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Interaction of Cosmic-Ray Primaries with Sunlight and Starlight*

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This paper discusses collision processes between cosmic-ray primaries (protons and electrons) and the thermal photons of sunlight and starlight. In particular, electron-positron pair production and Compton scattering in interplanetary, intragalactic, and intergalactic space are treated in detail. It is found that the number of collisions between primary particles and thermal photons in single traversals of the solar system and the local galaxy is not sufficiently large to cause either appreciable energy loss to the particles or appreciable production of secondary pairs and energetic scattered photons. The same statement holds for the primary protons even on an intergalactic scale. On the other hand, energetic primary electrons may experience a sufficient number of Compton collisions in intergalactic space (travel time of the order 2×10^9 years) to eliminate them effectively from the cosmic radiation reaching the neighborhood of the earth.

The stopping power for electrons of the observed interstellar radiofrequency spectrum is also estimated and found to be comparatively large.

1. INTRODUCTION

THE primary cosmic radiation incident on the earth's atmosphere is now thought to consist largely of protons.[1] In addition, highly energetic photons and positive and negative electrons must inevitably accompany the primary protons, if only because they are produced when the protons collide with interstellar diffuse matter and with the low energy "thermal" photons which constitute the major portion of starlight and sunlight. The present study is concerned chiefly with collisions between thermal photons and primary cosmic-ray particles taken as either protons or electrons.[2] Such collisions are found to produce, under certain conditions, large changes both in the composition and in the energy distribution of the original primary radiation. A variety of interaction processes between the primaries and the thermal photons may occur, depending on the energies and momenta of the colliding particles. Thus one may anticipate (for primary protons):

1. Electron-positron pair production (when the energy ϵ^* of the thermal photon in the rest frame of the proton exceeds $2\,mc^2$.[3]
2. Production of energetic recoil photons by Compton scattering (no restriction on ϵ^*).
3. Double Compton scattering—one photon incident, two recoiling—a bremsstrahlung process (no restriction on ϵ^*).
4. Single meson creation ($\epsilon^*>200\,mc^2$).
5. Meson pair creation ($\epsilon^*>400\,mc^2$).
6. Nucleon pair creation ($\epsilon^*>6000\,mc^2$).[4]

Each such process reduces the energy of the original primary particle and at the same time contributes a new component to and possibly removes an original component from the cosmic radiation. In the present paper the emphasis is on processes (1) and (2). We compute the energy losses from Compton scattering and electron-positron pair production, as well as the numbers of scattered photons and pairs, (i) for a particle (proton or electron) falling radially through the sun's radiation field to the earth's orbit, (ii) for a particle traversing the local galaxy, and (iii) for a particle traveling the distance 1.85×10^9 l.y. in intergalactic space (the distance parameter in the empirical red shift formula).[4a]

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[1] T. H. Johnson, Rev. Mod. Phys. 11, 208 (1939); M. Schein, W. P. Jesse, and E. O. Wollan, Phys. Rev. 59, 615 (1941); 59, 930 (1941).

[2] Collisions between high energy photons, considered as cosmic-ray primaries, and thermal photons, with resultant electron-positron pair creation have been considered by G. Breit and J. A. Wheeler, Phys. Rev. 46, 1087 (1934); 45, 134 (A) (1934). Extensive calculations similar to the present have been carried out by J. W. Follin, Bull. Am. Phys. Soc.July 11, 1947, Abstract D5. Through the courtesy of Dr. J. R. Oppenheimer, we have seen a manuscript copy of Dr. Follin's paper.

[3] Because of the relativistic Doppler and aberration effects, a more or less isotropic distribution of low energy photons ($\epsilon\sim1$ ev) in the earth's frame of reference appears as an extremely anisotropic distribution of high energy photons ($\epsilon^*\gg1$ ev) in the rest frame of the primary. Thus from the viewpoint of an observer traveling with the primary, the electromagnetic field in interstellar and intergalactic space consists of extremely energetic photons filling a small cone of directions opposite to that in which the primary itself is moving.

[4] H. Feshbach and L. Schiff, Phys. Rev. 72, 254 (1947).

[4a] For glossary of symbols used, see end of paper.

2. THE PHOTON POPULATIONS OF INTERPLANETARY, INTERSTELLAR (INTRAGALACTIC), AND INTERGALACTIC SPACE

We require estimates of the total number and energy distribution of the thermal photons occurring in the spaces traversed by the primary particles. Considering case (i) first, it is convenient to refer to the number of solar thermal photons, $n_s(\epsilon/kT)$, per unit volume and unit energy range, at the earth's orbit.[5] Evidently n_s can be expressed in terms of the solar constant, η, and the equivalent blackbody spectral distribution function. Thus,

$$n_s(u)d\epsilon = A\frac{u^2du}{e^u-1}, \quad u\equiv\frac{\epsilon}{kT}; \quad A\equiv\frac{15}{\pi^4}\frac{\eta}{ckT}. \quad (1)$$

With $\eta=1.94$ cal./min. cm^2 [6] there are 2×10^7 photons/cm^3 of mean energy $2.7kT\cong 1.35$ ev at the earth's orbit corresponding to a total radiation energy density $\cong 2.7\times 10^7$ ev/cm^3 $(=\eta/c)$. The total number Q_s of thermal photons in a cylinder of unit cross section extending radially from the earth's orbit (radius R_e) to infinity has the value[7]

$$Q_s=\int_0^\infty d\epsilon\int_{Re}^\infty n_s(u)\left(\frac{R_e}{R}\right)^2 dR = 2.4AR_e$$
$$= 3.1\times 10^{20}\ \text{photons/cm}^2. \quad (2)$$

Case (ii) is treated next. The galaxy is represented by a collection of N stars all of the same absolute magnitude and spectral type as the sun. To further simplify the calculation the average number of stars per unit volume, $p(x, y, z)$, is given the simple form

$$p(x, y, z) = \frac{N}{R_g^2R_g'\pi^{\frac{3}{2}}}\exp\left\{-\frac{x^2+y^2}{R_g^2}-\frac{z^2}{(R_g')^2}\right\};$$
$$R_g\cong 5R_g'.^8 \quad (3)$$

If one now considers the number of photons (per unit energy range) in an infinitely long cylinder of unit cross section enclosing the x axis, one finds that the contribution $\Delta Q_g(u)$, to this number from a single star located at x, y, z is given by

$$\Delta Q_g(u) = \frac{\pi R_e^2 n_s(u)}{(y^2+z^2)^{\frac{1}{2}}}. \quad (4)$$

Thus the total number of photons in the cylinder in question is

$$Q_g=\int_0^\infty d\epsilon\int\int\int \Delta Q_g(u)p(x, y, z)dxdydz$$
$$=6.1\pi^{-\frac{1}{2}}\frac{NR_e}{R_g}Q_s. \quad (5)$$

Inserting the values $N\cong 8.5\times 10^7$ and $R_g\cong 10^4$ l.y.,[9] one obtains

$$Q_g\cong 1.5Q_s = 4.5\times 10^{20}\ \text{photons/cm}^2.$$

Again using Eq. (3), the ratio of the intragalactic and solar (at earth's orbit) total photon energy densities or number densities is

$$N\left(\frac{R_e}{R_g}\right)^2 2\ln\left(\frac{2R_g}{R_g'}\right)\cong 10^{-9}.^{9a} \quad (5a)$$

The magnitudes of Q_s, Q_g and of the cross sections expected for the collision processes (1)–(6) indicate that neither the radiation of the sun nor that of the local galaxy possesses an effective stopping power for the primary particles in a single traversal.

We turn now to the photon population in intergalactic space. A simple treatment will be presented, based on general assumptions of uniformity in space and static conditions in time. The cosmic-ray primaries are pictured as traveling in a uniform, unbounded, and static universe, meeting everywhere and always the same

[5] Here ϵ is the energy of the thermal photon in the earth's frame of reference, and T is the surface (equivalent blackbody) temperature of the sun$\cong$6000° Abs.

[6] See *Smithsonian Physical Tables* (Smithsonian Institution, Washington, D. C., 1933), eighth revised edition, p. 608.

[7] The inverse square law used in the derivation of Eq. (2) holds rigorously for the photon energy current density, but not for the photon energy or number densities. However, the deviations from the inverse square law are inappreciable at distances from the center of the sun greater than two or three solar diameters.

[8] B. J. Bok and P. F. Bok, *The Milky Way* (The Blakiston Company, Philadelphia, 1941), p. 18.

[9] E. Hubble, *The Realm of the Nebulae* (Yale University Press, New Haven, Connecticut, 1936).

[9a] Equation (5a) predicts a total intragalactic energy density of $10^{-9}\times 2.7\times 10^7$ ev/cm^3 $=4.3\times 10^{-14}$ erg/cm^3. This value seems to be some ten times smaller than the estimate given by T. Dunham, Proc. Am. Phil. Soc. 81, 277 (1939). Also the spectral distribution observed by Dunham is not exactly blackbody. Our choice for the intragalactic spectral distribution thus probably underestimates the magnitude of the effects here considered.

average density and spectral distribution of thermal photons—namely, that observed at present on the earth—and losing energy only through encounters with these photons or with diffuse matter. Such a treatment should be useful for a preliminary orientation and may be of help in formulating a more rigorous discussion within the framework of non-static cosmological models.[10] More specifically we assume:

(a) The average density $\mathfrak{N}$ of galaxies or nebulae (independent isolated stellar systems comparable to the local galaxy) is constant in Euclidean space.

(b) The individual nebulae radiate like N stars of the same spectral type and absolute magnitude as the sun.

(c) At the distance R from a particular nebula, the photon number density produced by it, in the energy range $d\epsilon$, is

$$\Delta n_n(u, R)d\epsilon = N\left(\frac{R_e}{R}\right)^2 g(R)A\frac{(u/f(R))^2du/f(R)}{e^{u/f(R)}-1}, \quad (6)$$

so that $n_n(u)d\epsilon$, the photon number density in the energy range $d\epsilon$ due to all the nebulae, is the same at all points in space and has the value

$$n_n(u)d\epsilon = d\epsilon\int_0^\infty \Delta n_n(u, R)\mathfrak{N}4\pi R^2dR = 4\pi\mathfrak{N}NR_e^2Au^2du\int_0^\infty\frac{g(R)f(R)^{-3}dR}{e^{u/f(R)}-1}. \quad (7)$$

In Eqs. (6) and (7), the constant A and the nebular temperature T (in $u=\epsilon/kT$) are identified with the corresponding quantities in Eq. (1). The function $f(R)$ measures the red shift in the photon's energy, while $g(R)$ is an associated reduction factor in the radiation intensity and, therefore, in the photon number density at R. Thus, if the red shift is interpreted as a Doppler effect arising from nebular recession, one has $g=f\cong 1-v(R)/c$, where $v(R)$ is the radial velocity of the nebula at the distance R from the point of observation. On the other hand, the observations of Hubble[11] (out to $R\cong 2.5\times 10^8$ l.y. from the earth) show that

$$f(R)\cong 1-R/R_0;\quad R_0=1.85\times 10^9 \text{ l.y.}, \quad g(R)\cong 1.^{12} \quad (8)$$

In the present discussion the behavior of f and g out to $R\cong R_0$ is important, but cannot be determined from the observational material now available. The red shift and the (still uncertain) reduction in intensity associated with the red shift determine the linear terms in R/R_0 in the power series expansions of f and g and yield no evidence for higher order terms. It is therefore necessary, if one wishes to evaluate the photon number density, to extrapolate $f(R)$ and $g(R)$ to distances of the order of R_0. Two simple, but extreme, procedures for such an extrapolation will be discussed. In the first, the red shift is neglected except insofar as it suggests an "effective" radius, R_0, for the matter in the universe. The conditions $g=f=1$ for $R<R_0$, and $g=0$, for $R\geqq R_0$ describe a universe with no red shift and no nebulae outside of a sphere of radius R_0. In this case

$$n_n(u)d\epsilon = [4\pi\mathfrak{N}NR_e^2R_0]A\frac{u^2du}{e^u-1}. \quad (9)$$

The second procedure utilizes the relations[13]

$$g=1,\quad f=\exp(-R/R_0), \quad (10)$$

in agreement with Eq. (8) for $R\ll R_0$.

Equations (7), (10) and the change of variable $y=u\exp(R/R_0)$ now yield

$$n_n(u)d\epsilon = [4\pi\mathfrak{N}NR_e^2R_0]A\frac{du}{u}\int_u^\infty\frac{y^2dy}{e^y-1}. \quad (11)$$

The "red shifted" spectral distribution of Eq. (11) predicts a strong concentration of thermal photons at very low energies ($\epsilon\ll kT$) and thus

[10] See, for example, H. P. Robertson, Rev. Mod. Phys. 5, 62 (1933); E. Hubble and R. C. Tolman, Ap. J. 82, 302 (1935); E. A. Milne, *Relativity Gravitation and World Structure* (Oxford University Press, London, 1935).

[11] See reference (9) and also Hubble's article in *Science in Progress* (Yale University Press, New Haven, Connecticut, 1942), third series.

[12] This relation (described by Hubble as the absence of the number effect) is still in dispute. J. L. Greenstein, Ap. J. 88, 605 (1938) presents evidence for the inadequacy of both $g=f$ and $g=1$.

[13] Within the framework of a uniform static cosmological model, the exponential form for $f(R)$ can be derived from the assumption that the frequency red shift in the distance ΔR is proportional to the frequency and to ΔR. The same expression for $f(R)$ is deduced by G. J. Whitrow, Phil. Mag. 37, 469 (1946), on the basis of considerations involving kinematic relativity.

exhibits an entirely different behavior than the "no shift" blackbody distribution of Eq. (9). Other reasonable extrapolations of Hubble's f and g functions to large R result, in general, in spectral distributions intermediate between those of Eqs. (9) and (11); in what follows only the more or less extreme cases of Eqs. (9) and (11) are considered explicitly.[14]

We note that the "red shifted" and the "no shift" spectral distributions yield the same total photon energy density:

$$\int_0^\infty \epsilon n_n(u)d\epsilon=[4\pi\mathfrak{N}NR_e{}^2R_0]\eta/c. \qquad (12)$$

With $N\cong 8.5\times 10^7$, $\mathfrak{N}\cong 2\times 10^{-19}$ (l.y.)$^{-3}$, and $R_0=1.85\times 10^9$ l.y.[11] one has

$$[4\pi\mathfrak{N}NR_e{}^2R_0]\cong 10^{-10}, \qquad (13)$$

so that the ratio of the radiation energy density in intergalactic space to the radiation density produced by the sun at the earth's orbit is 10^{-10}. This same number is also the ratio of the intergalactic "no shift" photon number density and the solar photon number density at the earth's orbit. For the present discussion, however, a better comparison is provided by the ratio of the radiation energy in intergalactic space in a cylinder of volume 1 $\text{cm}^2\times R_0$ to the energy of solar radiation in a cylinder of unit cross section extending radially from the earth's orbit to infinity. This ratio is

$$[4\pi\mathfrak{N}NR_eR_0{}^2]\cong 10^4. \qquad (14)$$

Finally, in the special case of "no shift," the total number of photons in the cylinder of volume 1 $\text{cm}^2\times R_0$ has the value[15]

$$Q_n=[4\pi\mathfrak{N}NR_eR_0{}^2]Q_s\cong 3\times 10^{24}. \qquad (15)$$

The connection of Q_* with the yield from processes (1)–(6) will be treated below.

In the whole above discussion we have supposed that the photon populations in the various spaces considered are well represented (apart from red shift effects) by blackbody distributions appropriate to a temperature of some 6000° Abs. On the other hand, observation shows that a "radiofrequency tail" is present in the radiation from the local galaxy (and presumably in the radiation from intergalactic space as well) which arises from proton-electron recombination in interstellar space.[16] A discussion of the contributions to process 1–2 from the intragalactic and intergalactic "radiofrequency" photons appears in Appendix A; the possible importance of such contributions was first recognized by Follin.[2]

3. ELECTRON—POSITRON PAIR CREATION IN PROTON—PHOTON AND ELECTRON-PHOTON COLLISIONS

We shall now consider the creation of electron-positron pairs as a result of collisions between the cosmic-ray primaries (protons) and the thermal photons of starlight and sunlight. The processes involved are described in two coordinate systems: (a) in the rest frame of the proton where some of the incident photons have energies large enough to produce pairs in collisions with originally stationary particles and (b) in a frame of reference fixed in the earth where the electromagnetic field of the rapidly moving proton may be replaced by a set of virtual photons, some of which possess enough energy to create pairs by collisions with thermal photons. As might be expected, the two treatments give essentially the same results; however, it will be helpful to discuss them both, particularly since the second is especially well suited to describe any possible effect on pair creation of the anamolous magnetic moment of the proton, or, in general, of any

[14] A more general discussion can be based on the one-parameter family of functions

$$f(R)=(1+R/R_0p)^{-p};\quad p\neq 0,$$

with the special restriction $f=0$, $R>-pR_0$ for $p<0$, and, either $g=1$ or $g=f$. The range $-\frac{1}{2}<p<0$ may be excluded by the observed linearity of the red shift out to the limit of observation ($R\cong R_0/6$), while the requirement of finite energy density excludes the special cases $g=1$, $0<p\leq 1$ and $g=f$, $0<p\leq\frac{1}{2}$. The exponential form occurs for $p=\pm\infty$. It will be observed that the paired conditions $g=1$, arbitrary p, and $g=f$, $p'=p/(p+1)$ yield identical values for n_n.

[15] Equation (15) and the relation $Q_g\sim Q_s$ (Eq. (5)) show that the number of thermal "no shift" photons encountered by a primary particle in the distance R_0 in intergalactic space is of the same order as the number of photons encountered by a primary in a time 10^8 years in the local galaxy. The primary may actually spend such a time in the galaxy if it is originally produced there and if an intragalactic magnetic field of sufficient magnitude exists and prevents its escape. See H. Alfvén, Zeits. f. Physik **107**, 579 (1937); L. Spitzer, Phys. Rev. **70**, 777 (1946); H. Babcock, Phys. Rev. **72**, 83 (1947).

[16] G. Reber, Ap. J. 91, 621 (1940); L. G. Henyey and P. C. Keenan, *ibid.*, 91, 625 (1940).

manifestation of the meson charge cloud surrounding the latter.

We begin with the treatment appropriate to the rest frame of the proton. Here a photon of energy $\epsilon^* > 2\ mc^2$ [17] incident on a fixed proton may be converted into an electron-positron pair with energies ϵ_+^* and ϵ_-^* ($\equiv \epsilon^* - \epsilon_+^*$) and directions of motion falling in the elements of solid angle $d\Omega_+^*$ and $d\Omega_-^*$. The differential cross section for this process with ϵ_+^* falling in the range $d\epsilon_+^*$ is denoted by

$$\phi_\pm^*(\epsilon^*; \epsilon_+^*, \theta_+^*, \theta_-^*)d\epsilon_+^*d\Omega_+^*d\Omega_-^*.$$

The average rate at which pairs are produced per proton, $dN^\pm/dt$, as measured in the earth's reference frame, is then given directly in terms of the total cross section,

$$\phi_\pm^*(\epsilon^*) \equiv \int_{mc^2}^{\epsilon^*-mc^2} \int\int \phi_\pm^*(\epsilon^*; \epsilon_+^*, \theta_+^*, \theta_-^*) \times d\epsilon_+^*d\Omega_+^*d\Omega_-^* \quad (16)$$

by the relation

$$\frac{dN^\pm}{dt} = \gamma^{-1}\frac{dN^\pm}{dt^*} = c\gamma^{-1}\int_{2mc^2}^{\infty}\int n^*(\epsilon^*, \theta^*) \times \phi_\pm(\epsilon^*)d\epsilon^*d\Omega^* \quad (17)$$

in which $\gamma = (1-\beta^2)^{-\frac{1}{2}} = E/Mc^2$, E and M are the total energy and rest mass, respectively, of the proton, m is the rest mass of the electron and $n^*(\epsilon^*, \theta^*)d\epsilon^*d\Omega^*$ is the number of photons per unit volume in the energy range $d\epsilon^*$ which are traveling in directions lying in the element of solid angle $d\Omega^*$. The polar axis from which θ_+^*, θ_-^*, and θ^* are measured is directed opposite to the velocity of the proton. Also, if dW^*/dt^* and dP^*/dt^* denote the average rates (in the proton's rest frame) at which energy and momentum are transferred to the pairs, then, from the transformation properties of energy and momentum and from the time dilatation, one has for $-dE^\pm/dt$, the average rate at which the proton loses energy because of pair creation (in the earth's coordinate system), the expression,

$$\frac{-dE^\pm}{dt} = \frac{dW}{dt} = \frac{dW^*}{dt^*} + c\beta\frac{dP^*}{dt^*}$$
$$= c\int_{2mc^2}^{\infty}\int n^*(\epsilon^*, \theta^*)d\epsilon^*d\Omega^* \int_{mc^2}^{\epsilon^*-mc^2} \times \int\int \phi_\pm^*(\epsilon^*; \epsilon_+^*, \theta_+^*, \theta_-^*)$$
$$\times[\epsilon_+^* + \epsilon_-^* - c\beta(p_+^*\cos\theta_+^* + p_-^*\cos\theta_-^*)]d\epsilon_+^*d\Omega_+^*d\Omega_-^*. \quad (18)$$

Finally, the average energy of a pair has the value

$$\langle\epsilon_+ + \epsilon_-\rangle = -\frac{dE^\pm}{dt}\Big/\frac{dN^\pm}{dt}. \quad (19)$$

We first evaluate the rate of pair production. With the aid of the relativistic transformation formula[18]

$$\frac{n^*(\epsilon^*, \theta^*)d\epsilon^*d\Omega^*}{n(\epsilon, \theta)d\epsilon d\Omega} = \frac{\epsilon^*}{\epsilon} = \gamma(1+\beta\cos\theta), \quad (20)$$

the integral in Eq. (17) can be reduced to the form

$$\frac{dN^\pm}{dt} = c\int d\Omega\int_{\epsilon(\theta)}^{\infty} n(\epsilon, \theta)(1+\beta\cos\theta) \times \phi_\pm^*(\gamma[1+\beta\cos\theta]\epsilon)d\epsilon, \quad (21)$$

where $\epsilon(\theta) = 2mc^2/\gamma(1+\beta\cos\theta)$. Thus, in the earth's coordinate system $\phi_\pm(E, \epsilon, \theta) = \phi_\pm^*(\epsilon^*)$ plays the role of the total cross section for pair creation in the collision of a proton and a photon with energies E and ϵ, respectively, and angle $\pi - \theta$ between their directions of motion. The factor $c(1+\beta\cos\theta)$ multiplying $n(\epsilon, \theta)$ arises from the relative velocity of proton and photon in the earth's reference frame. The actual expression employed for $\phi_\pm^*(\epsilon^*)$ is based on the Bethe-Heitler formulae for the differential and total

[17] In what follows, starred and unstarred symbols always refer to quantities measured in the proton's rest frame and in the earth's frame, respectively. Most of the relativistic transformation formulae used (connecting corresponding quantities in the two frames) may be found in W. Pauli, *Relativitatstheorie* (Teubner, Leipzig, 1921).

[18] Equation (20) follows from $n(\epsilon, \theta)d\epsilon d\Omega \sim A^2/\epsilon$, where A is the amplitude of the corresponding electromagnetic wave, and from $A/\epsilon = A^*/\epsilon^*$, a consequence of the transformation properties of the electric and magnetic field strengths. Alternatively, Eq. (20) can be derived from the transformation law for the time component of the null radiation energy current-density four-vector.

pair creation cross sections,[19] which treat the proton as a point electric charge, so that for the time being, any possible effect of the latter's anomalous moment (or more generally, of its enveloping meson charge cloud) is disregarded. For convenience an analytic approximation to the Bethe-Heitler $\phi_\pm^*(\epsilon^*)$ is used, *viz.*,

$$\phi_\pm^*(\epsilon^*) \cong \frac{28}{9}\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2\left[\ln\left(\frac{\epsilon^*}{6.7mc^2}\right) + \frac{1}{6}\left(\frac{\epsilon^*}{mc^2}-2\right)^3 \exp\left(-0.6\frac{\epsilon^*}{mc^2}\right)\right],$$
$$\text{for } \epsilon^* \geqq 6.7mc^2$$
$$\cong \frac{14}{27}\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2 \times\left[\left(\frac{\epsilon^*}{mc^2}-2\right)^3 \exp\left(-0.6\frac{\epsilon^*}{mc^2}\right)\right] \tag{22}$$
$$\text{for } 2mc^2 \leqq \epsilon^* < 6.7mc^2$$

where exact numerical values of $\phi_\pm^*$ at $\epsilon^*/mc^2 = 3$, 4, 5, 6, 10, 20, and 50 have been employed to fit the exponential correction term.

Consider now the case of a proton falling through the sun's radiation field. If the fall is radial, θ is effectively zero, i.e.,

$$n(\epsilon, \theta) \cong (R_e/R)^2 n_s(u)\delta(\theta)/4\pi \text{ [20]}$$

so that Eq. (21) yields

$$(c\beta)^{-1}\frac{dN^\pm}{dt} = \frac{dN^\pm}{dR} \cong 2\left(\frac{R_e}{R}\right)^2 \int_{mc^2/\gamma}^{\infty} \times n_s(u)\phi_\pm^*(2\gamma\epsilon)d\epsilon. \tag{23}$$

Equation (23) can be integrated at once to yield, $N_s^\pm$, the number of pairs produced by a proton falling radially in the sun's radiation field from infinity to the earth's orbit. With the aid of Eqs. (1) and (22), the result is,

$$N_s^\pm = 2R_e \int_{mc^2/\gamma}^{\infty} n_s(u)\phi_\pm^*(2\gamma\epsilon)d\epsilon \tag{24}$$
$$= \frac{280}{3\pi^4}\frac{e^2}{\hbar c}\left(\frac{e^2}{mc^2}\right)^2 [\Psi_{1s}(u_\pm)+\Psi_{2s}(u_\pm)]\left(\frac{\eta R_e}{kTc}\right)$$
$$\cong 2.5\frac{e^2}{\hbar c}\left(\frac{e^2}{mc^2}\right)^2 [\Psi_{1s}(u_\pm)+\Psi_{2s}(u_\pm)]Q_s,$$

in which

$$u_\pm \equiv \frac{mc^2}{kT}\frac{Mc^2}{E}$$

and

$$\Psi_{1s}(u_\pm) = \sum_{p=1}^{\infty}\frac{1}{p^3}\left[(3+3.35pu_\pm)\exp(-3.35pu_\pm) + 2\int_{3.35pu_\pm}^{\infty}\frac{e^{-x}}{x}dx\right], \tag{25}$$

$$\Psi_{2s}(u_\pm) = 8u_\pm^3 \sum_{p=1}^{\infty}\frac{1}{(1.2+pu_\pm)^6}[(pu_\pm+1.2)^2 + 8(pu_\pm+1.2)+20]\exp(-pu_\pm-1.2). \tag{26}$$

In the "extreme relativistic" case ($\epsilon^* \gg mc^2$, i.e., $u_\pm \ll 1$ or $E \gg mc^2Mc^2/kT \cong 10^6 Mc^2 \cong 10^{15}$ ev) the infinite sums in Eqs. (25) and (26) can be transformed into simple closed expressions (which are listed below in Table II) and one obtains from Eq. (24).

$$N_s^\pm(R_e) \cong 1.1\times 10^{-6}\ln\left(\frac{0.6}{u_\pm}\right). \tag{27}$$

According to Eq. (27), a proton with energy $E = 10^8 Mc^2$ produces, on the average, 5×10^{-6} electron-positron pairs in falling radially through the radiation field of the sun to the earth's orbit.[21]

[19] See, for example, W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, pp. 194–204. One sees from Eq. (20) that $\epsilon^* \sim (E/Mc^2)\epsilon \sim (E/Mc^2)kT$, so that, for example, energies E as high as 2×10^{18} ev (protons) or 10^{15} ev (electrons) correspond to rest frame photon energies ϵ^* of only 10^9 ev. Such photon energies lie in the range where the Bethe-Heitler pair creation cross section (and presumably also the Klein-Nishina scattering cross section) are at least in approximate agreement with observations on cascade showers.

[20] $\delta(\theta)$ is a Dirac delta-function normalized according to $\int\delta(\theta)d\Omega = 4\pi$.

[21] The 5×10^{-6} pairs of Eq. (27) are created in the encounter of a proton with the 3×10^{20} photons/cm² of Eq. (2). This number of photons encountered by the proton in passing (essentially) across the solar system may be compared with the number, 10^{15}, of material particles (mostly hydrogen atoms) encountered in the same passage. The last number is calculated assuming the number density of diffuse interplanetary atoms as 1/cm³. Also, protons passing through the solar system in directions other than radial generally meet numbers of photons comparable with 3×10^{20}; an exceptional case is that of grazing passage near the sun's edge where the number of photons encountered may be greater by a factor of several hundred.

More generally, without restriction on $u_{\pm}, \Psi_{1s}$, and Ψ_{2s} can be evaluated in numerical form. The results for $N_s^{\pm}$ appear in Table I. The numbers in the last column are proportional to the number of pairs per unit energy range of the primary spectrum, the latter being assumed to follow an inverse third power law.

We now estimate the integral in Eq. (18) for the average rate of energy transfer from the protons to the pairs which it creates. The difficult angular integration over θ_+^*, θ_-^* can be reduced to a known result by using the inequality

$$\epsilon_+^*+\epsilon_-^*-\beta c(p_+^*\cos\theta_+^*+p_-^*\cos\theta_-^*) > (mc^2)^2\left[\frac{1}{\epsilon_+^*+cp_+^*}+\frac{1}{\epsilon_-^*+cp_-^*}\right], \quad (28)$$

where, near the threshold for pair production, the left- and right-hand sides of Eq. (28) are each $2\ mc^2$, while far from the threshold $\langle\cos\theta^*\rangle \cong \langle\cos\theta_+^*\rangle \cong \langle\cos\theta_-^*\rangle \cong 1$[19] and the left- and right-hand sides are again equal. Consequently, the left- and right-hand sides of Eq. (28) are always of comparable magnitude, differing by no more than a factor of 2 for average deviations of θ^*, θ_+^*, and θ_-^* from the polar axis. Equation (28) yields an easily evaluated lower limit to the rate of energy loss:

$$-\frac{dE}{dt} > c(mc^2)^2\int_{2mc^2}^{\infty}\int n^*(\epsilon^*, \theta^*)d\epsilon^* d\Omega^* \times\int_{mc^2}^{\epsilon^*-mc^2}\int\int\phi^*(\epsilon^*, \epsilon_+^*, \theta_+^*, \theta_-^*) \quad (29)$$
$$\times\left[\frac{1}{\epsilon_+^*+cp_+^*}+\frac{1}{\epsilon_-^*+cp_-^*}\right]d\epsilon_+^* d\Omega_+^* d\Omega_-^*.$$

At "low" proton energies ($E \ll mc^2Mc^2/kT$, say $\cong 10^5 Mc^2$) only the far ultraviolet portion of the thermal photon spectrum is capable of creating pairs, and these are mostly produced near the threshold. Equations (17), (18), and (19) then show that the average energy of a pair is approximately $2\gamma\ mc^2$ or 10^{11} ev for $E=10^5$ Mc^2.

TABLE I. Number of pairs as a function of the primary energy.

E/Mc^2	Ψ_{1s}	Ψ_{2s}	$N_s^{\pm}\times 10^6$	$(10^8Mc^2/E)^3N_s^{\pm}\times 10^6$
10^5	——	0.00001	6×10^{-6}	6×10^3
$\frac{1}{4}10^6$	——	0.01	6×10^{-3}	4×10^5
$\frac{1}{2}10^6$	0.01	0.13	7×10^{-2}	5×10^5
10^6	0.24	0.34	3×10^{-1}	3×10^5
10^7	4.45	0.05	2	2×10^3
10^8	10.0	——	5	5
10^9	15.5	——	7	7×10^{-3}

In the "extreme relativistic" case ($\epsilon^* \gg mc^2$)

$$\int\int\phi^*(\epsilon^*, \epsilon_+^*, \theta_+^*, \theta_-^*)d\Omega_+^* d\Omega_-^* \cong 4\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2\frac{\epsilon_+^{*2}+\epsilon_-^{*2}+\frac{2}{3}\epsilon_+^*\epsilon_-^*}{\epsilon^{*3}}\cdot\left[\ln\frac{2\epsilon_+^*\epsilon_-^*}{\epsilon^* mc^2}-\frac{1}{2}\right], \quad (30)$$

and

$$-\frac{dE^{\pm}}{dt} > c\left(\frac{2e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2(mc^2)^2\int d\Omega \times\int_{2\epsilon(\theta)}^{\infty}\frac{n(\epsilon, \theta)}{\epsilon}\left(\ln\frac{\gamma\epsilon(1+\beta\cos\theta)}{4mc^2}\right)^2 d\epsilon, \quad (31)$$

where the approximations made in the derivation of Eq. (31) from Eqs. (29) and (30) have been such as to strengthen the inequality. In particular the lower limit of the integration with respect to ϵ does not extend down to the actual threshold. Also, in view of the fact that the approximate integrated cross section $\phi^*(\epsilon^*)$ (the logarithmic terms in Eq. (22)) computed from the right-hand member of Eq. (30) falls below the exact numerical values, particularly for small values of ϵ^*, we infer that Eq. (30) can be treated as an inequality ($\geqq$ replacing $\cong$) valid for all values of ϵ^* above the threshold for pair production. It follows then that Eq. (31) retains its validity for low values of the primary energy.

Equation (31) will now be applied to compute the average energy loss, $\Delta E_s^{\pm}$ experienced by a proton with $E \gg mc^2Mc^2/kT$ in the radial fall through the sun's radiation field to the earth's orbit. A simplification is possible just as in Eq. (22) since the angle θ is effectively zero, and one

TABLE II. Electron-positron pair production formulae.

$u_\pm$	$\Psi_{1n}+\Psi_{2n}$; Eq. (9)*	$\Psi_{1n}+\Psi_{2n}$; Eq. (11)	$\Psi_{1s}+\Psi_{2s}$; Eqs. (1) and (24)
$\ll 1$	$1.2\ln(0.4/u_\pm)$	$0.6(\ln 0.4/u_\pm)^2$	$2.4\ln(0.6/u_\pm)$
1	0.16	0.060	0.60
2	0.029	0.006	0.15
$\gg 1$	$(2.4/u_\pm^2)e^{-u_\pm}$	$(2.4/u_\pm^3)e^{-u_\pm}$	$(2.4/u_\pm)e^{-u_\pm}$

* The "no shift" blackbody directionally isotropic distribution applies in the case of a primary traversing the local galaxy. From Eq. (5) and Table II one has $N_g^\pm \sim N_s^\pm$. Hence the total pair production by a proton during a single traversal of the local galaxy is negligible.

finds

$$\Delta E_s^\pm = \int_{R_e}^{\infty} (c\beta)^{-1}\left(\frac{dE}{dt}\right) dR > R_e\left(\frac{2e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2$$

$$\times (mc^2)^2 \int_{2mc^2/\gamma}^{\infty} \frac{n_s(u)}{\epsilon}\left[\ln\frac{\gamma\epsilon}{2mc^2}\right]^2 d\epsilon$$

$$\cong 0.2\ln\left(\frac{1}{u_\pm}\right)\frac{(mc^2)^2}{kT} N_s^\pm$$

$$\cong 10^{12} N_s^\pm \text{ ev at } E = 10^8 Mc^2. \quad (32)$$

Thus in the "extreme relativistic" case, the average energy of a pair varies only logarithmically with the energy of the primary particle and has a value in the neighborhood of 10^{12} ev. Recalling the result for low proton energies we conclude that the average energy with which pairs are produced remains essentially constant at 10^{12} ev in the range $E \geqq 10^6 Mc^2$.

We turn now to the problem of electron-positron pair production in intergalactic space. The quantities of interest are $N_n^\pm$, the average number of pairs per proton resulting from proton-photon collisions in the distance R_0, and $\Delta E_n^\pm$, the average energy lost by the proton in producing pairs in the same distance. Since in this case the photons are isotropically distributed over direction ($n(\epsilon, \theta) = 1/4\pi \cdot n_n(\epsilon/kT)$), Eq. (21) yields,

$$N_n^\pm = \frac{R_0}{4\pi}\int d\Omega \int_{\epsilon(\theta)}^{\infty} n_n(\epsilon/kT)(1+\beta\cos\theta)$$

$$\times \phi_\pm^*(\gamma(1+\beta\cos\theta)\epsilon) d\epsilon$$

$$\cong 2R_0 \int_{\pm}^{\infty} du(kT)\phi_\pm^*(2\gamma kTu)u$$

$$\times \int_u^\infty n_n(y)\frac{dy}{y^2}, \quad (33)$$

where the second integral of Eq. (33) can be derived from the first by the series of transformations: (1) change variables of integration replacing ϵ by $u = 1/2(1+\beta\cos\theta)\epsilon/kT$ and θ by $\omega = 2(1+\beta\cos\theta)^{-1}$, (2) interchange order of integration (now possible because lower limit of integration with respect to u is independent of ω), (3) replace upper and lower limits of integration with respect to ω by ∞ and 1, respectively, (4) Put $y = u\omega$. These transformations bring to light an effective "unidirectional" spectral distribution of photons,

$$[n_n(u)]_{\text{eff}} \equiv u\int_u^\infty n_n(y)\frac{dy}{y^2}, \quad (34)$$

for the pair-production problem in intergalactic space.

We evaluate $N_n^\pm$ for the "no shift" blackbody spectral distribution of Eq. (9) and the "red shifted" spectral distribution of Eq. (11). Equation (33) can be represented as a generalization of Eq. (24) in the form

$$N_n^\pm \cong 2.5\frac{e^2}{\hbar c}\left(\frac{e^2}{mc^2}\right)^2 [\Psi_{1n}(u_\pm)+\Psi_{2n}(u_\pm)]Q_n. \quad (35)$$

With the "no shift" distribution

$$\Psi_{1n} = \sum_{p=1}^{\infty}\frac{1}{p^3}\left[\exp(-3.35pu_\pm) + \int_{3.35pu_\pm}^{\infty}\frac{e^{-x}}{x}dx\right], \quad (36)$$

$$\Psi_{2n} = 8u_\pm^2\sum_{p=1}^{\infty}\frac{1}{p}\left[\frac{4}{(1.2+pu_\pm)^5}+\frac{1}{(1.2+pu_\pm)^4}\right] \times \exp(-1.2-pu_\pm). \quad (37)$$

The "red shifted" distribution yields

$$\Psi_{1n} = \sum_{p=1}^{\infty}\frac{1}{p^3}\int_{3.35pu_\pm}^{\infty}\frac{1}{x}\left(1+\ln\frac{x}{3.35pu_\pm}\right)e^{-x}dx, \quad (38)$$

$$\Psi_{2n} = 8\sum_{p=1}^{\infty}\frac{1}{p^3}\left[\frac{pu_\pm}{(1.2+pu_\pm)^4}\exp(-1.2-pu_\pm) + \int_{1.2+pu_\pm}^{\infty}\frac{e^{-x}}{x}dx\right]. \quad (39)$$

TABLE III. Electron-positron pair production by proton-photon collisions in intergalactic space.

E/Mc^2	"No shift;" Eq. (9) $N_n^\pm$	$(10^8Mc^2/E)^3N_n^\pm$	"Red shifted;" Eq. (11) $N_n^\pm$	$(10^8Mc^2/E)^3N_n^\pm$
10^5	1.1×10^{-6}	4.8	1.1×10^{-7}	0.5
5×10^5	1.3×10^{-4}	1000	2.6×10^{-5}	210
10^6	7.0×10^{-4}	700	2.6×10^{-4}	260
10^7	7.3×10^{-3}	7.3	5.1×10^{-3}	5.1
10^8	2.0×10^{-2}	0.02	3.6×10^{-2}	0.04
10^9	3.2×10^{-2}	3×10^{-5}	9.6×10^{-2}	10^{-4}

These infinite sums have been evaluated for small, intermediate, and large values of $u_\pm$, with results which are summarized in Tables II and III.

Evidently the total number of pairs (created by all the primary particles) is not especially sensitive to the thermal photon spectral distribution. Near $E=10^8\ Mc^2$, $N_n^\pm$ attains the value 0.01; however, its variation with E is at best not more rapid than $\{\ln(E/Mc^2)\}^2$.

The analog of Eq. (32) in the present calculation is

$$\Delta E_n^\pm > R_0\left(\frac{2e^2}{\hbar c}\right)\left(\frac{e^2}{mc^2}\right)^2(mc^2)^2 \times\int_{2u_\pm}^\infty \frac{du}{u}\left[\ln\frac{u}{2u_\pm}\right]^2[n_n(u)]_{\text{eff}}, \quad (40)$$

derived from Eq. (31) with the help of the transformations listed in the discussion following Eq. (33). Inserting the spectral distributions of Eqs. (9) and (11) into Eq. (40), one gets

$$\Delta E_n^\pm > \frac{e^2}{\hbar c}\left(\frac{e^2}{mc^2}\right)^2 Q_n\frac{(mc^2)^2}{kT}\begin{cases}\sum_{p=1}^\infty \frac{1}{p}\int_{2u_\pm}^\infty\left[\ln\frac{u}{2u_\pm}\right]^2 e^{-pu}du & \text{(no shift)}\\ \sum_{p=1}^\infty \frac{1}{p^3}\int_{2u_\pm}^\infty\left[\ln\frac{u}{2u_\pm}\right]^2\left[\frac{p}{u}+\frac{1}{u^2}\right]e^{-pu}du & \text{(red shifted)}\end{cases} \quad (41)$$

which in the extreme relativistic case ($u_\pm\ll1$) becomes[22] (in ev units)

$$[\Delta E_n^\pm]^{\text{no shift}} > 5\times10^8\left[\ln\frac{2}{u_\pm}\right]^2 \cong 10^{11}\ln\left(\frac{2}{u_\pm}\right)N_n^\pm, \quad (42)$$

$$[\Delta E_n^\pm]^{\text{red shifted}} > 10^3\frac{E}{Mc^2} \cong 10^6\frac{E}{Mc^2}\left(\ln\frac{2}{u_\pm}\right)^{-2}N_n^\pm. \quad (43)$$

Both formulae agree in predicting an average energy per pair of $\sim10^{12}$ ev at $E=10^8\ Mc^2$. At higher energies, however, the "red shifted" distribution produces both an appreciably greater total energy loss and an appreciably greater average energy per pair than the "no shift" distribution.[23] For example, at $E=10^{11}\ Mc^2=10^{20}$ ev, each proton traversing a distance R_0 in the "red shifted" distribution produces, on the average, 3×10^{-1} electron-positron pairs of energy 3×10^{14} ev each.

We now discuss electron-positron pair creation in proton-photon collisions by the virtual photon method. Here one adopts the viewpoint of the earth's frame of reference and replaces the electromagnetic field of the primary proton by a set of virtual photons traveling parallel to the original direction of motion of the proton:[24] these virtual photons create pairs in collisions with thermal photons. The cross section for pair creation in the collision of a proton of energy E with a thermal photon of energy ϵ is then given by

$$\phi_{\text{pair}}(E,\epsilon,\theta)=\int_0^\infty d\epsilon_v Q(\epsilon_v)\sigma(\epsilon_v,\epsilon,\theta), \quad (44)$$

in which $Q(\epsilon_v)d\epsilon_v$ is the number of virtual photons with energies between ϵ_v and $\epsilon_v+d\epsilon_v$, and $\sigma(\epsilon_v,\epsilon,\theta)$ is the cross section for pair creation in the collision of two photons, with energies ϵ_v

[22] The sums in Eq. (41) approach, for $u_\pm\ll1$ the values $\pi/6\cdot(\ln2/u_\pm)^2$ and $1.2/u_\pm$, respectively.

[23] Considering electrons in the role of cosmic-ray primaries Eq. (43) (with M replaced by m) yields $\Delta E_n^\pm>2\times10^{-3}E$ and an average energy loss per pair

$$>2E\{\ln(10^{-6}E/mc^2)\}^{-2}.$$

These inequalities hold in the "extreme relativistic" case ($E\gg(mc^2)^2/kT\cong5\times10^{11}$ ev). For a discussion of the applicability of the above equations to electrons, see below.

[24] The general method of virtual photons is due to C. F. Weizsacker and E. J. Williams. See, for example, reference 24, pp. 263–266.

and ϵ, and with the angle $\pi-\theta$ between their original directions of motion. The cross section $\sigma(\epsilon_v, \epsilon, \theta)$ determines the probability for the process inverse to the two photon annihilation so that its value can be deduced from the Dirac formula for the annihilation probability by considerations based on microscopic reversibility.

The virtual photon distribution $Q(\epsilon_v)$, with inclusion of the effect of the proton's anomalous magnetic moment[25] as well as of its charge, can be computed as a function of b, the minimum transverse distance at which the electromagnetic field accompanying the proton still appears equivalent to a set of independent photons. It is then known that $b \cong \hbar/mc$ yields results in agreement with the Bethe-Heitler formula for a proton originally at rest and with no anomalous magnetic moment.[26] Adopting this value for b throughout the present calculation one finds that the contribution to $Q(\epsilon_v)$ from the anomalous moment is smaller than that from the charge by a factor $\sim(m/M)^2$ and thus may be disregarded. An equivalent conclusion can be reached by considering the quantum perturbation theory derivation of the cross section for pair creation in a proton-photon collision, with inclusion of the effect of the anomalous moment. Here the matrix element of the interaction potential between the current density of the created electron and the anomalous moment is smaller than the matrix element of the associated electrostatic potential by a factor $\hbar/Mcb \sim m/M$, so that the anomalous moment's contribution to the cross section is again smaller than that of the charge by the factor $(m/M)^2$. In general, the proton receives, for each pair created, a transverse momentum $\sim mc$, so that most of the pairs are created at transverse distances (from the proton's path) $\sim \hbar/mc$. This distance is large compared to the presumed dimensions of the meson cloud surrounding the proton, so that the cloud can have only a small effect.[27]

We now consider briefly the possibility of primary electrons in the cosmic radiation creating electron-positron pairs by collisions with thermal photons. If the Bethe-Heitler pair-creation formula (in the electron's rest frame) can be applied to this process then all the above formulae (Eqs. 16–43) are valid with M replaced by m. Thus, for example, an electron with energy $2\times10^8\ mc^2=10^{14}$ ev will create on the average 6×10^{-6} pairs, each with average energy $\cong10^1$ ev^2, in a radial fall through the sun's radiation field to the orbit of the earth. However, the applicability of the Bethe-Heitler formula to the description of pair creation in electron-photon collisions is questionable because the (originally stationary) electron may acquire (in its rest frame) a large recoil energy upon impact of the photon. Consequently the interaction between the primary and the created electron will not be purely electrostatic (as is necessary for the exact validity of the Bethe-Heitler formula), but will contain magnetic and retarded contributions; in addition the equivalence of the two electrons must be considered. A complete and detailed analysis of this problem has apparently not been published,[28] but a treatment by the virtual photon method of the related process of bremsstrahlung in the collision of the two electrons indicates that the order of magnitude predicted by the Bethe-Heitler formula is correct, at least in the extreme relativistic case when the primary

[25] The proton's total magnetic moment is $2.79\ e\hbar/2Mc$, so that the anomalous magnetic moment is $1.79\ e\hbar/2Mc$. From the point of view of current meson theories the anomalous moment is a manifestation of the meson charge cloud surrounding the proton. Formally, the addition of a Pauli term to the Dirac equation to describe this anomalous moment makes possible a complete phenomenological description of the interaction of the proton with an electromagnetic field, without explicit reference to the meson field.

[26] E. J. Williams, Kgl. Danske Vid. Sels. Math.-Fys. Medd 13, 17 (1935).

[27] The mean transverse momentum, $p_\pm \sin(\pi-\theta_\pm) \cong \epsilon_\pm/c\cdot(\pi-\theta_\pm)$, carried off by a pair and so received by the proton can be estimated from the relation $\theta_\pm^* \sim mc^2/\epsilon^*$; $\epsilon_+ + \epsilon_- \sim (mc^2)^2/kT$ which are consequences of the Bethe-Heitler formula and of Eq. (32). From these relations and from the transformation formula, $\tan\theta_\pm = \gamma^{-1}\sin\theta_\pm^*/(\cos\theta_\pm^* - \beta)$, one immediately obtains $\pi-\theta_\pm \sim kT/mc^2 \sim 10^{-6}$ and $p_\pm \sin(\pi-\theta_\pm) \sim mc$. The same result follows directly from the virtual photon picture

$$(\pi-\theta_\pm \sim mc^2/(\epsilon_v+\epsilon) = mc^2/(\epsilon_+ + \epsilon_-) \sim kT/mc^2).$$

[28] However, see A. Borsellino, Helv. Phys. Acta 20, 136 (1947); P. Nemirovsky, J. Phys. U.S.S.R. 11, 94 (1947); J. A. Wheeler and W. Lamb, Phys. Rev. 55, 858 (1939). The Bethe-Heitler formula (for pair creation in proton-photon collisions) is, of course, not applicable to pair creation in electron-photon collisions near the threshold ($4mc^2$) of the latter process. *Note added in proof:* K. W. Watson, Phys. Rev. 72, 1060 (1947), has just given a thorough discussion of electron-positron pair creation in photon-electron collisions, and has found a cross section which, in the extreme relativistic case, is approximately double the Bethe-Heitler cross section of Eq. (22).

energy greatly exceeds the energy given the electron-positron pair.

The possibility of meson and nucleon creation in proton-photon collisions (processes 4–6) should also be mentioned. Three types of meson processes are conceivable in such collisions: (a) the transformation of a proton into a neutron and a positive meson, (b) the creation of mesons in (odd as well as even) multiples, and (c) the creation of a meson pair as a result of electromagnetic interactions alone. The cross section for (*c*) in a proton-photon collision is probably smaller than the cross section for the creation of an electron-positron pair by a factor $\sim(m/\mu)^2$ and is therefore negligible.[29] A further safeguard here (and particularly for (b)) is the high threshold energy which makes the overwhelmingly large proportion of the primary spectrum ineffective in producing meson (and, *a fortiori*, nucleon) pairs. However, in the extreme relativistic case, (a) (and/or (b)) may conceivably occur with a cross section comparable in magnitude to that for electron-positron pair creation.[30]

Finally, it may be remarked that the eventual (time dilated) radioactive decay of any neutrons and positive mesons formed in proton-photon collisions gives rise to a "delayed" electron-positron pair production.[31] Detailed investigation of such transformations must await a better understanding of the whole process of meson creation in proton-photon collisions.

[29] See R. F. Christy and S. Kusaka, Phys. Rev. **59**, 405 (1941); J. R. Oppenheimer, H. Snyder, and R. Serber, *ibid.* **57**, 75 (1940); μ is the meson mass, $\sim$200 m.

[30] For example, J. Hamilton and H. W. Peng, Proc. Roy. Ir. Ac. **49A**, 197 (1944); C. Morette and H. W. Peng, Nature **160**, 60 (1947), give 5×10^{-27} cm² as the asymptotic value (for $\epsilon^*\gg\mu c^2$) of the cross section for (a), while results of the same general order of magnitude are obtained by M. Lax and H. Feshbach (private communication); this value is matched by the $\phi_\pm^*$ of Eq. (22) for $\epsilon^*\cong\mu c^2$. If, however, there is no negative meson capture by light nuclei, one must conclude that single meson creation in proton-photon collisions is an extremely rare process, at least if only one kind of meson exists and/or if nucleons have no excited "meson-pregnant" states. In this connection see, E. Fermi, E. Teller, and V. Weisskopf, Phys. Rev. **71**, 314 (1947); J. A. Wheeler, *ibid.* **71**, 320 (1947); R. E. Marshak and H. A. Bethe, *ibid.* **72**, 506 (1947); V. Weisskopf, *ibid.* **72**, 510 (1947); S. Sakata and T. Inoue, Progress of Theoretical Physics **1**, 143 (1946).

[31] "Delayed" electron-positron pairs also arise from the creation (and subsequent decay) of neutrons and mesons in collisions of primary protons with the hydrogen nuclei of interstellar diffuse matter. See Appendix C.

4. COMPTON SCATTERING IN PROTON-PHOTON AND ELECTRON-PHOTON COLLISIONS

We shall now discuss those collisions of the primary cosmic-ray protons and electrons with thermal photons which result in Compton scattering. In the primary rest frame the photon energies ϵ^* and ϵ'^* before and after scattering, and the scattering angle χ^* are connected by the relation,[32]

$$\epsilon'^*=\frac{\epsilon^*}{1+\dfrac{(\epsilon^*)^2}{Mc^2}(1-\cos\chi^*)}. \tag{45}$$

Correspondingly, in the earth's frame,[33]

$$\begin{aligned}\epsilon'&=\gamma\epsilon'^*(1-\beta\cos\theta'^*)\\&=\frac{\gamma^2\epsilon(1+\beta\cos\theta)(1-\beta\cos\theta'^*)}{1+\dfrac{\epsilon^*}{Mc^2}(1-\cos\chi^*)}\\&=\epsilon\frac{1+\beta\cos\theta}{1+\beta\cos\theta'+\dfrac{\epsilon}{E}(1-\cos\chi)}\\&\cong\epsilon\frac{1+\beta\cos\theta}{1+\left\{\beta-\dfrac{\epsilon}{E}(1+\beta\cos\theta)\right\}\cos\theta'},\end{aligned} \tag{46}$$

where Mc^2 and E are the rest energy and total energy, respectively, of the primary particle.

We require general integral expressions for the average number of scattered photons per

[32] See, for example, reference 19, pp. 146–161.

[33] The transformation properties of the photon energy-momentum four vector yields,

$$1-\cos\chi^*=[\gamma^2(1+\beta\cos\theta)(1+\beta\cos\theta')]^{-2}(1-\cos\chi)$$

with $\pi-\theta$ and $\pi-\theta'$ the angles between the original directions of motion of the primary and the directions of motion of the photon before and after the scattering. This relation and the relativistic Doppler formulae connecting ϵ'^*, ϵ', and ϵ^*, ϵ make possible the derivation of the third expression for ϵ' from the second (in Eq. (46)). This third expression can also be deduced directly by an application of the energy-momentum conservation laws in the earth's frame to the particle-photon collision. The fourth simplified, but entirely adequate form for ϵ' can be derived by considering the behavior of the denominator in the third form for small values of $\pi-\theta'$.

primary per unit time (dN^c/dt) and the average energy loss per primary per unit time ($-dE^c/dt$). A procedure analogous to that employed in the derivation of Eqs. (17), (18), and (21) yields,

$$\frac{dN^c}{dt}=c\int d\Omega\int_0^\infty n(\epsilon,\theta)(1+\beta\cos\theta)\times\phi_c^*(\epsilon^*)d\epsilon, \quad (47)$$

$$-\frac{dE^c}{dt}=c\int d\Omega\int_0^\infty n(\epsilon,\theta)(1+\beta\cos\theta)d\epsilon\times\int\phi_c^*(\epsilon^*,\chi^*)(\epsilon'-\epsilon)d\Omega^*. \quad (48)$$

Here $\phi_c^*(\epsilon^*,\chi^*)d\Omega^*$ and $\phi_c^*(\epsilon^*)$ are the differential and total cross sections for Compton scattering evaluated in the rest frame of the primary. For the time being we neglect all effects connected with the anomalous magnetic moment of the proton, or, more generally, with its surrounding meson charge cloud, and evaluate Eqs. (47) and (48) with the aid of the Klein-Nishina cross sections (presumably rigorously valid for point electric charges):[34]

$$\phi_c^*(\epsilon^*,\chi^*)\begin{cases}\nearrow \dfrac{1}{2}\left(\dfrac{e^2}{Mc^2}\right)^2(1+\cos^2\chi^*), & \epsilon^*\ll Mc^2\\ \searrow \dfrac{1}{2}\left(\dfrac{e^2}{Mc^2}\right)^2\dfrac{\epsilon'^*}{\epsilon^*}, & \epsilon^*\chi^{*2}\gg Mc^2,\end{cases} \quad (49)$$

$$\phi_c^*(\epsilon^*)\begin{cases}\nearrow \dfrac{8\pi}{3}\left(\dfrac{e^2}{Mc^2}\right)^2, & \epsilon^*\ll Mc^2\\ \searrow 2\pi\left(\dfrac{e^2}{Mc^2}\right)^2\dfrac{Mc^2}{2\epsilon^*}\ln\dfrac{2\epsilon^*}{Mc^2}, & \epsilon^*\gg Mc^2.\end{cases} \quad (50)$$

Employing Eqs. (47) and (50) we obtain

$$\frac{dN^c}{dR}\cong\int d\Omega\int_0^{\epsilon(\theta)} n(\epsilon,\theta)(1+\beta\cos\theta)\times\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]d\epsilon+\int d\Omega\int_{\epsilon(\theta)}^\infty n(\epsilon,\theta)(1+\beta\cos\theta)\times\left[2\pi\left(\frac{e^2}{Mc^2}\right)^2\frac{Mc^2}{2\epsilon^*}\ln\frac{2\epsilon^*}{Mc^2}\right]d\epsilon \quad (51)$$

in which $\epsilon(\theta)\equiv Mc^2/2\gamma(1+\beta\cos\theta)$. A similar procedure applied to Eq. (48) with explicit utilization of Eqs. (46) and (49) yields

$$-\frac{dE^c}{dR}\cong\int d\Omega\int_0^{\epsilon(\theta)} n(\epsilon,\theta)(1+\beta\cos\theta)\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]\Delta E^{(1)}d\epsilon+\int d\Omega\int_{\epsilon(\theta)}^\infty n(\epsilon,\theta)(1+\beta\cos\theta)\left[2\pi\left(\frac{e^2}{Mc^2}\right)^2\frac{Mc^2}{2\epsilon^*}\ln\left[1+\frac{2\epsilon^*}{Mc^2}\right]\right]\Delta E^{(2)}d\epsilon, \quad (52)$$

where the quantities

$$\Delta E^{(1)}=\frac{\gamma\epsilon^*}{1+\epsilon^*/Mc^2},\qquad \Delta E^{(2)}=E\left[1-\frac{2\epsilon^*/Mc^2}{(1+2\epsilon^*/Mc^2)\ln(1+2\epsilon^*/Mc^2)}\right] \quad (53)$$

are approximately the average energy losses of the primary in a single Compton collision for $2\epsilon^*<Mc^2$ and $2\epsilon^*>Mc^2$, respectively.

Equations (51)–(53) and Eqs. (1) and (2) are now used to compute the average number of collisions, N_s^c, and the average energy loss, ΔE_s^c, experienced by a primary falling radially from infinity through the sun's radiation field to the orbit of the earth ($\theta\cong 0$). It is convenient to distinguish two extreme cases: "rest-frame non-relativistic" and "rest-frame extreme relativistic," depending on whether $\epsilon^*=\gamma\epsilon(1+\beta\cos\theta)$ is $\ll Mc^2$ or $\gg Mc^2$ for $\epsilon\cong 2.7\ kT$. An equivalent statement is

$$u_c=\frac{(Mc^2)^2}{EkT}\begin{cases}\gg 1 & \text{("rest-frame non-relativistic" } E\ll 2\times 10^{18}\text{ ev-protons; } E\ll 5\times 10^{11}\text{ ev-electrons)}\\ \ll 1 & \text{("rest-frame extreme relativistic").}\end{cases} \quad (54)$$

In the "rest-frame non-relativistic" case one

[34] See reference 19, pp. 146–160.

obtains

$$N_s{}^c \cong 2R_e\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]\int_0^\infty n_s(u)d\epsilon$$

$$=2\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]Q_s$$

$$\cong 1.2\times 10^{-10}(\text{protons})$$

$$\cong 4.0\times 10^{-4}(\text{electrons}), \tag{55}$$

$$\Delta E_s{}^c \cong 2R_e\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]\int_0^\infty n_s(u)2\gamma^2\epsilon d\epsilon$$

$$=2\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]5.4kT\gamma^2Q_s \cong \frac{10^9}{u_c{}^2}\text{ ev}. \tag{56}$$

The average energy of the scattered photons, $5.4kT\gamma^2=5.4E/u_c$, is comparatively large, but the total energy loss is scarcely sufficient to produce an observable intensity of scattered radiation. Recalling Eq. (5), it is clear that the same conclusion holds for a single traversal of the local galaxy.

In the "rest-frame extreme relativistic" case, Eqs. (1), (2), and (51)–(53) give

$$N_s{}^c \cong 2R_e\int_{u_c/4}^\infty\left[2\pi\left(\frac{e^2}{Mc^2}\right)^2\frac{u_c}{4u}\ln\frac{4u}{u_c}\right]n_s(u)kTdu$$

$$\cong\frac{16\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2Q_s\left[\frac{u_c}{8}\ln\frac{2}{u_c}\right]$$

$$\cong 5\times 10^7\frac{kT}{E}\ln\frac{2}{u_c}. \tag{57}$$

$$\Delta E_s{}^c \cong E\left(1-1/\ln\frac{4}{u_c}\right)N_s{}^c$$

$$\cong 2.5\times 10^7\ln\frac{0.7}{u_c}\text{ ev}. \tag{58}$$

On the average, a single Compton collision reduces the energy of the primary particle from $E(\gg(Mc^2)^2/kT)$ to $E/\ln(4/u_c)$; thus, for example, an eighty-five percent loss in a single collision requires $u_c\cong 10^{-2}$ or $E\cong 2\times 10^{20}$ ev (protons), $E\cong 5\times 10^{13}$ ev (electrons). It is clear that a considerable number of collisions are required to degrade a particle from "rest-frame extreme relativistic" to "rest-frame non-relativistic;" also the small value of $N_s{}^c$, even for electrons, and its decrease with E, shows that the occurence of many such collisions is extremely rare in a single traversal of the solar system or the local galaxy.

We now consider Compton collisions in intergalactic space. Here the distribution in direction of photons is isotropic $(n(\epsilon,\theta)=(1/4\pi)n_n(u))$, so that the transformations described after Eq. (34) are applicable and reduce Eqs. (51) and (52) to

$$R_0\frac{dN^c}{dR}\cong\left[\frac{16\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]\left[\int_0^{u_c/4}[n_n(u)]_{\text{eff}}(kTdu)+\frac{3}{4}\int_{u_c/4}^\infty\frac{u_c}{4u}\ln\frac{4u}{u_c}[n_n(u)]_{\text{eff}}(kTdu)\right], \tag{59}$$

$$-R_0\frac{dE^c}{dR}\cong\left[\frac{16\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2\right]\left[\frac{1}{2}E\int_0^{u_c/4}\frac{4u/u_c}{1+2u/u_c}[n_n(u)]_{\text{eff}}(kTdu)\right.$$

$$\left.+\frac{3}{4}E\int_{u_c/4}^\infty\frac{u_c}{4u}\ln\left(1+\frac{4u}{u_c}\right)\left\{1-\frac{4u/u_c}{(1+4u/u_c)\ln(1+4u/u_c)}\right\}[n_n(u)]_{\text{eff}}(kTdu)\right]. \tag{60}$$

The function $[n_n(u)]_{\text{eff}}$ is defined by Eq. (34); performing the integration there indicated we obtain the explicit formulae

$$[n_n(u)]_{\text{eff}}^{\text{no shift}}=\frac{Q_n}{2.4kTR_0}u\ln(1-e^{-u})^{-1}, \tag{61}$$

$$[n_n(u)]_{\text{eff}}^{\text{red shifted}}=\frac{Q_n}{2.4kTR_0}\frac{1}{u}\int_u^\infty y\ln(1-e^{-y})^{-1}dy. \tag{62}$$

Under the "rest-frame non-relativistic" condition the limit of integration $u_c/4$ in Eqs. (59) and (60) may be replaced by infinity, thus eliminating the second of the two integrals appearing in each equation. Equations (59)–(62) then give

$$R_0\frac{dN^c}{dR}\begin{cases}\nearrow\cong\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2Q_n\right] & \text{(no shift)}\\ \searrow\text{logarithmic infinity} & \text{(red shifted)},\end{cases} \tag{63}$$

and

$$-R_0\frac{dE^c}{dR}\cong\left[\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2Q_n\right]3.6kT\gamma^2 \cong\frac{3.5\times10^{12}}{u_c^2}\text{ ev.} \quad (64)$$

It is interesting that the average space rate of energy loss has the same value for both "no shift" and "red shifted" distributions since this average rate depends only on the total photon energy density—see Eqs. (60)–(62), and (12). On the other hand, Eq. (63) shows that the average space rate at which collisions occur is critically dependent on the choice of spectral distribution. In the "no shift" case, the average number of collisions per primary in the distance R_0 is

$$N_n^c\cong\frac{8\pi}{3}\left(\frac{e^2}{Mc^2}\right)^2Q_n \begin{cases}\cong5\times10^{-7}\text{ (protons)}\\ \cong2\text{ (electrons).}\end{cases} \quad (65)$$

According to Eq. (65), the probability that an electron with $u_c\gg1$ suffers a collision in traveling the distance R_0 through the "no shift" distribution is $1-e^{-2}=0.85$. The corresponding average energy of the scattered photons is $3.6kT\gamma^2=3.6E/u_c$ or two-thirds the value computed earlier for unidirectional radiation moving directly towards the primary particle.

Regarding numerical values of the primary's energy loss ΔE_n^c, Eq. (64) yields $-R_0dE/dR\cong3.5\times10^{10}$ ev at $u_c=10(E\cong2\times10^{17}$ ev (protons), $E\cong5\times10^{10}$ ev (electrons)). Thus, in the distance R_0, the total energy loss of a proton is negligible; on the other hand, an electron loses a good fraction of its original energy. For an accurate evaluation of the energy loss of an electron we integrate Eq. (64), obtaining

$$\frac{mc^2}{E-\Delta E_n^c}-\frac{mc^2}{E}\cong\left[\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2Q_n\right]3.6kT/mc^2 \cong7\times10^{-6}. \quad (66)$$

At $E=5\times10^{10}$ ev, this loss amounts to 40 percent. It is clear that the energy loss experienced by such electrons as a result of their interaction with starlight (i.e., thermal photons in intergalactic space) over long periods of time ($\cong1.85\times10^9$ years) must be considered in interpreting the energy spectrum exhibited by these particles in the neighborhood of the earth (if they are indeed found to occur in the primary cosmic radiation).

We turn now to the "rest-frame extreme relativistic" case: $u_c\ll1$. It will be helpful to discuss the results for the "no-shift" and "red shifted" distributions separately beginning with the former. Introducing Eq. (61) into Eqs. (59) and (60) the first integrals in each equation may be omitted and the second readily evaluated with the results

$$R_0\frac{dN^c}{dR}\cong\frac{Q_n}{Q_s}N_s^c\cong10^4N_s^c,$$
$$-R_0\frac{dE^c}{dR}\cong E(1-1/\ln4/u_c)N_n^c. \quad (67)$$

The average number of collisions, N_n^c, and the average energy loss, ΔE_n^c, per primary in the distance R_0, predicted by Eq. (67), are quite negligible for protons ($E\gg2\times10^{18}$ ev); for electrons $N_n^c\cong0.4$ at $E=10^{12}$ ev, but decreases rapidly as E increases (compare Eq. 57). Results from Eqs. (64)–(67) may be summarized by the statement that electrons in the energy range 10^{10} to 10^{12} ev traveling a distance R_0 in intergalactic space (or a distance $\sim10^8$ l.y. in the galaxy—see reference 15a) experience a substantial fractional energy loss if the space is filled with the "no shift" photon distribution; however, if the initial energy falls outside the above-mentioned range the fractional loss of energy is relatively small.

Results of an entirely different character follow from the "red shifted" thermal photon distribution of Eqs. (11) and (62). Here $n_n(u)$ and $[n_n(u)]_{\text{eff}}$ vary inversely as u for small u so that the first integral in Eq. (59) is again infinite (as in Eq. (63)), signifying that an infinite number of "small energy transfer" collisions ($\Delta E^{(1)}$ of Eq. (53) $\ll E$) occur. The corresponding average number of "large energy transfer" collisions ($\Delta E^{(2)}$ of Eq. (53) $\sim E$) is given by the second integral in Eq. (59) with the approximate value

$$R_0\left[\frac{dN^c}{dR}\right]^{\text{red shifted}}_{\substack{\text{large energy}\\\text{transfer}}} \cong2\pi\left(\frac{e^2}{Mc^2}\right)^2Q_n \begin{cases}\cong4\times10^{-7}\text{ (protons)}\\ \cong1.6\text{ (electrons).}\end{cases} \quad (68)$$

It is particularly interesting that Eq. (68) predicts an average number of "large energy transfer" collisions per primary independent of the primary's energy (compare with the inverse dependence on E in the "no shift" case (Eq. (67)) and in the solar problem (Eq. (57)). This behavior is a consequence of the fact that the falling off of the Klein-Nishina cross section with increasing E (Eq. (50)) is compensated for by the associated increase in the number of photons capable of producing large energy transfers

$$\left([n_n(u_c)]_{\text{eff}}^{\text{red shift}} \sim \frac{1}{u_c} \sim E\right).$$

Both integrals in Eq. (60) make finite contributions to the average energy loss $\Delta E_n{}^c$ in the distance R_0. The primary particle loses energy partly in an infinite number of small dribbles (first integral in Eqs. (52) and (60)) and partly in an occasional catastrophic collision (second integral in Eqs. (52) and (60)). Evaluating the integrals we get

$$-R_0\left[\frac{dE^c}{dR}\right]_{\substack{\text{small energy}\\ \text{transfer}}}^{\text{red shifted}} \cong \pi\left(\frac{e^2}{Mc^2}\right)^2 Q_n E \begin{cases} \nearrow \cong 2\times 10^{-7}E \text{ (protons)} \\ \searrow \cong 0.8E \text{ (electrons)}, \end{cases} \tag{69}$$

$$-R_0\left[\frac{dE^c}{dR}\right]_{\substack{\text{large energy}\\ \text{transfer}}}^{\text{red shifted}} \cong 2\pi\left(\frac{e^2}{Mc^2}\right)^2 Q_n (E\ln 2) \begin{cases} \nearrow \cong 3\times 10^{-7}E \text{ (protons)} \\ \searrow \cong 1.1E \text{ (electrons)}, \end{cases} \tag{70}$$

$$\begin{aligned} [\Delta E_n{}^c]^{\text{red shifted}} &\cong E\left[1-\exp\left(-2.4\pi\left(\frac{e^2}{Mc^2}\right)^2 Q_n\right)\right] \\ &\cong 5\times 10^{-7}E \text{ (protons)} \\ &\cong 0.85E \text{ (electrons)}. \end{aligned} \tag{71}$$

Equations (68)–(71) show that the distance R_0 in an intergalactic space filled with "red shifted" photons is almost perfectly transparent to protons, but is beginning to exhibit appreciable stopping power for electrons. As computed, the stopping power is not quite large enough to cut off completely a possible primary electron (or positron) spectrum above 10^{12} ev. However, the assumption that the particles reaching the earth have traveled a distance of just R_0 (in intergalactic space) is, of course, only an order of magnitude estimate. Possibly, also, the assumption of uniform conditions in space and time is seriously at fault. Thus for example, if all uncertainties in the evaluation of the energy loss are equivalent to replacing the distance traveled by $3.5R_0$, primary electrons would lose 99.9 percent of their initial energy before reaching the earth rather than 85 percent as given by Eq. (71). Thus the significant fact about the numerical coefficient for electrons in Eq. (71) is that it does not differ by an order of magnitude from the least value required to eliminate effectively the electron spectrum above 10^{12} ev.

In the light of the above remarks one sees a possible explanation for the apparent small number of high energy electrons ($E > 10^{12}$ ev) among the particles striking the earth's atmosphere.[35] Even on the natural supposition that energetic protons and electrons were originally produced in comparable numbers, the attenuation due to Compton scattering, of the latter component in a travel time $\cong 1.85\times 10^9$ years (i.e., travel distance $\cong R_0$) may be sufficient to account for the observed preponderance of protons.[1]

The scattered photons produced in "large energy transfer" collisions give rise to an intergalactic cascade process. Such photons have energies of the order $0.7E$ and are therefore capable of producing electron-positron pairs by collision with the thermal photons. The cross section for this process, $\phi_{p-p}{}^{\pm}$,[2] may be ap-

[35] I. Pomeranchuk, J. Phys. U.S.S.R. **2**, 65 (1940) shows that electrons with original energy $\gg 10^{17}$ ev will be degraded to $\cong 10^{17}$ ev by radiation losses in the earth's magnetic field, while the radiation losses for electrons of initial energy below 10^{17} ev are relatively insignificant.

proximated by

$$\phi_{p-p}^{\pm}(\epsilon_{sc}, \epsilon, \theta) \begin{cases} 2\pi\left(\dfrac{e^2}{mc^2}\right)^2 \dfrac{\ln 2x}{2x}, & \text{if } 2x>1 \\ 0, & \text{if } 2x<1, \end{cases} \quad (72)$$

in which $x=\epsilon_{sc}\epsilon(1+\cos\theta)/(mc^2)^2$, ϵ_{sc} and ϵ are the energies of the scattered photon and of the thermal photon, respectively, and $\pi-\theta$ is the angle between their directions of motion. Applying once more the methods used in deriving Eqs. (47), (51), and (59), we obtain

$$R_0\frac{dN^{\pm,\, p-p}}{dR}=R_0\int d\Omega\int_{\epsilon(\theta)}^{\infty} n(\epsilon,\theta) \times(1+\beta\cos\theta)\phi_{p-p}^{\pm}d\epsilon \cong 2\pi\left(\frac{e^2}{mc^2}\right)^2 Q_n \cong 1.6, \quad (73)$$

where $\epsilon(\theta)\equiv(mc^2)^2/2\epsilon_{sc}(1+\cos\theta)$, and the second line holds only for the "red shifted" distribution of thermal photons (compare with Eq. (68)). Thus the probability of pair creation by the high energy photon-thermal photon process is the same (about 0.8 in the distance R_0) as the probability of the "large energy transfer" collision between the original electron and the thermal photons. One can now visualize the formation of a cascade of electrons, positrons, and photons in intergalactic space with R_0 playing the role of the characteristic radiation length; the degradation of the energy of the original primary electron occurs in this cascade *via* the process of "large energy transfer" to scattered photons,[36] the subsequent and equally probable process of pair creation by these energetic photons in collisions with thermal photons[37] and the simultaneous and slightly less probable process (compare Eqs. (69) and (70)) of "small energy transfer" in electron-thermal photon collisions.[38]

The double Compton scattering (bremsstrahlung) process listed in the introduction (process 3) requires a brief discussion. In this case, there is initially a primary particle and a thermal photon; after the collision there are two energetic photons and a particle with reduced energy. The cross section for this process has been estimated by Heitler and Nordheim,[39] who find that it is smaller than the Klein-Nishina cross section by factors $e^2/\hbar c(\epsilon^*/Mc^2)^2$ and $e^2/\hbar c$, according to whether $\epsilon^*/Mc^2 \ll$ or $\gg 1$. This estimate involves the restriction that both scattered photons have comparable energies, and in the case $\epsilon^*/Mc^2 \gg 1$, the additional restriction that the two angles of scattering (in the rest frame) are not too small; in general these restrictions are satisfied if an appreciable energy transfer occurs. We may thus conclude that energy losses by the bremsstrahlung process are less probable than by Compton scattering in a ratio which does not exceed 1/137, so that the bremsstrahlung process is unimportant.

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APPENDIX A. THE "RADIOFREQUENCY" SPECTRUM

We have so far considered only the radiation originating from the individual stars under the simplifying assumption that the corresponding frequency distribution conforms closely to that of blackbody radiation at 6000°K. Actual measurements of extra-terrestrial radiofrequency noise, however, reveal an intensity with a pronounced maximum in the direction of the galactic center and greatly in excess of that computed from our assumption of dilute blackbody radiation.[16] The following mechanism has been proposed to account for this observation:[16] hydrogen atoms in interstellar space are ionized by stellar radiation. The resulting electron gas possesses a mean kinetic energy corresponding to a temperature ≅10,000° Abs. Inelastic collisions between the free electrons and protons and

[36] Analogous to photon creation by the bremsstrahlung process in an ordinary cascade.

[37] Analogous to pair creation by photons colliding with nuclei and electrons in an ordinary cascade.

[38] Analogous to the energy loss by ionization in an ordinary cascade.

[39] Reference 19, p. 178; W. Heitler and L. Nordheim, Physica **1059** (1934). See also O. Halpern and N. Kroll, Phys. Rev. **72**, 82 (1947); C. J. Eliezer, Proc. Roy. Soc. **A187**, 210 (1946); **191**, 133 (1947); R. Jost, Phys. Rev. **72**, 815 (1947).

recombination produce a radiation spectrum extending from the near infra-red down to 3×10^7 cycles/sec. with essentially constant intensity. Below 3×10^7 cycles/sec. the spectrum coincides with that of blackbody radiation at 10,000°K. With reasonable values for the electron-proton density, the computed intensity at 5 meters, for example, is then found to agree with the observations.

Guided by the plot of the theoretical spectrum we represent the intragalactic radiofrequency photon density by the simple distribution

$$n_g'(u)d\epsilon = B\frac{du}{u}e^{-\alpha u},\quad u \geqq 10^{-7}\ \text{ev}/kT = 2\times10^{-7} \equiv u_m$$
$$= B\frac{udu}{u_m^2},\quad u<u_m,\qquad \text{(A1)}$$

in which $B\sim2\Omega/(10^3)^2\cdot 1$ ev$/hc\sim(\Omega/60)$ cm^{-3}. Here Ω is the effective solid angle subtended by the radiation incident on the earth. It is clear from the directional characteristics of the observed intensity that Ω is a small fraction of 4π. The value $\Omega\sim1$ steradians appears reasonable and certainly not in error by an order of magnitude. With $\Omega\sim1$ the total radiofrequency energy density has the value

$$U_g' = \int_0^\infty \epsilon n_g'(u)d\epsilon = \frac{BkT}{\alpha}\sim\frac{1}{120\alpha}\ \text{ev/cm}^3.\qquad \text{(A2)}$$

We have estimated the total energy density U_g of intragalactic starlight at the solar system as 3×10^{-2} ev/cm^3 (see reference 9a). Since the energy source producing the free electron gas is stellar radiation it appears necessary to require $U_g'\ll U_g$. Consequently $\alpha\gg1$. The calculations presented below are based on the choice $\alpha=10$, which effectively cuts off the spectrum at $10u\sim1$ or $\epsilon\sim5\times10^{-2}$ ev. Fortunately, in the application to the cosmic-ray problem, the results do not depend in a critical manner on the choice of α.

The interaction of cosmic-ray primaries with the intragalactic "radiofrequency" spectrum is particularly interesting in conjunction with the hypothesis that the primaries are produced within the galaxy and prevented from escaping by an intragalactic magnetic field.[15] Adopting this hypothesis for the moment consider first the "rest-frame extreme relativistic" Compton scattering by an electron constrained to spiral indefinitely within the galaxy. Equations (60) and (34) can be adapted to the present problem by the substitution $n_g'(u)$ for $n_n(u)^{\text{no shift}}$; in this way one gets

$$[n_g'(u)]_{\text{eff}} = \frac{B}{kT}u\int_u^\infty \frac{e^{-\alpha y}}{y^3}dy,\quad u>u_m$$
$$\cong \frac{B}{kT}\frac{u}{u_m^2}\ln\frac{1.7u_m}{u},\quad u<u_m,\qquad \text{(A3)}$$

and

$$-R_0\frac{dE^c}{dR}\sim2.5\pi\left(\frac{e^2}{Mc^2}\right)^2R_0BE\sim17E\qquad \text{(A4)}$$

in the range $u_m\ll u_c/4\ll\alpha^{-1}\sim0.1$. For $u_c/4\ll u_m$, the second line of Eq. (A3) implies

$$-R_0\frac{dE}{dR}\sim4\pi\left(\frac{e^2}{mc^2}\right)^2R_0BE(u_c/4u_m)$$
$$\times\ln(1+4u_m/u_c)\ln(7u_m/u_c)$$
$$\sim2\times10^{19}\ln(1+4u_m/u_c)\ln(7u_m/u_c)\ \text{ev}.\qquad \text{(A5)}$$

From these relations it is apparent that the intensity of an electron component in the energy range 10^{12} ev$\ll E\ll5\times10^{17}$ ev is reduced by an extremely large factor in a time $\cong1.85\times10^9$ years. It will also be noted that α appears only in the determination of the upper limit on u_c. Thus, if α has been underestimated by a factor of 10, the lower limit on the energy range, 10^{12} ev, should be replaced by 10^{13} ev.

At the present stage of observational and theoretical study of the intragalactic "radiofrequency" noise and of a possible intragalactic magnetic field one may properly hesitate to draw far reaching conclusions from these numerical results. But granting the various assumptions one can understand why the cosmic radiation incident on the earth's atmosphere should contain few electrons (and positrons) in the energy range 10^{12} ev $\ll E\ll5\times10^{17}$ ev. On the other hand, this theory does not provide a mechanism for the elimination of electrons with energies $\gg5\times10^{17}$ ev.[35] Electrons of such high energy, however, must, in any event, be considered separately since the galactic magnetic field which has been proposed[15] is not strong enough to constrain within the galaxy particles with energies in excess of 10^{16} ev.[40]

We now apply a principle of uniformity and postulate the existence of the "radiofrequency" spectrum as a feature common to all or nearly all galaxies. There must then exist an intergalactic photon number density $n_n'(u)d\epsilon$ related to the intragalactic "radiofrequency" photons in the same way that $n_n(u)d\epsilon$ is related to the intragalactic thermal photons. These considerations suggest the relations (in the range $u>u_m$),

$$n_n'(u)d\epsilon \sim \frac{1}{10}B\frac{du}{u}e^{-\alpha u} \text{ (no shift)} \tag{A6}$$

$$\sim \frac{1}{10}B\frac{du}{u}\int_u^\infty \frac{e^{-\alpha y}}{y}dy \text{ (red shifted)}, \tag{A7}$$

and, using Eqs. (A6), (A7), and (34),

$$[n_n'(u)]_{\text{eff}}^{\text{no shift}} \sim \frac{B}{20kT}\frac{1}{u}, \quad u_m \ll u \ll \alpha^{-1}, \tag{A8}$$

$$[n_n'(u)]_{\text{eff}}^{\text{red shifted}} \sim \frac{B}{20kT}\frac{1}{u}\ln\left(\frac{1}{3\alpha u}\right), \quad u_m \ll u \ll \alpha^{-1}. \tag{A9}$$

The factor 1/10 has been taken from the estimated ratio of the thermal photon energy densities in intragalactic and intergalactic space (Eqs. (5a) and (13)); the same factor is assumed to give the ratio of the corresponding radiofrequency energy densities. We substitute $[n_n'(u)]_{\text{eff}}$ for $[n_n(u)]_{\text{eff}}$ in Eq. (60) and compute the following results for electrons:

$$-R_0\frac{dE^c}{dR} \cong 1.7E \text{ (no shift)} \tag{A10}$$

$$\cong 1.7E\ln\left(\frac{1}{3\alpha u_c}\right) \text{ (red shifted)} \tag{A11}$$

in the range $u_m \ll u_c/4 \ll \alpha^{-1}$.

Because of the accidental numerical coincidence ($1.9E$ from Eqs. (69) and (70) and $1.7E$ in Eq. (A10)) the discussion of the stopping power of the "red shifted" thermal photon distribution following Eq. (71) can be immediately applied to the "no shift radiofrequency" distribution with the added restriction $E<5\times10^{17}$ ev in the latter case. Further, the presence of a "red shifted radiofrequency" spectrum renders distances R_0 in intergalactic space practically impenetrable to electrons with energies between 10^{12} and 5×10^{17} ev. For example, Eq. (A11) shows that

$$[\Delta E_n^c]_{\text{radiofreq.}}^{\text{red shifted}} = E\left[1-\left(\frac{3\alpha(mc^2)^2}{kTE}\right)^{0.82}\right]$$

so that at $E=10^{17}$ ev the electron loses 99.99 percent of its original energy in the distance R_0.

At still higher energies, however, the rate of energy loss becomes approximately proportional to the logarithm of the energy squared (as in Eq. (A5)), so that above $E=10^{20}$ ev, the energy loss in the distance R_0 is relatively small.

APPENDIX B. ANOMALOUS COMPTON SCATTERING BY THE MESON CHARGE CLOUD SURROUNDING THE PHOTON

We attempt in this appendix to estimate the effect of the meson cloud surrounding the proton on the scattering of photons. At small photon energies ($\epsilon^* \ll \mu c^2 \ll Mc^2$; $\mu \cong \frac{1}{10}M$) the effect of the meson cloud can presumably be expressed as an interaction between the anomalous magnetic moment of the proton and the electromagnetic field of the photons. This interaction can be described phenomenologically by a Pauli term in the Dirac equation of the proton, *viz.*:

$$-el[\beta\boldsymbol{\sigma}\cdot\mathbf{H}-i\beta\boldsymbol{\alpha}\cdot\mathbf{E}],^{41} \tag{B1}$$

and the resulting additional scattering estimated by a calculation similar to the Casimir (spur summation) derivation of the Klein-Nishina formula.[19] In this way, one obtains the Compton (anomalous) magnetic moment (cmm) cross

[40] The cascade formation discussed earlier is not an immediate complicating factor in the preceding discussion, since the cosmic "radiation length" is much greater than the galactic dimensions. However, the energetic photons produced within the galaxy collide with thermal photons in intergalactic space and in turn produce energetic electron-positron pairs. These pairs then constitute a particle component in intergalactic space having energies smaller than, but still comparable with, those of the primary galactic electrons.

[41] Here, $el=1.79e\hbar/2Mc$ is the anomalous magnetic moment of the proton; β, $\boldsymbol{\alpha}$, $\boldsymbol{\sigma}$ are Dirac matrices; $\mathbf{E}$, $\mathbf{H}$ are the external electric and magnetic fields.

section,[42]

$$\phi_{\text{cmm}}^*(\epsilon^*) \sim \begin{cases} \nearrow \pi\left(\dfrac{e^2}{Mc^2}\right)^2\left(\dfrac{\epsilon^*}{Mc^2}\right)^2, \text{ if } \dfrac{\epsilon^*}{Mc^2} \ll 1 \\ \searrow \pi\left(\dfrac{e^2}{Mc^2}\right)^2\left(\dfrac{\epsilon^*}{Mc^2}\right), \text{ if } \dfrac{\epsilon^*}{Mc^2} \gg 1 \end{cases} \tag{B2}$$

The continued linear increase of $\phi_{\text{cmm}}^*(\epsilon^*)$ with ϵ^* in the extreme relativistic region is usually considered physically unreasonable. Certainly the interaction between the meson cloud and an external electromagnetic field cannot be represented adequately at all frequencies by the phenomenological device of a Pauli term with a constant l. In reality the effective value of the anomalous magnetic moment may well decrease with increasing frequency (for $\epsilon^* > \mu c^2$ or perhaps Mc^2) giving a scattering cross section which decreases (or remains constant) with increasing photon energy. Qualitatively such a behavior is indeed obtained in a classical non-relativistic theory if electromagnetic radiation damping is included in the calculation of the cross section. Thus, consider a point magnetic dipole with magnetic moment $e\mathbf{l}$ and spin angular momentum $\mathbf{s}$ (both of constant magnitude), the dipole being subjected to a driving torque $e\mathbf{l}\times\mathbf{H}$ (arising from an external electromagnetic field of angular frequency ω^*) and a damping torque

$$-\frac{2}{3c^3}\left(\frac{e|\mathbf{l}|}{|\mathbf{s}|}\right)\frac{d^3}{dt^3}(e\mathbf{l})$$

(arising from radiation resistance); the resulting cross section is then,[43]

$$\phi_{\text{cmm}}^*(\omega^*)] = 4\pi\left(\frac{c}{\omega^*}\right)^2 \frac{\dfrac{4}{9}\dfrac{(el)^4}{(\mathbf{s})^2}\dfrac{(\omega^*)^4}{c^6}}{1+\dfrac{4}{9}\dfrac{(el)^4}{(\mathbf{s})^2}\dfrac{(\omega^*)^4}{c^6}} \tag{B3}$$

which, with the replacements $\epsilon^* = \hbar\omega^*$, $(\mathbf{s})^2 = \frac{3}{4}$, $el \cong 0.9e\hbar/Mc$, becomes

$$\phi_{\text{cmm}}^*(\epsilon^*)]_{\text{class}} \cong \begin{cases} \nearrow 1.6\pi\left(\dfrac{e^2}{Mc^2}\right)^2\left(\dfrac{\epsilon^*}{Mc^2}\right)^2, & \text{if } \epsilon^* \ll \left(\dfrac{\hbar c}{e^2}\right)^{\frac{1}{2}} Mc^2 \\ \searrow 4\pi\left(\dfrac{\hbar c}{e^2}\right)^2\left(\dfrac{e^2}{Mc^2}\right)^2\left(\dfrac{Mc^2}{\epsilon^*}\right)^2, & \text{if } \epsilon^* \gg \left(\dfrac{\hbar c}{e^2}\right)^{\frac{1}{2}} Mc^2. \end{cases} \tag{B4}$$

If one now supposes that essentially the same damping denominator occurs in the quantum relativistic as in the classical non-relativistic case[44] one sees from Eq. (B2) that the maximum value of $\phi_{\text{cmm}}^*(\epsilon^*)$ is

$$\phi_{\text{cmm}}^*(\epsilon^*) \sim \pi\left(\frac{e^2}{Mc^2}\right)^2\left(\frac{\hbar c}{e^2}\right)^{\frac{1}{2}} \quad \text{at } \epsilon^* \sim \left(\frac{\hbar c}{e^2}\right)^{\frac{1}{2}} Mc^2 \tag{B5}$$

and is thus larger than the Klein-Nishina cross section (at the same energy) by a factor $\sim\hbar c/e^2 = 137$. Since Compton scattering effects for protons, calculated from the Klein-Nishina formula (Eq. (50)) are quite negligible, even on an intergalactic scale (see Eqs. (65)–(71)), it is clear that the above factor of 137 can hardly alter the situation.

However, in addition to the effects just estimated, there is probably present at high photon energies ($\epsilon^* \gg \mu c^2$) a manifestation of the meson charge cloud which cannot be reduced to an

[42] H. C. Corben, quoted in W. Pauli, Rev. Mod. Phys. **13**, 203 (1941), Tables III and IIIa, has given exact formulae for the Compton scattering cross sections of particles of spin $\frac{1}{2}$ and arbitrary magnetic moment which check the estimates in Eq. (B2) to within numerical factors of the order one. See also S. B. Batdorf and R. Thomas, Phys. Rev. **59**, 621 (1941).

[43] See, for example, H. J. Bhabha and H. C. Corben, Proc. Roy. Soc. A**178**, 273 (1941).

[44] The argument of Heitler (see reference 20, p. 250), anent the negligible influence of the radiation resistance damping on the various cross sections in quantum electrodynamics, is applicable only to processes involving the interaction of the electric charge with the electromagnetic field, in which case the interaction energy matrix elements are proportional to $(\epsilon^*)^{-\frac{1}{2}}$. On the other hand, a (magnetic) dipole interacting with the field possesses interaction energy matrix elements proportional to $(\epsilon^*)^{\frac{1}{2}}$. This last fact vitiates the use of the perturbation theory in the derivation of Eq. (B2) for $\epsilon^* > (\hbar c/e^2)Mc^2$ (see J. R. Oppenheimer, quoted in Pauli, reference 50a) which however is greater than the ϵ^* at which damping first becomes important in Eqs. (B3) and (B4).

effective frequency dependent anomalous magnetic moment. This manifestation involves an additional photon scattering arising from the virtual creation and destruction of mesons by the photons ("virtual meson scattering" (vms) processes); thus the original photon may be absorbed by the proton with the emission of a virtual meson—the latter is then reabsorbed by the resulting neutron with emission of a photon different in general from the original one. The corresponding cross section, $\phi_{vms}^*(\epsilon^*)$, has been estimated by means of Heitler's quantum theory of damping,[45] and found to have the value,

$$\phi_{vms}^*(\epsilon^*)\cong\frac{\pi^2}{2^{\frac{1}{2}}}\left(\frac{eg}{\hbar c}\right)\left(\frac{\hbar}{\mu c}\right)^2\cong 5\times 10^{-27}\text{ cm}^2. \quad \text{(B6)}$$

for $\epsilon^*\gg\mu c^2$; other estimates of the same cross section (e.g., in a weak coupling meson theory) involve other more or less arbitrary cut-off procedures, and seem to yield smaller results.

In the absence of more certain information we shall use the $\phi_{vms}^*(\epsilon^*)$ of Eq. (B6) to estimate the number of scattered photons per primary N_s^{vms} and the corresponding energy loss ΔE_s^{vms} in the radial fall through the sun's radiation field. Equations (47) and (B6) yield, for $\epsilon^*\gg\mu c^2$

$$\left(\text{i.e., } E\gg\frac{Mc^2\mu c^2}{kT}\cong 2\times 10^{17}\text{ ev}\right),$$
$$N_s^{vms}\cong 2\left[\frac{\pi^2}{2^{\frac{1}{2}}}\left(\frac{eg}{\hbar c}\right)\left(\frac{\hbar}{\mu c}\right)^2\right]Q_s\cong 3\times 10^{-6}. \quad \text{(B7)}$$

On the other hand, the calculation of ΔE_s^{vms} from Eq. (48) requires knowledge of the angular dependence of the "virtual meson scattering" which has not yet been computed. We observe that an angular dependence more pronounced than that of the Klein-Nishina cross section is unlikely since the matrix elements for the virtual emission and absorption of the mesons have a moderate, e.g., cosine, dependence on the angle between the directions of motion of the incident and scattered photons. It is thus probable that the scattered photon carries away a substantial fraction of the proton's energy (in the earth's frame) over the entire range of validity of Eq. (B7). Moreover the "substantial fraction" may well increase to over 80–90 percent in the range $E\gg(Mc^2)^2/kT=2\times 10^{18}$ ev.

It is clear that the above number of collisions per primary and the associated energy loss may well be important on an intergalactic scale (replacement of Q_s in Eq. (B7) by $Q_n\cong 10^4Q_s$). There is thus need for an adequate theory of the differential and total scattering cross sections in proton-photon collisions at high energies.

We conclude this appendix by raising the question of the effect of the meson cloud on the double Compton scattering (bremsstrahlung in proton-photon collisions—process 3 of introduction). No calculation of this effect exists, but it seems possible that for $\epsilon^*\gg\mu c^2$ (or Mc^2?) the ratio of the double and single Compton scattering cross sections exceeds 1/137, the value which is appropriate to lower energies.

APPENDIX C. COLLISIONS OF ENERGETIC PROTONS, ELECTRONS, AND PHOTONS WITH INTERSTELLAR AND INTERGALACTIC DIFFUSE MATTER

The amount and chemical composition of diffuse matter in space is not readily accessible to measurement; nevertheless rough estimates have been obtained from the interpretation of astronomical observations. For the sake of definiteness in the statement of our results we adopt the values:

$$\text{Interstellar(Intragalactic)}\sim 1\text{ H atom/cm}^3. \qquad \text{Intergalactic }\sim 10^{-5}\text{ H atom/cm}^3. \quad \text{(C1)}$$

These estimates at least do not contradict any of the available astrophysical evidence.[46]

We also need the following cross sections:

Meson creation in primary proton—H atom collisions

$$\phi_m(E)\cong 10(\hbar/\mu c)^2\cong 4\times 10^{-25}\text{ cm}^2.^{47} \quad \text{(C2)}$$

Photon creation (bremsstrahlung) in primary electron—H atom collisions—

[45] See Hamilton and Peng, reference 30. The theory which gives this comparatively large value for ϕ_{vms}^* also predicts an equally large cross section for the creation of single mesons in photon-proton collisions—see, however, reference 34.

[46] L. Spitzer, Ap. J. 93, 369 (1941), and private communication.

[47] See W. Heitler and P. Walsh, Rev. Mod. Phys. 17, 252 (1945). Other estimates of $\phi_m(E)$ give results of more or less the same order of magnitude, essentially the cross-sectional dimensions of the meson charge cloud surrounding the proton. The observed distance traveled by primary protons in the atmosphere before absorption caused by meson creation is of the order of 100 g/cm², in agreement with Eq. (C2).

$$\phi_p(E)\cong 20\frac{e^2}{\hbar c}\left(\frac{e^2}{mc^2}\right)^2\cong 1.2\times 10^{-26}\ \mathrm{cm}^2.^{48} \qquad (\mathrm{C}2')$$

These cross sections are considerably larger than those for any of the other processes which may occur in the collisions under consideration, e.g., electron-positron pair creation in primary proton—H atom collisions.

We can now estimate the average number of mesons N_g^m and N_n^m created in the passage of a primary proton, across the distance $R_g(\cong 10^4$ l.y.) in the local galaxy, and across the distance $R_0(\cong 1.85\times 10^9$ l.y.) in intergalactic space. One has from Eqs. (C1), (C2),

$$N_g^m\sim 4\times 10^{-3},\quad N_n^m\sim 10^{-2}. \qquad (\mathrm{C}3)$$

The neutrons and mesons produced in such collisions eventually decay, leaving as final products protons and electron-positron pairs. Also, the number of bremsstrahlung photons produced in primary electron—H atom collisions is smaller than the corresponding number of mesons by a factor $\cong 35$ (cf. Eq. (C2)). Thus it seems that the intragalactic and intergalactic matter does not constitute a serious obstacle to the cosmic-ray primaries,[49] subject, of course, to the order of magnitude validity of the matter densities and cross sections listed in Eqs. (C1) and (C2); conversely, any definite information about the nature and energy distribution of the primary particles may be employed to set limits on the cross sections and on the amount of intragalactic and, particularly, intergalactic matter.

For completeness we estimate the frequency of the various possible secondary processes. First of all, the preceding results for primary electrons can be applied immediately to secondary electron-positron pairs produced, for example, in primary proton-thermal photon collisions. Equations (C2), (C1) then show that photon creation by these electrons and positrons in collisions with diffuse matter is unimportant. Finally the very energetic Compton scattered photons (arising from primary particle—thermal photon collisions) may create electron-positron pairs in collisions with the diffuse matter; however, the pair creation cross section is of the same order as that from bremsstrahlung so that the number of such pairs, even on an intergalactic scale, is small.

[48] See reference 19, pp. 161–177.

[49] From Eq. (C3) it appears that protons and electrons constrained to move within the local galaxy by intragalactic magnetic fields (see reference 15a) would experience many meson producing and photon producing collisions in a period $\cong 1.85\times 10^9$ years. One thus finds another mechanism to eliminate the primary electron component. (The meson producing collisions of the protons ultimately only degrade the energy of the latter.) The high energy bremsstrahlung photons would, on this picture, escape from the galaxy and in turn produce energetic electron-positron pairs by collisions with thermal photons in intergalactic space (see reference 48b).

GLOSSARY OF FREQUENTLY USED SYMBOLS

η—solar constant

R_e—radius of the earth's orbit

R_g—linear dimension of a galaxy ($\sim 10^4$ light years (l.y.))

R_0—1.85×10^9 l.y. (red shift parameter)

N—effective number of stars in a galaxy

$\mathfrak{N}$—average number of nebulae per unit volume

T—effective temperature of the sun's surface

ϵ—energy of a thermal photon in the earth reference frame; also $u\equiv\epsilon/kT$

$n_{s,g,n}(u)d\epsilon$—number of thermal photons per unit volume in the energy range $d\epsilon$ from solar radiation at the earth's orbit (s), starlight averaged over the galaxy (g), and nebular radiation in intergalactic space (n). The spaces s, g, and n are defined precisely in the text.

$N^{\pm}_{g,s,n}$—number of collisions between a primary particle and thermal photons resulting in electron-positron pair production in the spaces s, g, and n.

$N^c_{s,g,n}$—number of Compton collisions between a primary particle and thermal photons in the spaces s, g, and n.

$Q_{s,g,n}$—number of photons in cylinder of unit cross section in the spaces s, g, and n

M—rest mass of the cosmic-ray primary (electron or proton)

m—electron rest mass

E—energy of the cosmic-ray primary

$\gamma\equiv(1-\beta^2)^{-\frac{1}{2}}\equiv E/Mc^2$

$u_{\pm}\equiv mc^2Mc^2/EkT\cong 5\times 10^5/\gamma$

$u_c\equiv(Mc^2)^2/EkT\cong 10^9/\gamma$ (protons) or $\cong 5\times 10^5/\gamma$ (electrons)