweezers) on the outside along the entire assembly from end to end. The weak spot or spots will now crack electrically. This technique frequently fails to work in cases where the gap volume is very small, apparently because the gamma-rays or their ionizing secondaries miss the tiny volume. However, the method has been found well worth trying and often obviates the primitive necessity of drilling holes or putting in windows to *see* where the spark is passing.

[10] D. Q. Posin, Phys. Rev. **69**, 541 (1946).

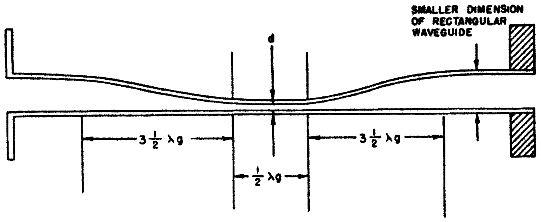

FIG. 1. Generalized drawing of "swayback" section. The breakdown gap is at *d*.

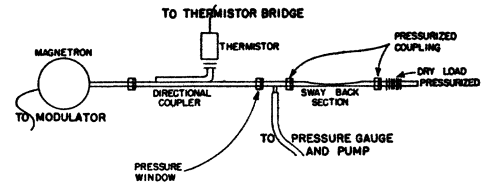

FIG. 1A. Assembly for breakdown at increased pressure. (For reduced pressure, pressure pump and pressure gauge are replaced by vacuum pump and vacuum gauge. For atmospheric pressure, pressurized windows and couplings are not used. For calibration of thermistor, dry load is replaced by water load.)

aspects of reflection of microwaves by strongly ionized layers.[11] In general, it is felt that the use of centimeter waves and the impending production of high intensity millimeter waves opens the field of gas discharge phenomena anew in the same refreshing way that the classical work of Thomson, Crookes, and Townsend did, when their efforts welcomed a host of other pioneers to investigations in the d.c. and low frequency domain.

3. APPARATUS AND EXPERIMENTAL PROCEDURE

To obtain a controlled measurable field at microwave frequencies one must either measure the field directly or use some method from which the field may be computed. In the experiments reported in this paper the second of these procedures was followed. Since it was desired that the microwave breakdown take place across a definite gap of known height, the gap within a section of wave guide had to be narrower than the rest of the wave guide. To accomplish this the wave guide was gradually narrowed in its shorter dimension, extended for a short distance with this minimum dimension, and then it was caused to taper back to normal size gradually. A typical "swayback" of the kind used is shown in Fig. 1. It is obvious that the gradual taper minimizes reflections, and since the same Poynting power now passes through a smaller guide, the field is increased in proportion to the square root of the factor by which the guide is decreased.[12] A measured amount of power was fed into the section and thus it became possible to compute the field at any power level. The procedure during the experiments was thus to increase the power from the magnetron until a spark passed in the swayback; observation of the power level at that critical point then permitted the calculation of the field corresponding to dielectric rupture.

A number of magnetrons, each of appropriate fixed frequency were utilized in turn, driven either by a hard tube or hydrogen-thyratron modulator.

4. METHODS OF MEASURING THE POWER FOR CALCULATION OF FIELD STRENGTH

A water load was used as the standard of power measurement. This type of load consists of a glass tube inserted in a wave guide; the tube is given some form of taper in order to minimize reflections, and water is circulated through the tube at an accurately known rate of flow,[13] the difference between the input and output temperatures being indicated by thermocouples. Errors due to radiation and conduction are reduced by making the temperature of the input water as close as possible to the room temperature; for this reason high flow rates, resulting in small temperature differences, are used.

A sensitive Rubicon galvanometer (0.5-microampere full-scale deflection) was used to measure the small thermocouple voltages. The voltage sensitivity of this galvanometer was varied by

[11] A somewhat related problem, treated theoretically, is that on conduction and dispersion of ionized gasses at high frequencies, by H. Margenau, Phys. Rev. **69**, 508 (1946).

[12] It may be readily shown from an integration of Poynting vector over area in wave guide that the peak power, P_m, and peak field strength, E_m, are related as follows:

$$P_m = KE_m^2(\lambda_0/\lambda_g)ab,$$

where K is a constant, λ_0 is the wave-length of the waves in free space, λ_g is their length in the wave guide, and ab is the cross-sectional area of the wave guide at the particular plane where one is contemplating E_m.

[13] For a description of an accurate pump designed in the later stages of the work by H. F. Clarke, see his Radiation Laboratory Report 53.1, November 7, 1945. This also contains details relevant to measurements in microwave breakdown work.

means of a series resistance (G.R. decade, 0–1000 ohms). In order to calibrate the thermocouples and the galvanometer, it was necessary to substitute a tube containing a coiled heating element for the glass tube in water load. The coil was then connected to the 60-cycle line, and the power recorded by a wattmeter.

Although the water load is a highly desirable power measuring device, being an absolute method and giving reasonable accuracy, it nevertheless suffers from the following defects: It has to be used to terminate the line after the energy has passed through the device being studied, and this is often inconvenient. It is rather slow in its response, perhaps taking half a minute to record the reading on the lagging galvanometer. In view of the fact that discharge phenomena are practically instantaneous in their development, it is desirable that the power measuring device (ultimately field strength) be as fast as possible, especially in order to follow any fluctuations in the magnetron output. For this reason, it was found desirable to make use of a thermistor—essentially a resistance wire enclosed in a glass bead, forming one arm of a type of Wheatstone bridge. The thermistor was coupled to the main wave-guide line by means of a shunt—directional coupler—which coupled a known fraction of the r-f energy out of the line and allowed it to impinge upon the thermistor. The thermistor bridge was known at Radiation Laboratory as a "W" bridge, type TBN 3EV, and could be individually calibrated by the application of a d.c. voltage to the thermistor wire itself. Although the termistor power monitor may be used in an absolute manner, by measuring the coupling of the directional coupler, and calibrating the bridge for a known sensitivity at the thermistor, Mr. H. F. Clarke found discrepancies when using this system with the water load, especially for 1-cm waves. Accordingly, it was decided to calibrate all directional-coupler monitors against water loads. Arrangement for achieving this calibration, as well as other typical arrangements used in the study here reported, are indicated in Fig. 1A and its explanatory caption.

Inasmuch as all methods of power measurement using thermal devices measure average power, it becomes necessary to calculate the peak power from a knowledge of the repetition rate of the pulsed signals and the width of the particular pulse being generated. The determination of the repetition rate is achieved by using an oscilloscope, placing the modulator trigger voltage pulse on the vertical plates, and a sine wave from a calibrated audio oscillator on the horizontal plates. The frequency of the sine wave is then varied until one observes on the scope that one trigger occurs each cycle.

The pulse width, for purposes of power measurement,[14] is taken as the time from the center of the rise of the current pulse to the center of the fall of the pulse, as in standard procedure for pulse width measurements. The measurements were made on synchroscopes calibrated with the standard range calibrators of the Indicator Group at Radiation Laboratory.

In those experiments in which a single pulse was sent down the wave guide by pressing a button, thermal methods of power measurement could not be used, and it sufficed merely to view the height of the current pulse from the magnetron on the oscilloscope, and compare it to the heights of the pulses when repetition rate performance was employed (and power was measured by the thermistor). The reason why it suffices to study the size of the current pulse in the calculation of power is the following: the power output of a magnetron operating at a given magnetic field is found to be approximately proportional to the magnitude of the current pulse. Furthermore, it seems reasonable to assume that current pulses of the same magnitude correspond to the same power, whether single, or at some repetition rate. Therefore, one can, by using a water load with a steady string of pulses, calibrate the height of the current pulse against the power output as measured by the water load. As a matter of fact, in those portions of the present experiments where single pulse breakdown was compared to repetition rate

[14] It was clear to the writer that, insofar as ionization effects and breakdown are concerned, the width of the pulse could not be considered as the value indicated above. The problem essentially was to calculate the effective equivalent *ionization* width of the r-f voltage pulse, when the latter was shaped as a trapezoid. The ionization width of the rectangular equivalent pulse of the same amplitude as the maximum of the trapezoid was found by the writer by utilizing the ionization equation developed in the present paper. The calculation of this effective width may be obtained from the writer upon request.

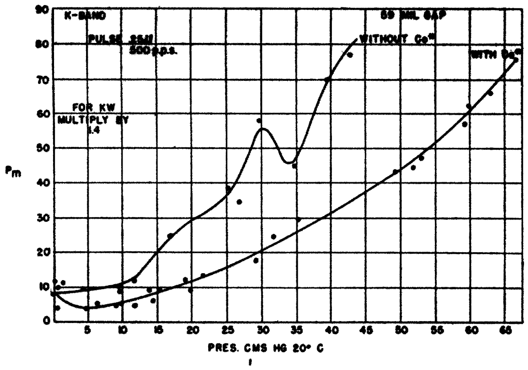

FIG. 2. One run with Co* compared to one run without Co*.

breakdown, and also in other single pulse experiments, only relative powers were of interest, and therefore it sufficed to make comparisons merely of the heights of the magnetron current pulses.

5. THE DETECTION OF THE SPARK

Ordinarily, the occurrence of the spark was evident by its sound; occasionally, when the section which terminated the wave-guide line after the swayback was an open horn, the spark could be detected also by viewing the emitted light. In other cases, especially at low pressure when the sound was extremely faint and when it was impossible to gaze down the horn because much of the assembly was under an evacuating bell-jar, the breakdown was detected by other methods. A stethoscope or a contact microphone was attached to the swayback section immediately over the enclosed gap and the sound could be detected in that way, or else a directional coupler with a power detecting device was used to pick up the sudden increased reflections due to the spark acting as an abrupt termination. This latter technique promises to develop into a form from which one might establish by quantitative definition the fact that a spark has occurred, for the amount of reflection from a cumulatively ionized gas is a function of the intensity of the ionization. In utilizing the reflection method for spark detection it is desirable that a contrivance be used which will indicate the instantaneous energy reflected, inasmuch as the sporadic reflections from an occasionally sparking gap would not register on an average power indicator, such as a resistance wire. For this reason, in these reflection method cases use was made of the set of apparatus comprising a crystal, amplifier, and synchroscope, with inverted directional-coupler pick-up inserted between the prepared sparking gap and the magnetron in position to pick up reflections from the gap should it begin to spark. The spark would thus be detected by the sudden increase in height of the pulse appearing on scope. (Even though the directional coupler in inverted position should theoretically pick up only energy reflected from the termination end of the line, in practice one always has some direct pick-up from the Poynting energy pouring outward from the source. But this may be small and rather steady until it is suddenly augmented by the reflection energy from an occurring spark.) As has been mentioned, the passage of a spark could also be detected by observing the increase in the standing-wave ratio in a slotted section located between the magnetron and the spark.

6. RESULTS

Typical experimental results are shown in Figs. 2 to 11. These graphs reveal the effects of the following parameters (after repeatability in procuring data is achieved through the use of the radioactive capsule): 1. Intensity of initial ionization. 2. Gas pressure. 3. Height of breakdown gap. 4. Repetition rate of pulses. 5. Pulse width.

In addition to presenting the typical graphs, it is felt worth while to present also an empirical summary of a portion of the results.

Figure 2 shows that, ordinarily sparking data at microwave frequencies tend to be unreliable until extra initial ionization is provided; that is in addition to the natural erratic ionization due to such sources as cosmic rays.

The use of a strong radioactive source, such as 3.2 millicuries of Co*, to provide extra ionization, brings with it also the following changes: (a) the breakdown field strength is always lower than when the radioactive source is not used; (b) the "law" of breakdown is changed from approximately straight line[15] when breakdown power ($P_m \sim E_m^2$) is plotted against pressure, to parabolic, when the Co* is used. (This effect becomes evident in the equations given in this paper.)

[15] When many non-Co* curves are averaged.

Figure 3 indicates to what extent it becomes easier to produce a microwave spark when the number of high frequency pulses running through the gap per second increases fourfold. The following is a possible interpretation of such a "repetition rate effect": If the spark does not occur on the first pulse, then some of the electrons created during the first pulse may still be roaming freely in the gas by the time that the second pulse starts (although perhaps most of the electrons have been lost through attachment, recombination, and diffusion during the rest period between two pulses). The second pulse then has an advantage, and builds on top of its inheritance; the third pulse works on a still bigger initial supply, etc., until at some pulse down the line a spark passes.

It was found that, for a given field strength, a spark that just manages to occur when the repetition rate is R_1 does not occur when the repetition rate is $R_2 < R_1$ even if we wait for it indefinitely. Certain experiments especially designed to test this matter were performed as follows: for 1.25-cm waves a certain gap broke down at 14 units of average power when the repetition rate was 1000 pulses per sec., the field was then lowered and the repetition rate set at 500 p.p.s., then the field was raised until the average power read 14/2 or 7 units; this made the two peak field strengths equal but no spark passed now even after a wait of three minutes. When the field was raised a certain amount, the spark came. From these and a number of other experiments a fairly safe conclusion is this: a fourfold increase in repetition rate (500–2000 p.p.s.) lowers the peak breakdown field strength by about 15 percent.

Figure 4 shows how the breakdown field strength for 1.25-cm waves varies with gap width at a given pulse width and repetition rate over a large range of pressures. The fact that a stronger field is necessary to crack a narrower gap might be explainable as follows: the removal of electrons by the alternating instantaneous anodes robs the gap of part of its electron density, and is more serious on a percentage basis for narrow gaps than for the wider ones, thus necessitating a higher ionizing field to make up the loss for the narrow gaps. It is worth recalling that even at d.c. the field varies with gap width especially in the lower pressure regions. That is Paschen's law is not a straight line at the lower pressures, but merely states that breakdown voltage is a function of mass between the electrodes.

Figure 4A is a plot of Paschen curves using a number of gap widths over a fairly extended pressure range. These curves are derived from those of Fig. 4 and show that the Paschen law may be considered as being approximately valid at microwave frequencies, at least for the higher products of *pxd*, that is, for the grosser air masses. (One recalls that in Paschen's law the temperature is assumed constant; otherwise a correction must be made of the pressure value.) The only unique Paschen curve at microwave frequencies may be considered to be that for a very wide gap and long pulse, or for a pulse of infinite width—that is, continuous wave. As a practical matter, at atmospheric pressure, for a gap somewhat larger than 60 mils, and for any repetition rate, a pulse greater than about 6 microseconds yields values of breakdown field substantially equal to those for a continuous wave. (This may be inferred from the rapid flattening seen in the curves of the breakdown field strength *versus* pulse width; the flattening takes place after about 6 μsec., see Fig. 8.)

Figure 5 gives a curve of breakdown power *versus* pressure up to two atmospheres, for two different repetition rates at 1 μsec. These curves were obtained with Co* in use, and again reveal the non-linear (on the average nearly parabolic) rise of peak tolerable power with pressure.

Figure 6 shows the variation of breakdown field strength with pulse width at a given repetition rate, from 1 cm to 76 cm Hg in pressure.

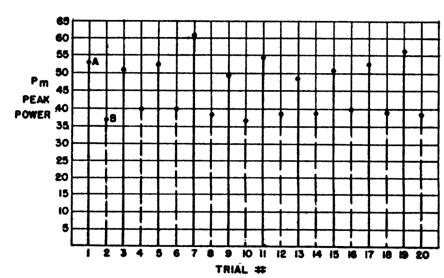

FIG. 3. Repetition rate effect on peak breakdown power at atmospheric pressure. Upper dots are for a repetition rate 500 p.p.s. Lower dots are for 1960 p.p.s. Average P_m, 30 percent.

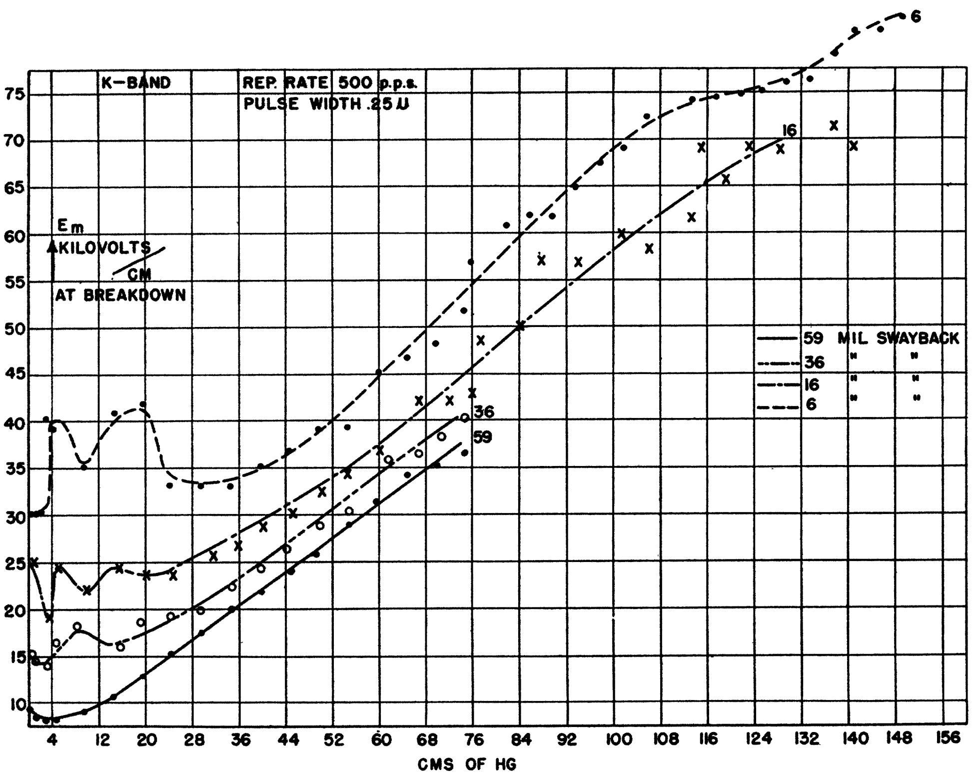

FIG. 4. Peak field strength *vs.* pressure for various swaybacks.

In Figure 7 is presented a breakdown "spectrum," showing the variation of peak power at breakdown *versus* pressure for a number of pulse widths and repetition rates. Also shown is one non-Co* curve contrasting with a Co* curve

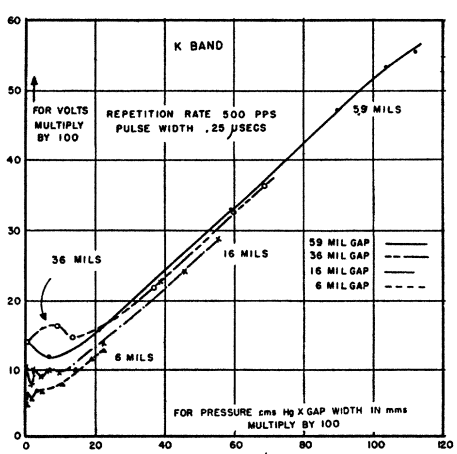

FIG. 4A. Paschen curves for various swaybacks (gap widths).

(those at 1 μsec.). The Co* curves are all parabolic and the non-Co* is practically linear.

Figure 8 reveals the dependence of breakdown power on pulse width and indicates that at atmospheric pressure, after about 4 or 5 μsec., the breakdown power is substantially independent of pulse width. That is, the "minimum formative field"** at atmospheric pressure is attained with a pulse width of about 5 μsec. (It is later shown mathematically that at lower pressures the minimum formative field becomes greater; in fact, in inverse ratio.)

It is seen from the graph that for pulses of less than about 0.3 μsec. the breakdown field becomes relatively very great.

It is perhaps possible to regard the pulse width effect in the following manner: for a narrow pulse the available and accumulating electrons may not have sufficient time to generate a sparking density of ions; hence it would be

** Here arbitrarily defined as the field which is 103 percent of its minimum cw value at a given pressure.

necessary to raise the field which brings about high electron ionization by collision. In the limit, as the narrowness of the pulse approaches zero, the height of the breakdown field approaches infinity. At the other extreme, as the pulse is made broader and broader, the field necessary for breakdown becomes smaller and smaller in amplitude, the interpretation conceivably being this: although the electrons are oscillating and ionizing less vigorously, they still can generate a sparking quantity of ions if given enough time, that is, enough pulse width. There exists a limit, however, to how low the field may be (even though the width may be extraordinarily great), and this limit depends on the rate of electron production compared to the rate of electron loss by recombination, attachment, diffusion, and removal at the instantaneous anode. At atmospheric pressure the lowest breakdown field was very nearly obtained with the relatively small width of about 4 μsec. After this value of pulse width little lowering of breakdown field was discernible, and pulses greater than 4 μsec. were essentially wasting their time, literally, after 4 μsec. In principle, the lowering of field continues to a definite non-zero value asymptotically as we approach infinite pulse width or continuous wave. However, this asymptotic tapering goes on at a sub-microscopic rate, and the value of breakdown field at CW is substantially the same as that at about 5 or 6 μsec.

At lower gas densities the maximum width of the pulse after which no lowering of breakdown field is grossly discernible may be expected to be greater than the value at atmospheric pressure inasmuch as the electronic ionizing rate[16] decreases at the low pressures.

Figure 8A gives further studies of the effect of pulse width on breakdown power (or field strength) at atmospheric pressure, especially revealing the approximate minimum formative fields through the use of single pulse breakdown experiments.

Figure 9 gives a 10-cm power-pressure curve (with Co*), for the range of 1 to 1.52 atmospheres. In Fig. 10, the effect of repetition rate on breakdown is seen for the case of the 10-cm waves, the range in repetition rate being 200 to 2000 pulses per sec. The data were taken at a pulse of 0.76 μsec., at atmospheric pressure, for two different gap widths, with Co* as well as without it. One might expect the curve to flatten out at repetition rates somewhat higher than 2000, whereupon the gap would perform as though CW were being applied, or as though a pulse of 6 μsec. were active with any repetition rate whatever. For, as was indicated previously,

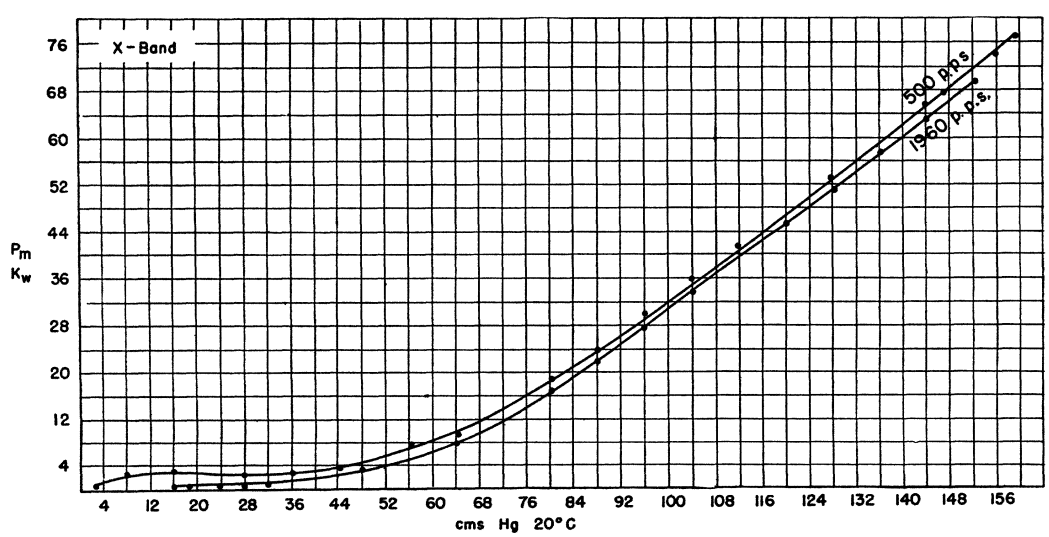

FIG. 5. Breakdown power *vs.* pressure up to two atmospheres for two different repetition rates at a pulse of 1 μsec.

[16] D. Q. Posin, Phys. Rev. 50, 650 (1936).

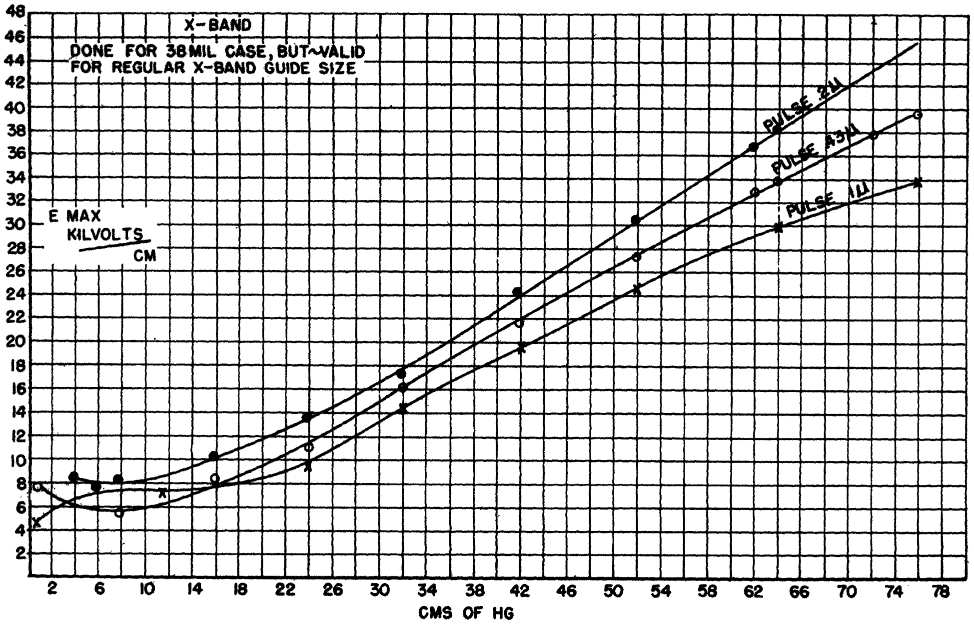

FIG. 6. Peak field strength at breakdown, and pressure, and pulse width.

CW very likely brings with it practically the same minimum breakdown field that a 6 μsec. pulse does (at atmospheric pressure).

Figure 11 shows a curve derived mathematically and predicts the manner in which the practical minimum formative field varies with pressure. Thus far, only one experimental point exists to be matched to this curve. Reference to this curve is made again in the section on mathematical formulation.

7. EMPIRICAL SUMMARY

(Peak breakdown field, E_m, in wave guide is related to peak breakdown power, P_m, by the formula

$$P_m = 6.63 \times 10^{-4} E_m{}^2 \cdot \frac{\lambda_0}{\lambda_g} ab,$$

P_m in watts, E_m in volts/cm, a and b dimensions of rectangular wave guide in cm.)

1. *For 1.25-cm Waves*

$$P_m = \frac{3.74}{(\mu)^{\frac{1}{3}}} \times 10^{-2}(5500 - R)p^n \frac{W}{170}$$

μ is the pulse width in microseconds;
p is the pressure in atmospheres;
P_m is the peak power in kilowatts;
W is the height of the gap in mils.

Limits of verified validity: μ, 0 to 2 μsec.; for $\mu > 2$, consult pulse width curves. R, 500 to 2000 pulses per sec.;

p, 5/76 to 2 atmospheres.

The exponent n is about 1 for pressures above about half an atmosphere. For the range 5-cm Hg to ½-atmosphere, $n \simeq 2$.

For atmospheric pressure, 500 p.p.s. and 1-μsec. pulse,

$E_{max} \simeq 27$ kv/cm,
$P_{max} \simeq 187$ kw
(full-size guide, 170×420 mils, inner).

2. *For 3-cm Waves*

$$P_m = \frac{4W}{(\mu)^{\frac{1}{3}}} p^n (8000 - R) \times 10^{-4} \qquad \text{(kw)}.$$

Limits of verified validity, including values of n: same as for the 1.25-cm waves.

$$E_m = \frac{0.38(p^n)^{\frac{1}{2}}(8000 - R)^{\frac{1}{2}}}{(\mu)^{\frac{1}{6}}} \text{ kv/cm}.$$

At atmospheric pressure, 1-μsec. pulse, and 500 p.p.s.,

$E_{max} \simeq 34$ kv/cm,
$P_{max} \simeq 1.2$ megawatts
(full-size guide, 400×900 mils, inner).

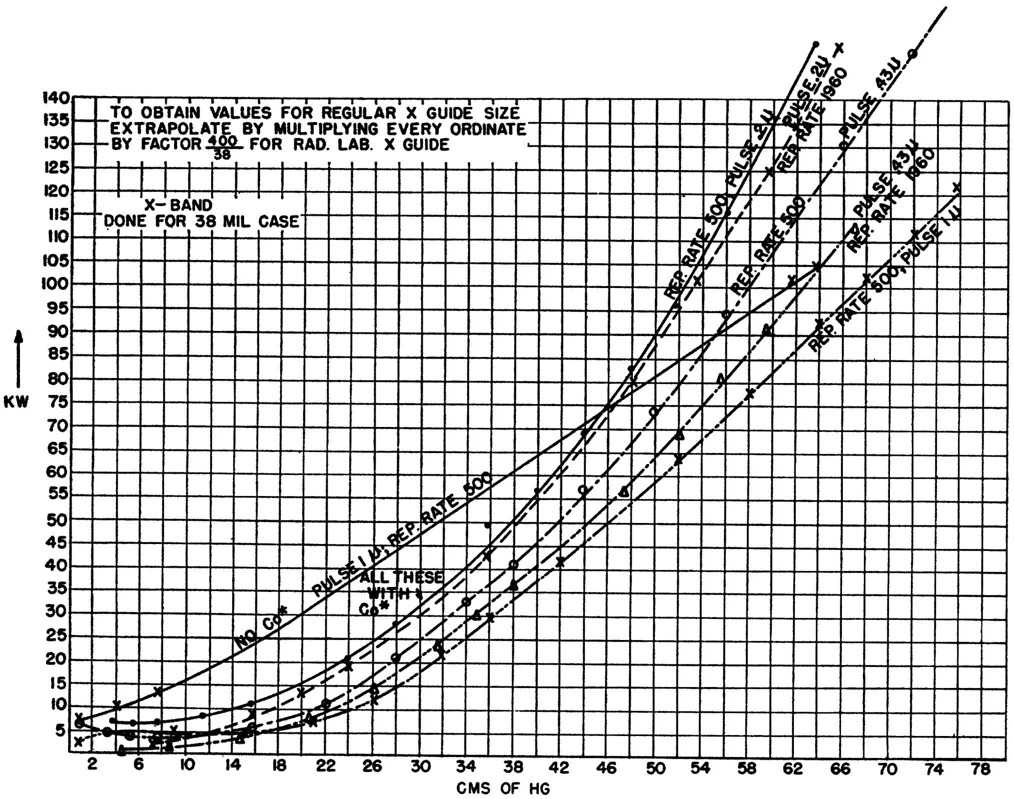

FIG. 7. Peak power at breakdown, and pressure, repetition rate, and pulse width.

3. *For 10-cm Waves*

Limits of verified validity:
μ, 0.76 to 1.86 μsec.;
p, 1 to 1.52 atmos.;
R, 500 to 2000 p.p.s.;
$P_m = 3.45 \times 10^{-4} p^2 (6300 - R) \cdot (6.4 - \mu)$;
P_m in megawatts;
p in atmospheres;
μ in microseconds.

At atmospheric pressure, 500 p.p.s., 1 μsec.; and for full-size wave guide (1.34 in. $\times$ 2.84 in., inner), $P_m = 10.8$ megawatts, $E_m = 30$ kv/cm.

8. MATHEMATICAL CONSIDERATIONS

Using the Townsend coefficient of ionization α as the electron generating mechanism and an electron attachment coefficient B as a loss mechanism, we may write for the net gain of electrons in a time dt

$$dn = \alpha n v dt - B n dt,$$

v being the electronic velocity.

One may now use to advantage $K_e = c/p$, where K_e is the electron mobility, and $|v| = K_e |E|$, or $v = K_e E_0 \cdot |\sin \omega t|$, together with α as a function of p and E/p. From experiment, in the region $E/p = 0$ to $E/p = 20$ (E in volts/cm, p in mm of Hg) no data are available in air; from 20 to 38 Sanders'[17] data were fitted by him, to α/p

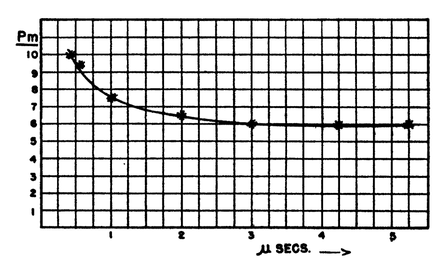

FIG. 8. Peak power as a function of pulse width (atmos. pres., rep. rate 100 p.p.s.; 17-mil gap; 3-cm waves).

[17] F. H. Sanders, Phys. Rev. 41, 667 (1932); 44, 1020 (1933); see also Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).

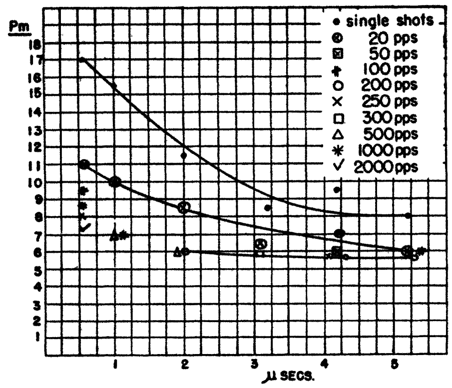

FIG. 8A. Peak breakdown power *vs.* pulse width.

$=a\exp b(E/p)$. From 38 to 140, Jodelbauer[18] fitted Sanders' data to the form $\alpha/p=[A_1(E/p)-B_1]^2$. Near atmospheric pressure Paavola[19] found $\alpha/p=A_2[(E/p)^2-B_2]^2$. The writer[20] has found similar relations in N_2, in d.c. work.

In the present paper the expression $\alpha/p=A(E/p)^2$ is used for the entire region $E/p=0$ to 140, and A is chosen to fit Sanders' data as closely as possible.

Thus,

$$dn/n=(\alpha/p)cE_0|\sin\omega t|dt-Bdt,$$

which gives, after substitution and integration in half-cycles,

$$n=n_0\exp\left(4/3AE_0{}^3\cdot\frac{c}{\pi p^2}-B\right)\mu$$

(the pulse width μ consists of $N=(\mu/\pi/\omega)$ half-cycles).

We might now assume that for any pulse width μ, the sparking condition is attained when for a given n_0, n/n_0 becomes large enough. Frequently, the sparking n/n_0 is equal to $K^*p/K_0p=K^*/K_0$, where K^* is defined as the sparking number of electrons per cc per mm Hg.

Then,

$$\left(\frac{4ACE_0{}^3}{3\pi p^2}-B\right)\mu=\ln\left(\frac{K^*}{K_0}\right),$$

$$\therefore\quad E_0=(3\pi/4AC)^{\frac{1}{3}}p^{\frac{2}{3}}\times[(1/\mu)\ln(K^*/K_0)+B]^{\frac{1}{3}}.\quad(1)$$

This sparking equation reduces to the following expression for a CW pulse, $\mu=\infty$:

$$E_0=(3\pi/4AC)^{\frac{1}{3}}B_0{}^{\frac{1}{3}}p,$$

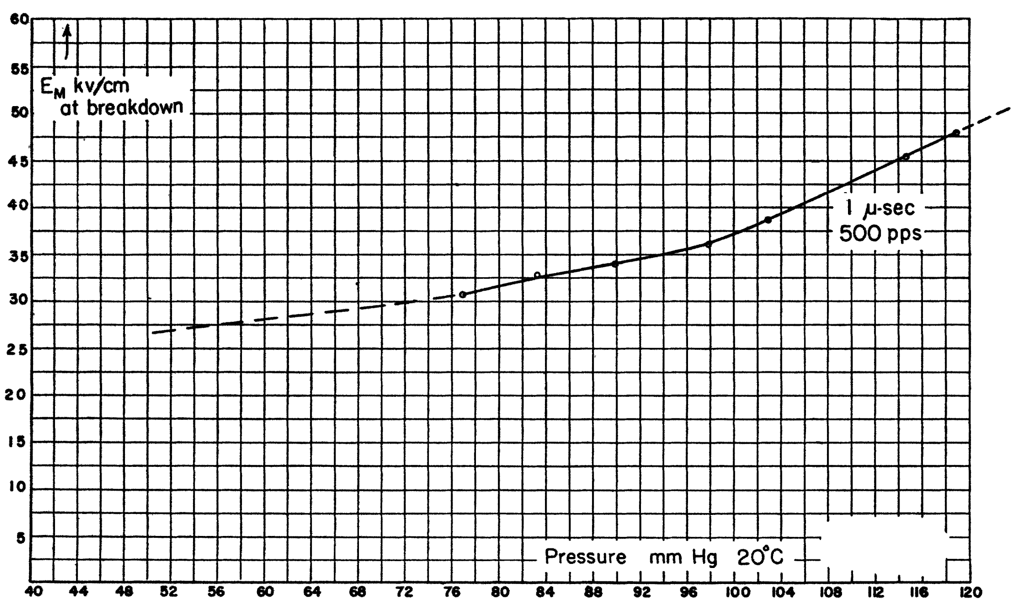

FIG. 9. Peak field *vs.* pressure (10-cm waves).

[18] Jodelbauer, Zeit. f. Physik **92**, 116 (1934); or Loeb, reference 17, p. 360.
[19] M. Paavola, Archiv. f. Elektrotechnik **22**, 443 (1922).
[20] D. Q. Posin, Phys. Rev. **50**, 650 (1936); values and formulation of α/p *vs.* E/p in N_2, from E/p 20 to 1000; or see Loeb, reference 17.

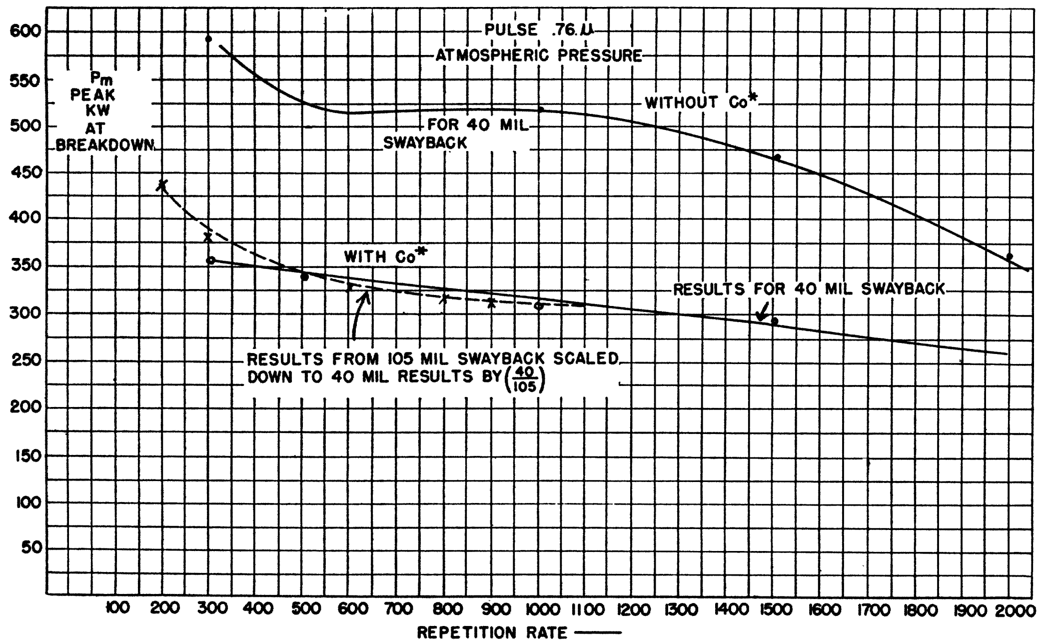

FIG. 10. Peak breakdown power *vs.* repetition rate (10-cm waves).

showing that breakdown field strength at high pressure and for long pulses is proportional directly to the pressure.

The voltage V_s across the gap is $E_0 \times d$,

$$V_s = (3\pi/4AC)^{\frac{1}{2}} B_0^{\frac{1}{2}} \cdot pd,$$

or, a Paschen law of type

$$V_s = K(pd).$$

We now see that the Paschen law may be expected to be valid at alternating frequency for continuous wave and at high gas pressure.

At lower pressures the electronic velocity is in general not in phase with the field. The equation of motion of the electron may be written as follows:

$$m\ddot{x} + f\dot{x} = E_0 e \sin\omega t,$$

where f is the friction factor and will be taken here in its average value independent of the field strength.

Then,

$$\dot{x} = \frac{efE_0 \sin\omega t}{m^2\omega^2 + f^2} - \frac{meE_0\omega \cos\omega t}{m^2\omega^2 + f^2},$$

provided that we choose the following initial condition: at $t=0$, $\dot{x} = (-E_0 e/m\omega)$. We have chosen here only electrons which have a special velocity when the field is zero. These electrons will oscillate in the gap, whereas all others will travel to an instantaneous anode and will be removed. When the oscillating electrons create new electrons by collision, again only those which have an appropriate initial velocity will oscillate in the gap. Eventually we may build up a sparking quantity of oscillating electrons. To continue analytically,

$$dx = |v|\,dt = |\dot{x}|\,dt,$$

where again we are only interested in the absolute value of the velocity, inasmuch as the new electrons formed add up whether the ionization occurs going up or down.

Integrating in half-cycles as before, we obtain for the breakdown field

$$E_0 = \left[\frac{3\pi(m^2\omega^2 + f^2)}{2Ae(m\omega - 2f)}\right]^{\frac{1}{2}} p^{\frac{1}{2}} \left[\frac{1}{\mu}\ln\left(\frac{K^*}{K_0}\right) + B_0 p\right]^{\frac{1}{2}},$$

For f, the friction term, $\rightarrow 0$,

$$E_0 = \left(\frac{3\pi m\omega}{2Ae}\right)^{\frac{1}{2}} p^{\frac{1}{2}} \left[\frac{1}{\mu}\ln\left(\frac{K^*}{K_0}\right) + B_0 p\right]^{\frac{1}{2}}. \quad (2)$$

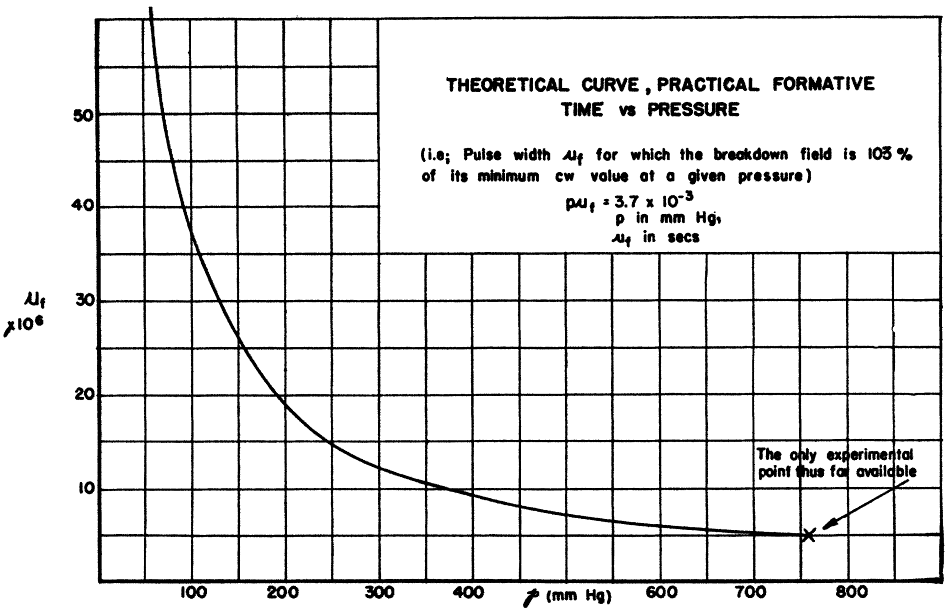

FIG. 11. Curve of "practical formative time."

For infinite pulse, or CW, this gives

$$E_0=\left(\frac{3\pi m\omega}{2Ae}\right)^{1/3}B_0^{1/3}p^{2/3}.$$

The sparking voltage in this case would be

$$V_s=\left(\frac{3\pi m\omega}{2Ae}\right)^{1/3}B_0^{1/3}p^{2/3}d,$$

and thus we note a deviation from the Paschen law at the lower pressures, inasmuch as $V_s \neq F(pd)$.

For still lower pressures we no longer use $\alpha/p=A(E/p)^2$ (with $E=E_0 \sin\omega t$), but rather $(\alpha/p)^2=b^2(E/p)$ (with $E=E_0 \sin\omega t$).

This general type of relation (but without the sinusoidal E) is known[21] from d.c. work, valid for $E/p=200$ to $E/p>1000$ (E in volts/cm, p in mm of Hg). Since this range is so great we will use the above α/p relationship in spite of the fact that E/p runs through zero values as the $\sin\omega t$ fluctuates; i.e., in spite of the fact that for relatively small stretches of E/p the other α/p relation is indicated.

After substitution, integration in half-cycles of $dn=\alpha n dx-Bndt$, gives

$$E_0=\left(\frac{m}{eb(0.4)^{1/2}}\right)^{2/3}\frac{\omega^{2/3}}{p^{1/3}}\left[\frac{1}{\mu}\ln\left(\frac{K^*}{K_0}\right)+B_0p\right]^{2/3}. \quad (3)$$

For infinite pulse width, or CW,

$$E_0=\left(\frac{m}{eb(0.4)^{1/2}}\right)^{2/3}\omega^{2/3}B_0^{2/3}p^{1/3}.$$

The sparking voltage becomes

$$V_s=\left(\frac{m}{eb(0.4)^{1/2}}\right)^{2/3}\omega^{2/3}B_0^{2/3}p^{1/3}d,$$

and this is a still greater deviation from Paschen's law, and $V_s \neq f(pd)$.

For a given gap width, we have now the following sparking voltage relations insofar as pressure is concerned:

$V_s \sim p$, at high pressure (and here alone, $V_s = f(pd)$),
$V_s \sim p^{2/3}$, at lower pressure,
$V_s \sim p^{1/3}$, at the lowest pressures.

In the formulas (1), (2), and (3), using certain convenient values of the constants, we determine

[21] D. Q. Posin, Phys. Rev. 50, 650 (1936); for N_2.

the value of K^*/K_0 from the relation

$$36\times10^3=\left(\frac{3\pi}{4\times4\times10^{-6}\times10^3\times760}\right)^{\frac{1}{3}}760^{\frac{2}{3}}\times\left[\frac{1}{10^{-6}}\ln\left(\frac{K^*}{K_0}\right)+10^5\times760\right]^{\frac{1}{3}}.$$

(36 kv/cm is a conservative value for breakdown field strength at 1 μsec. for single pulse performance.)

We then have

$$\ln(K^*/K_0)=33;\quad (K^*/K_0)=e^{33}\text{ or }10^{14.2},$$

which is to be taken for all conditions so long as K_0 is constant.

Equations (1), (2), and (3), now become:

$$E_0=0.92p^{\frac{2}{3}}[(33/\mu)+10^5p]^{\frac{1}{3}}, \qquad (1^1)$$

E_0 in volts/cm, p in mm of Hg, μ in seconds. This is for the high pressure region. For the middle pressure region,

$$E_0=4.2p^{\frac{2}{3}}[(33/\mu)+10^5p]^{\frac{1}{3}}, \qquad (2^1)$$

for 1.25-cm waves. For 3-cm waves in this middle pressure region one obtains

$$E_0=4.2\left(\frac{10^{10}}{2.4\times10^{10}}\right)^{\frac{1}{3}}p^{\frac{2}{3}}\left(\frac{33}{\mu}+10^5p\right)^{\frac{1}{3}}$$

or

$$E_0=3.14p^{\frac{2}{3}}\left(\frac{33}{\mu}+10^5p\right)^{\frac{1}{3}}.$$

For 10-cm waves in this pressure range

$$E_0=4.2\left(\frac{3\times10^9}{2.4\times10^{10}}\right)^{\frac{1}{3}}p^{\frac{2}{3}}\left(\frac{33}{\mu}+10^5p\right)^{\frac{1}{3}}$$

or

$$E_0=2.1p^{\frac{2}{3}}\left(\frac{33}{\mu}+10^5p\right)^{\frac{1}{3}}.$$

For the lowest pressure range, for 1.25 cm waves one may use the value 0.2 for b, and thus readily determine E_0 in terms of known constants and the parameters of formula (3).

Calculations made with various of the formulas given above are generally in fair agreement with experimental values. Similar formulas involving the parameter R, repetition rate of pulses, may be submitted on another occasion. For the present we may conclude by making predictions as to the magnitude of the "practical formative time of the spark." By this term, as previously suggested, we mean the length of a single microwave pulse in seconds for which the breakdown field strength is about 103 percent of the minimum possible at the given pressure and initial ionization. Obviously the actual minimum field strength occurs for an infinitely wide pulse.

We form the ratio

$$\left[\frac{(33/\mu_f)+10^5p}{10^5p}\right]^{\frac{1}{3}}=1.03,$$

μ_f being the practical formative time of the spark. The formula gives

$$p\mu_f=3.7\times10^{-3},$$

and this is presented graphically in Fig. 11. For atmospheric pressure, p, of 760 mm Hg we obtain $\mu_f=4.9\times10^{-6}$ sec., which is about the value we have been obtaining experimentally. At a lower pressure, for example 20 cm of Hg, we get $\mu_f=(3.7\times10^{-3}/200)=1.9\times10^{-5}$ sec., or 19 μsec. Perhaps the means are at hand for some laboratory to test this inference; that is, the curve of Fig. 11 giving the indicated quantitative increase in the practical formative time of the spark at lower pressures.

In conclusion we would like to state that most of the apparatus for the experiments described was assembled and placed into expert order by Mr. H. F. Clarke and Mrs. Ina Mansur, who frequently assisted with the measurements. Also assisting was Miss Alice Reynolds. We appreciate the interest displayed in this work by Mr. George Yevick, Dr. George Ragan, and Professor A. G. Hill. However, any inadequacy in experiment or theory is the responsibility of the writer alone.