Development of the Frequency Modulated Cyclotron

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The experimental development of the phase stability principle as applied to the 37-inch f-m cyclotron is discussed. The ions were required to pass through an over-all radial decrease in magnetic field of 13 percent. Some theoretical estimates of the yield *vs.* modulation frequency are given and these were found to be in satisfactory agreement with the experimental results. It is found that some of the ions emerge as late as four or five modulation cycles after their initial acceleration. Observations were made of the time of flight of the ions and evidence was found of the actual phase oscillations in the shape of the ion current patterns. The ion current received by a probe at different radii and different pressures using a pulsed source was analyzed, and indicates a considerable loss of beam at dee voltages less the 60 kev. The data confirm the variation of ion beam loss as the reciprocal of the square of the ion energy and extend the validity of this relation to 600 kev for deuterons. The variation of ion current with dee voltage is quite steep. A deflected beam amounting to 10 percent of the circulating current was obtained, taking advantage of the precessional motion of the ion orbits. Circulating ion currents of 3 μa of 7.5-Mev deuterons and 2 μa of 14.6-Mev protons were obtained.

1. INTRODUCTION

THIS is a report on the experimental development[1] of the phase stability principle[2,3] as applied to the frequency modulated cyclotron. The purpose of this development was to ascertain the chances of the successful application of the phase stability principle to the 184-inch cyclotron[4] by using the 37-inch Berkeley cyclotron as a model.

The cyclotron frequency for an ion of kinetic energy W (units Mc^2) in a magnetic field H is

$$f=f_cH/(1+W)H_c, \qquad (1)$$

where f_c is the frequency the ion would have with very low energy in the field H_c, H_c being the field at the cyclotron center. This relation exhibits the difficulty one encounters in trying to accelerate ions to high energies ($W>0.02$) in an ordinary cyclotron. In order to maintain vertical focusing of the ions it is necessary that the magnetic field fall off as the ion energy increases. The decrease in H and increase in W combine to produce a lowering of the resonant frequency f as the ion energy increases. This means that after a certain number of turns, the ions will be so far out of phase with the accelerating dee voltage that they will be decelerated and will never attain the full energy desired. In the past, the usual solution to this problem has been to increase the dee voltage until the number of ion turns was relatively small (<100). Then the ions would never get so much out of phase with the dee voltage that they would be decelerated.

For really high energies this solution requires very large dee voltages. In designing the 184-inch cyclotron for the acceleration of 100-Mev deuterons, for example, it was clear that something of the order of one million volts on the dees would be required. Although by no means insoluble, the handling of such large r-f voltages is a serious technical problem, and since the required dee voltage varies as the square of the desired energy[5] this would soon place a ceiling on the possible ion energy produced by the cyclotron.

The possibility of using the phase stability principle together with frequency modulation on the cyclotron was suggested by McMillan, but it was not clear what would be required in the way of ion injection to get them into the region of stable phase. However, one of us (JRR) was able to show by means of graphical integration that no special ion injection technique would be required and that a yield of some one to five percent (depending upon various conditions of the acceleration) of the total ion current would

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[1] J. R. Richardson, K. R. MacKenzie, E. J. Lofgren, and B. T. Wright, Phys. Rev. 69, 669 (1946).
[2] E. M. McMillan, Phys. Rev. 68, 143 (1945).
[3] V. Veksler, J. Phys. U.S.S.R. 9, 153 (1945).
[4] Brobeck, Lawrence, MacKenzie, McMillan, Serber, Sewell, Simpson, and Thornton, Phys. Rev. 71, 449 (1947).
[5] R. R. Wilson, J. App. Phys. 11, 781 (1940).

be picked up normally in the phase stable region. Since yields of this magnitude would certainly be adequate for most experiments at very high energies, it was decided to make experimental tests on this technique.

Looking at Eq. (1) we see that it is possible to simulate a combined frequency change caused by energy increase and magnetic field decrease by a change in magnetic field alone. The field on the 37-inch cyclotron was accordingly tailored[6] to fit an over-all frequency change of 13 percent from ion source to exit radius, and the experimental results described in this paper were obtained under these conditions.

2. EXPERIMENTAL APPARATUS

The field from the 37-inch magnet was adjusted to conform to the relation

$$H=H_c/(1+9\times 10^{-4}r^2)^{\frac{1}{2}}, \qquad (2)$$

where H_c is the field at the center and r is the radius in inches. This relation was followed out to a radius of 18 inches. Equation (2) is derivable on the assumptions of a geometrical similitude of the resonant ion frequency between the 37-inch and the 184-inch, that the frequency change caused by mass change is negligible on the 37-inch, and that the frequency change on the 184-inch is entirely due to the mass change. The last assumption is true only as a first approximation. The radial variation in field was obtained by the use of shims of circular symmetry.

Figure 1 is a plan view showing the four-foot square steel vacuum tank with 41-inch pole tips $2\frac{1}{4}$ inches thick welded in the top and bottom. The atmospheric load is carried by heavy non-magnetic steel[7] corner posts welded to the $2\frac{1}{4}$-inch disks. The vertical walls of the tank are also non-magnetic (this was probably not necessary) while the top and bottom are of ordinary steel. This type of construction is very easy to build, and the welds if reasonably well done are almost certain to be vacuum tight. Magnetic shims to shape the field are screwed to the pole tip disks. The magnet gap where the shims are thickest is $5\frac{1}{2}$ inches. Two entire sides of the tank are open for ready access.

Since the dee voltage required in the f-m cyclotron is quite moderate, the primary reason for having two dees has disappeared. The plate covering the opening on the left of Fig. 1 carries a single dee on a stem passing through a zircon insulator. The dee stem is part of the half-wave line of the f-m radiofrequency system.[8] The inside dimensions of the dee are $2\frac{1}{2}$ inches in height and 19 inches in radius. The opposite face plate carries the arc source, probe, and deflector. The source consists of a d.c. heated tungsten 100-mil filament enclosed in a gas tight copper box with a graphite top. The graphite has the advantage over most metals in that it resists sputtering and it can run hot enough to dissipate its heat by radiation. The ion column emerges from the box through a 0.090-inch by 0.625-inch slot, the long dimension parallel to the dee edge.

Both the grounded and negative deflector electrodes, Fig. 1 and Fig. 2, are hinged at the entrance to the deflected ion channel and are provided with cam and flexible shaft mechanisms to permit adjustment to optimum positions from outside the tank. Internally, the entrance to the deflector channel may be set to a given distance from the geometrical center of the tank. This radius is usually $17\frac{3}{4}$ inches. The exit strip is 0.006-inch tantalum sheet. The deflector face plate also carries a probe which may be inserted

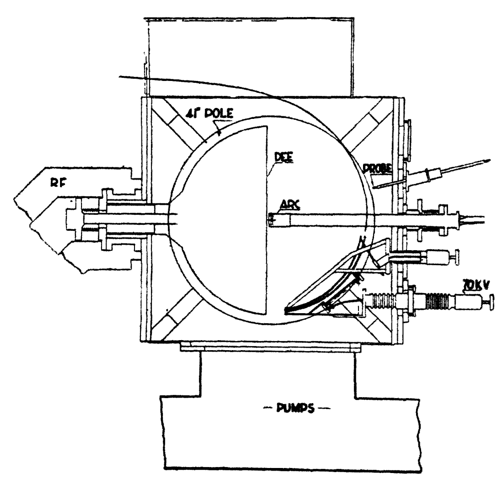

FIG. 1. A plan view of the cyclotron showing the single dee, arc source, and deflector assembly.

[6] Our thanks are due Mr. Duane Sewell for considerable assistance in this part of the work.

[7] Jessop non-magnetic steel Mn $10\frac{1}{2}$–$12\frac{1}{2}$ percent Ni 7–8 percent.

[8] K. R. MacKenzie and V. Waithman, to be published when patent release can be obtained.

along a radius to various distances from the center. A rectangular extension (top of Fig. 1) was added to the tank to give clearance to the deflected beam when it was brought outside the tank into the air.

The frequency modulation is produced by a rotating mechanical capacitor as previously described.[9] This capacitor is connected to the end of the half-wave resonant line which is opposite to that carrying the dee. The shape of the frequency time curve could be adjusted during operation by means of a moveable stator ring.

The ion source could be pulsed[10] with a condenser discharge which could be repetitiously tripped at any desired instant in the modulation cycle. The width of the pulse was variable from about three to 50 microseconds. Under steady operation, the 100-mil tungsten filaments in the ion source had an average useful life of about 60 hours. However, under pulsed operation this lifetime was increased by more than a factor ten.

A d.c. bias voltage was applied to the dee system, including the resonant line and the rotor of the capacitor. This voltage acted as a clearing field[8] to sweep out residual ionization which otherwise so loads up the oscillator that it is unable to establish a large dee voltage. For low r-f dee voltages a positive bias of 1000–2000 volts was definitely beneficial in beam output-giving a factor of ten at three kv r-f on the dee. As the

FIG. 2. A photograph showing the deflector and arc source face plate as it is removed from the tank.

[9] F. H. Schmidt, Rev. Sci. Inst. **17**, 301 (1946).

[10] We are indebted to James Vale for the design and construction of the ion pulser.

dee voltage was increased the benefit was less marked and was perhaps 20 percent at 10-kv dee voltage. This effect is not completely understood.

The problem of measuring the true ion beam current by means of a probe is a serious one. In addition to the usual difficulties with the rectification of r-f signals, and secondary emission, the probe currents sometimes acted similarly to those encountered in a plasma. It was usually necessary to check the probe current by putting both positive and negative voltages on the probe. These deleterious effects disappeared when the pulsed ion source was used. They could also be eliminated at the high ion energies by totally shielding the probe with aluminum foil.

Sufficient precautions were taken so that we are confident that the figures given in this paper are substantially correct for the ion beam currents.

Considerable experimenting was done with different arc source geometries, including round holes and slots of different dimensions for the arc column and graphite cones of different sizes and shapes. The one described was, however, the most successful under our particular conditions. It is possible that an arc column of larger cross section would be better if one had more r-f power available, but in our case it made the operation unstable. A dummy or vestigial dee was tried both at ground and plus and minus d.c. voltages, but it had little effect on the ion current. Some attempts were made to use small accelerating electrodes near the arc column with attractive voltages on them in order to increase the ion current, but these attempts, in general, were not successful. The principal difficulty encountered was in arranging the accelerating electrodes so that they would not be hit by the ions in subsequent revolutions.

3. THEORETICAL CONSIDERATIONS

The ions as picked up in the cyclotron from the ion source at the center are very quickly brought into phase (within ~ 0.1 radian) with the dee voltage after one or two r-f cycles.[11] This holds true independently of the actual phase angle at which they start being accelerated. Thus one can regard all the ions as being picked

[11] A. C. Helmoltz, private communication.

up with essentially zero phase. If α is the phase angle ($\alpha=0$ for maximum ion acceleration), V_D is the dee voltage, and f is the resonant frequency of the ion, then the kinetic energy gained by the ions (in electron volts) is

$$T=2\sum V_D f \cos\alpha\Delta t. \quad (3)$$

If the applied r-f frequency is f_a, we define $h=f-f_a/f_c$, where f_c is the resonant ion frequency at the center of the cyclotron. Then

$$\alpha=2\pi f_c\sum h\Delta t. \quad (4)$$

When the kinetic energy T can be considered small compared to the rest energy, W_0, we can use

$$h=(H/H_c)(T_a-T/W_0), \quad (5)$$

where H is the magnetic field at the ion path and H_c is the field at the center. T_a is the kinetic energy that an ion would have if it were exactly in resonance with the applied frequency f_a. Now we can define the phase stability factor, ρ, which is the value of $\cos\alpha$ which would be required to keep the ion of energy T_a always in step with the applied frequency f_a. If averaged over several phase oscillations, it will be true that

$$\rho=\frac{\text{average energy gain per turn}}{\text{maximum possible energy gain per turn}}. \quad (6)$$

Thus we can write

$$h=(2/W_0)(H/H_c)[\sum V_D f_a \rho\Delta t-\sum V_D f\cos\alpha\Delta t]. \quad (7)$$

If we are concerned only with the initial stages of the acceleration, we can simplify this to

$$h=(2/W_0)(H/H_c)V_D f\sum(\rho-\cos\alpha)\Delta t. \quad (8)$$

Equations (4) and (8) can be solved simultaneously by graphical integration to obtain the phase and frequency oscillations which occur in such a system, then Eq. (3) can be used to obtain the ion energy as a function of the time. The efficiency of ion pick-up, that is, the ratio of the number of ions accelerated to those available for acceleration, can be estimated from the ratio of the time during which ions are picked up to the total time of one complete modulation cycle. The area under the $h(t)$ curve is proportional to the phase angle α. Since $\Delta h/\Delta t$ is proportional to V_D, it follows that for a constant ρ the time over which ions are picked up is proportional to $1/(V_D)^{\frac{1}{2}}$. But for a constant ρ the modulation period is proportional to $1/V_D$. Therefore the efficiency of ion pick-up varies as $(V_D)^{\frac{1}{2}}$ for a constant ρ.

Table I. Variation of the ion pick-up efficiency with the value of the phase stability factor ρ.

ρ	1.0	0.9	0.8	0.7	0.6	0.5
ϵ (percent)	0	0.8	1.3	1.7	2.1	2.6

In general, if we neglect the change in magnetic field we will have

ϵ = efficiency

$$=\frac{(V_D W_0)^{\frac{1}{2}}}{T_f}\left(\frac{2}{\pi}\langle\cos\alpha-\rho\rangle_{\text{Av}}\cos^{-1}\rho\right)^{\frac{1}{2}}, \quad (9)$$

where T_f is the final kinetic energy of the ions and $\langle\cos\alpha-\rho\rangle_{\text{Av}}$ is an average value over the first quarter phase oscillation. For ρ between 1.0 and 0.5 the approximation

$$\langle\cos\alpha-\rho\rangle_{\text{Av}}=1-\rho-\frac{\alpha_1^2}{10},$$

where $\alpha_1=\cos^{-1}\rho$ is satisfactory. This gives

$$\epsilon=\frac{(V_D W_0)^{\frac{1}{2}}}{T_f}\left(\frac{2}{\pi}\cdot\alpha_1\cdot\left(1-\rho-\frac{\alpha_1^2}{10}\right)\right)^{\frac{1}{2}}. \quad (10)$$

In this expression for the efficiency it has been assumed that the period of acceleration occupies half of the total modulation cycle.

As an example we can consider the ion pick-up efficiency for the acceleration of deuterons to $T_f=200$ Mev with $V_D=50$ kev. The variation with different values of ρ is given in Table I.

A more elegant method of calculating these results was later introduced by McMillan and extended by Bohm and Foldy[12] to include the condition that the ions not return to the center. If the ions return to the center of the cyclotron they will be effectively lost, at least as far as that cycle is concerned. This condition applies for $\rho\leqslant 0.5$, and the results are shown in the theoretical curve of Fig. 3.

In the case of the 37-inch cyclotron we know H as a function of r and df_a/dt as a function of

[12] Bohm and Foldy, to be published.

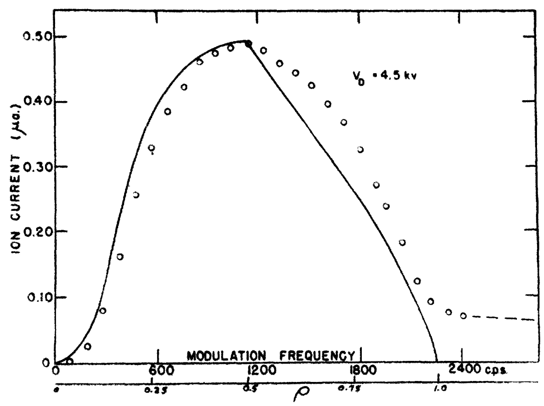

FIG. 3. The variation of ion current with modulation frequency. The solid curve is the theoretical expectation on the yield *vs.* ρ. The displacement of the experimental points from the curve is probably due to the variation of the pressure effect with ρ. See Fig. 9. The dashed line indicates the experimental trend at high modulation frequencies as indicated by the other data.

time. The latter can be evaluated at different values of r, and, then if one neglects the change in ion frequency caused by the change in mass, it is easily shown that

$$\rho=\frac{2\pi^2}{c^2}\frac{W_0}{V_D}\frac{df_a}{dt}r^2\left(1+\frac{1}{(r/H)(\partial H/\partial r)}\right), \quad (11)$$

where W_0 is here expressed in electron volts.

When the magnetic field is of the form (2) the expression

$$r^2\left(1+\frac{1}{(r/H)(\partial H/\partial r)}\right)$$

is a constant. Thus for ρ to be a constant throughout the ion motion we must have $(1/V_D)(df_a/dt)$ = constant. In our experimental arrangement only moderate efforts were made to have these relations hold so that a variation in the instantaneous value of ρ of almost 30 percent over the total ion path was sometimes encountered.

4. EXPERIMENTAL RESULTS

In Fig. 3 is shown a graph of the beam current received at a probe as a function of the angular speed of the rotating condenser. Since for a given radius and magnetic field variation the phase stability constant ρ is simply proportional to df_a/dt, this figure shows the experimental variation of the beam current with ρ. The theoretical curve is that predicted by Bohm and Foldy, and Eq. (10) for $\rho>0.5$.

Presumably the average value of ρ over the first half-cycle of phase oscillation is the important criterion in catching the ions for further acceleration. This first half-cycle of phase oscillation in our instrument corresponded to a radius up to about 6 inches. A precise determination of ρ would involve an exact knowledge of the variation of the dee voltage with the time and corresponding information concerning the radius and the quantity $(r/H)(dH/dr)$. If ρ is to be kept constant, then one has to tailor these quantities accordingly. Only moderate efforts were made to do this, since from the practical viewpoint it is quite unnecessary, and its only virtue would lie in an exact comparison with theory. As a result of these efforts, ρ was found to vary by 10 percent over the first six inches of radius. We estimate that the experimental value of ρ, which we assigned for comparison purposes, has an over-all uncertainty of about 20 percent. It is clear that in view of these facts the comparison with the theoretical curve is quite good. The conclusion is that the optimum value for the phase stability constant is near $\rho=0.5$. The absolute magnitude of the yield or efficiency is rather more uncertain. We estimate that our maximum efficiency is about one to three percent, which is in satisfactory agreement with the theoretical predictions.

Further evidence that the experimental variation in Fig. 3 is due to a change in ion pick-up efficiency is given by the following: If the ion source were to be pulsed for a duration short in comparison with the ion pick-up time involved, it would be expected that the total charge accelerated per pulse would be constant, and independent of the modulating frequency, as long as the dee voltage and other conditions were maintained the same. Therefore one would expect that under these conditions the time average accelerated current would be directly proportional to the frequency of the modulator until the limit of stability near $\rho=1$ was reached. This was confirmed experimentally to the extent that it was found that for short ion pulses the maximum ion beam current was moved toward the $\rho=1$ condition. Also for different modulating frequencies the minimum duration of arc pulse was determined which would just give the maximum possible beam at that modulating frequency. The qualitative variation of this duration with ρ was in agreement with the expected variation of ion "pick-up" time with ρ as predicted by the theory.

An interesting feature of the curves of beam current against ρ was that the beam did not decrease to zero $\rho > 1$ but actually remained at a value as much as 10 percent of the maximum even under these conditions when there was presumably no phase stability. Under the straightforward theory of the f-m cyclotron it should be impossible for ions to get out when the phase stability factor is greater than 1. However, if the ions can spend several modulating cycles in being accelerated, it might be possible for them to get out even when there is no phase stability present in the ordinary sense. With the aid of Eqs. (3), (4), and (8) we can obtain rather easily through graphical construction the kinetic energy of the ions as a function of the time for the case where $\rho > 1$. Figure 4 shows this function for the three cases where the ions start early ($f_a > f_c$), on time ($f_a = f_c$), or late ($f_a < f_c$). It is seen, therefore, that in a given modulating cycle as the applied frequency gets far out of resonance with the natural frequency of the ion, a large band of ions is left circulating in the chamber from zero radius up to a maximum radius determined by those ions which start early. In particular, the maximum radius will be attained by those ions which start just early enough so that they reach a phase lag of just $\pi/2$ before starting their large gain in phase which occurs when the applied frequency rapidly drops below their natural frequency. It can be estimated from curves such as those shown in Fig. 4 that in the case of the 37-inch f-m cyclotron and a ρ of 1.4 and V_D of 6 kev these ions would get out to a radius of about 8 inches on the first modulation cycle. Then they would circulate with approximately constant energy and radius until the applied frequency again approached the new natural frequency of the ions. Then, depending upon the phase in which they found themselves, the ions would be accelerated or decelerated for the period during which the applied frequency was within one percent, say, of the natural ion frequency. This would be true even when the applied frequency passes through as it is increasing in the unused part of the modulation cycle. Every time the applied frequency passes through the region the ions are "blurred out"—some being accelerated and some decelerated. Eventually after a series of such passes, some of the ions will receive the full energy and emerge from the cyclotron. This gives a picture of the way in which ions can be accelerated by the f-m cyclotron even when there is no phase stability.

This picture was investigated experimentally by pulsing the arc source once every ten modulation cycles and using a synchroscope to look at the probe current at different radii. At a radius of 17 inches and ρ of about one, it was found that beam came out over a series of four successive modulation cycles in pulses of somewhat decreasing magnitude and at times corresponding to the periods when f_a approximated the ion frequency at that particular radius. When this occurs on the decreasing part of the frequency-time cycle, we will call it a direct pass; when it occurs on the increasing frequency part of the modulation cycle, we will call it an inverse pass. For a radius of 5 inches and below, all ions were

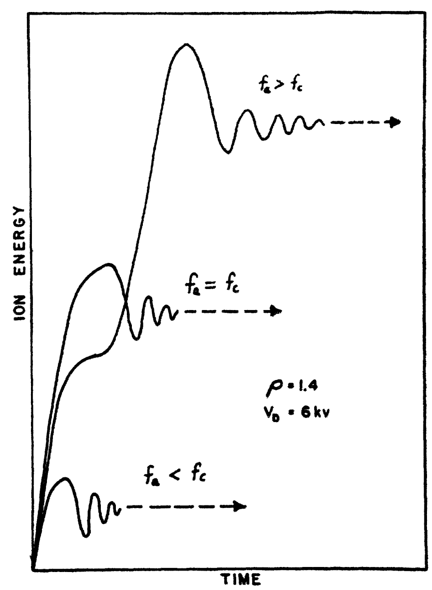

FIG. 4. This shows the kinetic energy of the ions as a function of the time for ions starting early ($f_a > f_c$), on time ($f_a = f_c$), or late ($f_a < f_c$). Particularly for the first two cases, the ions apparently remain circulating with the energy shown by the dotted arrows until the applied frequency again approaches the ion frequency, when they are either accelerated further or decelerated.

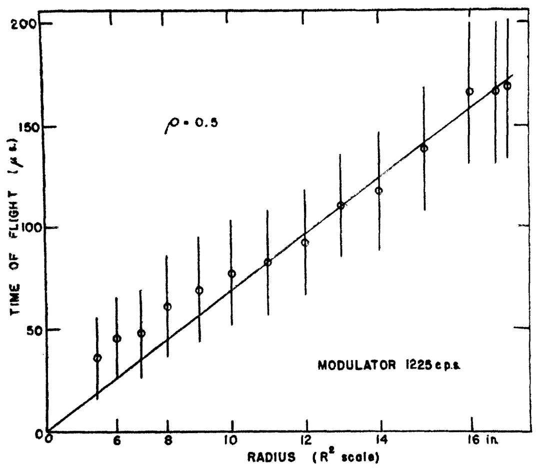

FIG. 5. The time of flight of the ions plotted against $(HR)^2$. The length of the vertical lines indicate the time over which current is received at the probe for that particular radius. The circles are just the centers of the vertical lines. The slanting line would be the expected curve if there were no phase oscillation.

brought out on the first direct pass. Ions appeared on the first inverse pass for radii between six and twelve inches. The ions from the second direct pass appeared at a radius of seven inches, and in general the evidence was in agreement with the idea that for $\rho \sim 1.2$ the ions came out to full radius in a series of four or five passes. One would expect that if this were true the order of magnitude of yield would be $1/2^4$ which is not out of line with the actual yield observed, which was 5 to 10 percent of the maximum beam.

5. TIME OF FLIGHT

Some observations were made using the synchroscope on the time of flight of the ions in normal phase stable operation. Figure 5 shows the results of a typical investigation with $\rho \sim 0.5$. It can be seen that the time over which beam is received at the probe is quite a large fraction of the time of flight of the ions.

This is probably due to the existence of the phase oscillations of the ions and their consequent oscillations about the frequency time curve. Precession of the orbits, an effect which we shall mention later, is too small by a factor of at least five to account for the breadth of the beam pulse. In the case of Fig. 5 we see that the ions must be receiving acceleration when the applied frequency is as much as 2 percent greater or smaller than the ion frequency. Since the total frequency change here is about 12 percent, this probably means that the energy oscillations have an amplitude in this case $2/12 \sim 17$ percent. At any instant, then, near the end of the acceleration period we must have a rather broad band of ions with a total energy spread of about one-third of the mean energy and in our case a total spread in radius of about 3 inches.

When the phase stability factor ρ is less than one-half, many of the ions undergo deceleration during part of their phase oscillation. It is presumed that this is the reason for the series of peaks observed in the beam pattern for low values of ρ as indicated in Fig. 6. This figure shows an oscilloscope pattern of the beam current to a probe as a function of the time. If during part of the time the ions are undergoing deceleration, this will show up as a reduction in the current reaching the probe during that period, and this should be followed by a period during which larger numbers of ions would reach the probe. A typical set of conditions for this type of observation was $\rho \sim 0.3$ and $V_D = 10$ kv. Under these circumstances one would expect the ions to undergo about 10 phase oscillations before attaining their full energy. At first sight one might expect that these oscillations would be smeared out by the time the full energy was attained, particularly since the ions which start late or early are in quite different portions of the energy oscillation. However, if one draws these oscillations out graphically one sees that there is a grouping tendency. This tendency is particularly marked for those ions which start early and on time; this persists throughout their path so that these ions are presumably the ones which produce the peaks in the beam pattern. The period of the oscillation, which is predicated by graphical integration to be 15μ sec. for these conditions, is in very good agreement with the experimentally observed value of 13μ sec. The fact that the time duration of the peaks was independent of the radius from 10 to 17 inches is in agreement with the supposition that they actually represent the oscillations in the natural frequency of the ions. The peaks then would represent successive scrapings by the probe of the ion beam as it oscillates outward. About twice as many peaks were found to be present at a radius of 17 inches as there were at 11 inches. The amplitude of the frequency oscillation is not damped very much in the small number of oscillations used in the f-m cyclotron so that we can say that it is approximately constant and that therefore the number of peaks present is approximately inversely proportional to df_a/dt. This was found to be the case since we generally operated on the lower section of the available frequency-time curve.

6. VARIATION OF ION CURRENT WITH DEE VOLTAGE

There are many reasons why the beam current will vary as the dee voltage is changed. Among them are:

1. Change in the source current. The accelerating field which pulls the ions out of the source is principally supplied by the dee voltage. One peculiar aspect of the d.c. dee bias is that its optimum value is positive and so tends to reduce this effect.

2. At constant modulation frequency, a change in the dee voltage results in a change in the phase stability factor ρ, which in turn means a change in the ion pick-up efficiency.

3. At constant ρ a change in the dee voltage will cause a change in the ion pick-up efficiency proportional to $(V_D)^{\frac{1}{2}}$.

4. If the beam is pressure sensitive, that is, if some ions are lost because of charge exchange, etc., with gas molecules in the chamber, then since the probability of these processes is very sharply dependent on the energy of the ions, a marked variation with dee voltage might be expected (see the next section). Because of this situation it is rather difficult to get strictly reproducible results on the variation of beam current with dee voltage.

However, some measurements were made which eliminated (2) by changing the modulation frequency in proportion to the dee voltage so that ρ was kept constant. Figure 7 shows the results of a typical set of observations of the ion beam current as a function of the dee voltage with the phase stability factor ρ held constant. It is seen that the dependence on dee voltage is quite steep. However, it is still not as steep as one might expect from reasons (1), (3), and (4), which would lead to a dependence of the form:

$$I \sim V_D^2 e^{-K/V_D}.$$

The presence of the d.c. bias certainly introduces some complication here.

7. VARIATION OF ION CURRENT WITH PRESSURE

In this section we wish to discuss the effect of the general chamber pressure on the ion current.[5] The presence of a large number of gas molecules in the path of the ion beam will cause a loss in the beam because of charge exchange, electron capture, and very large angle scattering. We shall assume in this discussion that the cross section for these processes varies inversely as the square of the energy of the ions. At least we shall assume that this is true above a certain energy, E_0, which in our experiment is taken to be about 25 kev.

If the cross section at the energy E_0 is σ_0 and the corresponding ion radius is r_0, then we assume

$$\sigma(E) \simeq \frac{\sigma_0}{(E_0+E^2)} \sim \frac{\sigma_0}{(r_0+r)^4}.$$

For our purposes we can ignore the variation of E with H in comparison with the variation with r. Then if I_0 is the value of the beam current I at r_0 we can write in the usual way $(dI)/I = -\sigma N ds$, where N is the number of gas molecules per cc and ds is the element of ion path length.

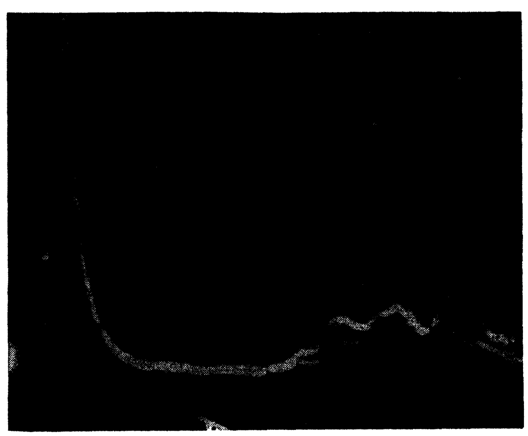

FIG. 6. This oscilloscope photograph shows the ion current to the probe as a function of the time. The markers are at 10-microsecond intervals. The peak on the left is the arc pulse, and those on the right represent the effect of the phase oscillations. There are three modulation cycles in this photograph.

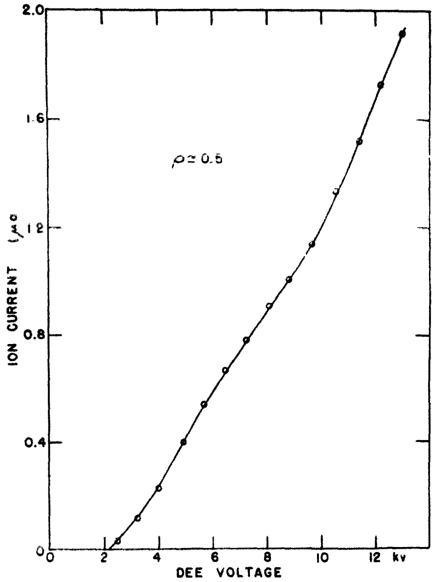

FIG. 7. The ion beam as a function of dee voltage, showing the rapid increase of current as the dee voltage is raised.

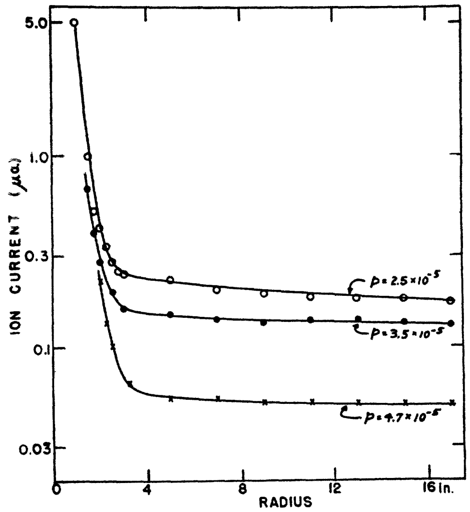

FIG. 8. The ion current to a probe as a function of the radius. The phase stable pick-up of ions does not enter here because the arc pulse was made short in comparison with the time of ion pick-up. Note that most of the losses occur at less than four inches radius.

We shall not investigate here the variation of the ion loss with the phase stability constant, ρ, but shall assume that the experiments are carried out at constant ρ. We shall also concern ourselves just with the radial variation of ion current over less than half of the first phase oscillation. Over this region we shall assume that the increase in path length per ion revolution is approximately proportional to the radius. Then in increasing the ion energy by ΔE we increase the ion path length by

$$\Delta S \sim 2\pi r \frac{\Delta E}{V_D},$$

and since

$$\frac{\Delta E}{E} = 2\frac{\Delta r}{r},$$

$$\Delta S \sim \frac{E}{V_D}\Delta r \sim \frac{(r_0+r)^2}{V_D}\Delta r.$$

If we substitute these results into the above differential equation and integrate, we obtain

$$\ln\frac{I_R}{I_0} \sim -\frac{N}{V_D}\int_0^{R-r_0} \frac{dr}{(r_0+r)^2} = \frac{KN}{V_D}\left(\frac{1}{r_0} - \frac{I}{R}\right),$$

or

$$\frac{I_R}{I_0} = \exp\left[-\frac{KN}{V_D}\left(\frac{1}{r_0} - \frac{1}{R}\right)\right],$$

where I_R is the ion current at the radius R and K is a dimensional constant.

It is interesting to note that for large radii the ion current approaches a constant value. This is, of course, a direct result of the assumption on the functional relationship between σ and E. The variation with pressure is what one would expect from this type of phenomenon. The variation of the ion current with dee voltage is of importance and expresses the fact that for low dee voltage the ions have a long path at low energies so that the ion loss is large. The effect on the ion loss of increasing the pressure in the chamber could be compensated for by increasing the dee voltage in proportion (provided that ρ is kept constant).

Figure 8 shows some experimental results on the ion beam at different radii and different pressures. These results were obtained with the arc source pulsed in a time short compared with the pick-up time for the ions into stable acceleration. Thus the radial loss of beam is presumably entirely due to the loss of charge of the ions which we have been considering. The pressures were measured by means of an ionization gauge on the pump line and therefore is only a rough approximation to the true pressures inside the chamber. The pressure was varied by letting air into the chamber.

If we plot the $\ln I_R/I_0$ *vs.* the reciprocal of the radius we should, according to the above equation, obtain a straight line if ρ, p, and V_D are held constant. Figure 9 gives such plots for these different sets of conditions. It is observed that within the expected experimental error the data are quite linear. The data are sufficiently good to be in definite disagreement with variations of the cross section σ either as the three halves or five halves power of the ion energy. This was checked by plotting $\ln I_R/I_0$ *vs.* $\ln R/r_0$ and *vs.* $1/R^2$ for the 3/2 and 5/2 assumptions, respectively. In both cases the deviation from linearity was very marked. It is important to note that even under our most favorable operating conditions ($p=2\times 10^{-5}$ mm and $V_D=11$ kv) fully 90 percent of the ion beam was lost in going from a radius of $1\frac{1}{4}$ inches to a radius of 5 inches. Of course, a larger dee voltage would improve this situation considerably, so that if $V_D=40$ kv one would expect only 50 percent ion loss in this region.

It is certainly unsafe to extrapolate these relations to lower energies and radii, but it seems clear that there must be additional large ion losses in the region below $1\frac{1}{4}$ (extrapolating from the data given here one would guess at least another 80 percent loss at 11 kv or a thirty percent loss at 40 kv). These losses would be the same, naturally, for a large cyclotron as for the 37-inch at equal dee voltage.

One can think of various injection schemes for getting around these difficulties by giving the ions a high initial energy, but the most straightforward solution is to increase the dee voltage until the effect becomes negligible. At a dee voltage of 60 kv, for example, only fifty percent of the ions should be lost over-all due to this effect. The same result, of course, could be attained by improving the vacuum, but this may be difficult to do in view of the desire for heavier arcs and larger gas flows in the ion source.

8. TIME VARIATION OF FREQUENCY AND DEE VOLTAGE

From Eq. (11) under conditions (2) we have:

$$\rho V_D = \text{const.} df_a/dt$$

for the 37-inch operation. It can be shown from phase stability considerations that either an increase in ρ or a decrease in V_D during the ion acceleration will result in some loss of ions. The theoretical curve in Fig. 3 for $\rho > 0.5$ is a measure of the loss of ions on increasing ρ, since the amount of damping of the phase oscillations in the case of the f-m cyclotron is rather small. (There are usually less than a dozen oscillations in the whole acceleration.)

Some qualitative tests were made on the effect of distortion of the frequency-time curve on the ion beam output. These were made by adjusting the variable stator ring of the rotating condenser (9) during operation. By this means the slope of the frequency time curve could be distorted over a range of perhaps 50 percent. The ion current was not very sensitive to this distortion, and the maximum loss in current was only 30 percent.

No systematic studies were made concerning the effect of differing V_D *vs.* time variations on the ion current. However, it was observed that the presence of sharp peaks or breaks in this function was definitely deleterious. As mentioned above, there would be some ion loss if the dee voltage were allowed to decrease during the acceleration. This loss would not be very large, and because of the great sensitivity of the ion

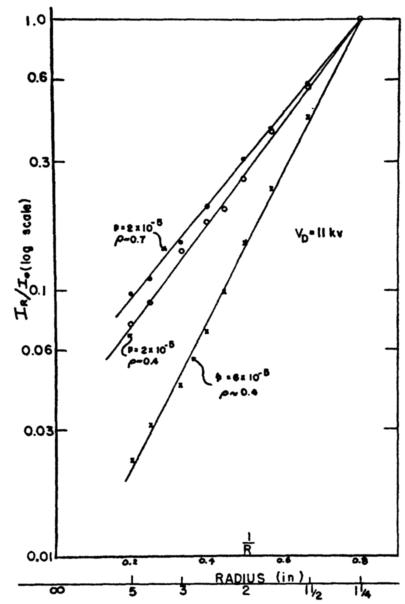

FIG. 9. This shows excellent verification of the expression developed in the paper and confirms the ion loss variation as one over the square of the energy up to deuteron energy of 600 kev. This also would indicate little ion loss from five inches radius to infinity.

current to dee voltage there might be some advantage in having the dee voltage abnormally high during the few microseconds of ion pick-up time. From the practical point of view this might be easier than holding V_D continuously at the higher value.

9. DEFLECTION OF THE BEAM

Shortly after it became clear that a substantial current of high speed ions was obtainable, a bombardment was planned for the purpose of comparing the actual energy of the beam with the expected value, as determined from the magnetic field and radius. A stack of Cu foils was placed on a probe, and the well-known 12.8-hour Cu^{64} activity was produced. The resulting excitation curve gave agreement between the actual and expected values of the beam energy to better than 5 percent, which is as good as could be expected because of the uncertain position of origin of the ions, and the uncertainties inherent in the method.

Of great interest was another result of such bombardments. Each foil was cut into several pieces so as to determine roughly the curve giving the *relative activity vs. radial distance back from the edge of the foil.* The result of one such experiment with four one-mil foils is shown in Fig. 10. Consider foil number 1. One sees that a quite appreciable amount of activity is found at distances greater than 7/64 inch from the edge of the probe.

In a cyclotron, if ΔE = maximum available energy increase per turn, at a radius R the maximum possible gain in radius per turn $=\Delta R = R\Delta E/2E$, where E is the energy of an ion at

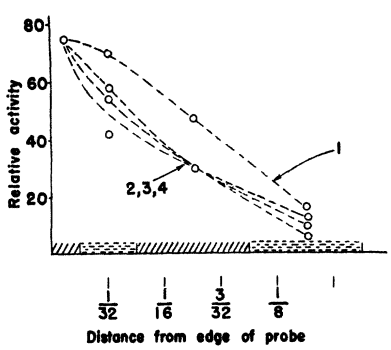

FIG. 10. Curves of *relative activity vs. radial distance from the edge of the probe* for a stack of four 0.001-inch Cu foils. The foils were sliced into the radial widths indicated.

radius R. In the case of the above experiment, $R=17$ inches, $E=7$ Mev, and $\Delta E=2$ kev, so $\Delta R\cong 5$ mils.

So we have a maximum possible increase of 5 mils per revolution for the radius of an ion trajectory, but an observed increase in the *distance from the cyclotron center* of more than 100 mils in one revolution.

The observed *fact* immediately offered, at least, a possibility of using a conventional d.c. deflector to withdrawn the beam; the apparent discrepancy posed an interesting problem.

That the curves of *relative activity vs. radial distance from probe edge* for the second, third, and fourth foils (Fig. 10) in the stack have the same shape proves that the ions falling some distance from the probe edge have the full energy. The different shape of curve 1 opens the possibility that the circulating current has a considerable relatively low energy tail.

The ability of an ion to increase its distance from the cyclotron center in one revolution as much as 20 or 30 times the amount of increase of its radius of curvature is based on the *precession* of the orbits.

In Fig. 11, O represents the geometric and magnetic center of the cyclotron. As mentioned previously, the magnetic field was made to decrease radially in such a fashion as to simulate a relativistic increase in mass. We may express the field near any radius r_0 in this fashion.

$$H(r)=H(r_0)(r_0/r)^n.$$

In Fig. 11 the dotted circle represents the path of a particle which has an energy, E_0, appropriate to the radius r_0 and the field $H(r_0)$. Now consider a particle which has the same E_0, but which, for one reason or another, exists at a distance x beyond the dotted circle and with its center of curvature at A on the radius OAJ (i.e., $OA=x$). It can be shown[13] that for $x\ll r_0$, and if n is constant in the region involved, that the distance of the particle from the dotted circle will vary in a simple harmonic manner. Moreover, if we call f_x the frequency of this motion and f the radiofrequency of the dee voltage, then $f_x=f(1-n)^{\frac{1}{2}}$, and the number of r-f cycles per cycle of radial oscillation $=1/(1-n)^{\frac{1}{2}}$. Consider

[13] McMillan, Bohm, and Foldy, unpublished.

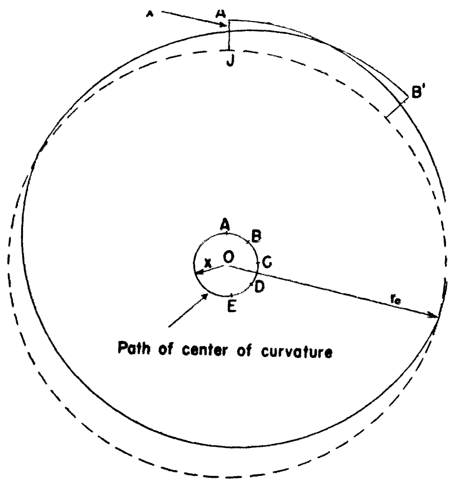

FIG. 11. The solid line represents the approximate path of an ion in a radially decreasing magnetic field during one cycle of radial oscillation.

a particular case in which $n=0.2$ in the region about r_0, then

$$\frac{\text{number of r-f cycles}}{\text{cycle of radial oscillation}}=1.12.$$

So after 1.12 r-f cycles, one cycle of radial oscillation would be completed. The particle follows the solid curve, Fig. 11, and in further cycles of radial oscillation the particle's path forms a rosette. The orbit is said to precess. In the above motion the center of curvature has moved from A to B. After subsequent cycles of radial oscillation the center of curvature takes the positions C, D, E, etc.

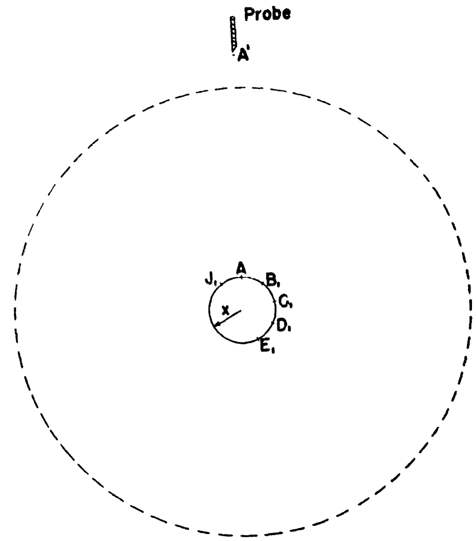

FIG. 12. The points, A, B_1, C_1, D_1, etc., represent the position of the center of curvature of an ion in a radially decreasing magnetic field after successive cycles of the radiofrequency voltage which is applied to the cyclotron dee.

Of more interest is the position of the center of curvature after successive r-f cycles.

$$\beta=\frac{\text{number of degrees of advance of center of curvature}}{\text{r-f cycle}}=360(1-(1-n)^{\frac{1}{2}}).$$

For our example $\beta=36°$.

In Fig. 12, then, we plot the position of the center of curvature after successive r-f cycles.

Now consider the case in which a particle at A', at the beginning of motion, just misses a probe P. Because of the progression of its center of curvature, it must certainly miss the probe on the next few r-f cycles. Now, depending on the ratio of r_0 to x, and depending on the amount of increase in the radius of curvature per cycle because the particle is being accelerated, we may obtain the case in which, after several cycles, the center of curvature is at J_1, with the particle just missing the probe, i.e., now having a radius of curvature J_1A'. Then during the *next* r-f cycle the center of curvature shifts to A. The particle after this cycle will strike the probe. Because of the shift in the center of curvature from J_1 to A, the distance back from the leading edge of the probe at which the particle strikes may be much greater than that which could be due to the increase in its radius of curvature in one turn.

It is clear that when one proceeds from a discussion of a probe to that of deflector, the question as to whether or not an appreciable fraction of the ions will enter the deflector channel under conditions such that they can be successfully deflected becomes involved. In the case of the 37-inch cyclotron these points bore consideration:

1. By the very nature of process making the

deflection possible, many of the ions could be expected to have relatively large radial components of velocity upon entering the channel.

2. In the range of radii from 17 inches to 19 inches, n varies from 0.25 to 1.1. The amplitude of the radial oscillation was expected to be at least an inch. With such a large variation of n over the range of the radial oscillation, the simple picture of the means whereby these oscillations might lead to a mechanism for deflection does not hold.

It was decided, however, to determine by direct experiment whether a conventional d.c. deflector would remove any ions. Accordingly, the deflector, whose mechanical details have been described, was installed. It operated very satisfactorily. With 7.5-Mev deuterons under optimum adjustment, about 10 percent of the circulating current was extracted. In a conventional cyclotron about 25 percent of the circulating current may be deflected.

With 14.5-Mev protons, it was possible to deflect 5 percent of the circulating current.

The above deflections amounted to a 11–14 percent increase in radius through a channel whose angular length was 60°.

The center of curvature could be shifted away from the geometric and magnetic center of the cyclotron by (1) simply adjusting the position of the ion source away from the cyclotron center, or (2) by the application of the positive d.c. bias on the dee. This bias produces an asymmetric acceleration, and tends to build up a radial oscillation.

It was found that the magnitude of the deflected beam was much more sensitive to the operating value of the magnetic field than was the circulating current. The value of n at a particular radius did depend somewhat on the magnitude of H, so by adjusting H it was apparently possible to adjust the average n at radii near that of the exit strip to a more or less favorable value. Note that $n=\frac{3}{4}$, which gives $\beta=180°$, would not be expected to be favorable for deflection, for after two r-f cycles the center of curvature would shift once around its circle, allowing an insufficient number of r-f cycles during which to increase the radius of curvature. Values of n greater or less than $\frac{3}{4}$ would be expected to be more effective.

10. PERFORMANCE

A steady ion current of three microamperes of 7.3-Mev deuterons was obtained at an internal target. Approximately 10 percent of this current was brought out through the deflecting channel.

At the conclusion of these experiments and tests on the f-m cyclotron it was decided to convert the 37 inch to the acceleration of protons. The f-m technique has a considerable advantage in simplicity even in the energy range of 14.6-Mev protons which was the energy produced in this case. It would probably require a dee voltage of 150 kv to produce a satisfactory beam of this energy without frequency modulation. In our case a dee voltage of 8 kv was sufficient to produce a proton beam of two microamperes to an internal target. The deflection of the 14.6-Mev protons was considerably more difficult than that of the 7.3-Mev deuterons, and the maximum current obtained down the deflector channel was five percent of the circulating current. This beam after it had emerged from between the coil tanks had a current density of 1.4×10^{-9} amp./cm^2.

It is a pleasure to express our thanks to Professors E. M. McMillan and R. L. Thornton for many helpful suggestions and criticisms, and our gratitude to Professor Ernest O. Lawrence whose inspiration and encouragement made this development possible.

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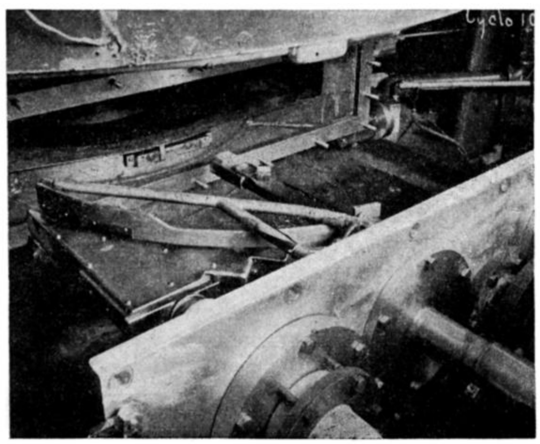

FIG. 2. A photograph showing the deflector and arc source face plate as it is removed from the tank.

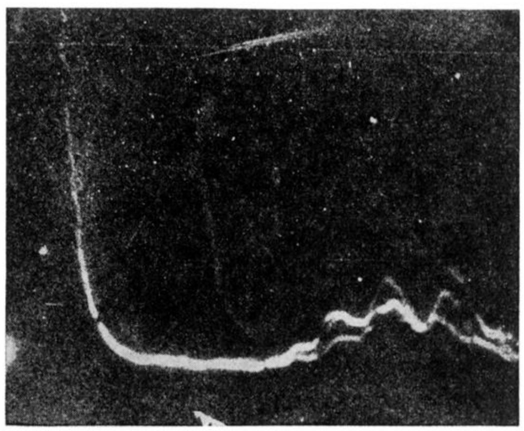

FIG. 6. This oscilloscope photograph shows the ion current to the probe as a function of the time. The markers are at 10-microsecond intervals. The peak on the left is the arc pulse, and those on the right represent the effect of the phase oscillations. There are three modulation cycles in this photograph.