pointing into the future are represented by spinvectors c^α:

$$a^{\dot{\alpha}\beta} = c^{\dot{\alpha}} c^{\beta}, \tag{2}$$

where c^1, c^2 are Gaussian integers, so that c^1 and c^2 are elatively prime and neither is divisible by $1+i$, or else so that $1+i$ is the greatest common factor of c^1, c^2 and one of them is divisible by 2. Such spinvectors will be called *integral.*

As is well known, the spin transformations

$$c'^{\alpha} = \lambda_\beta{}^\alpha c^\beta, \quad c'^{\dot{\alpha}} = \bar{\lambda}_\beta{}^\alpha c^{\dot{\beta}}, \quad |det(\lambda_\beta{}^\alpha)| = 1, \tag{3}$$

represent all proper homogeneous Lorentz transformations, $\lambda_\beta{}^\alpha$ being determined by $L_s{}^r$ to within an arbitrary phase factor $e^{i\theta}$. If this phase factor is chosen suitably, the integral Lorentz transformations are represented by exactly those spin transformations which, together with their inverse, map integral spinvectors into integral spinvectors. This theorem enables us to find the following spin representation of the integral Lorentz group:

$$\lambda_1{}^1\lambda_2{}^2 - \lambda_2{}^1\lambda_1{}^2 = 1, \tag{4}$$

where one of the following cases applies: I. $\lambda_\beta{}^\alpha$ are Gaussian integers such that $\Sigma_{\alpha\beta}\lambda_\beta{}^\alpha$ is divisible by $1+i$. II. $\lambda_\beta{}^\alpha = \mu_\beta{}^\alpha/(1+i)$, where $\mu_\beta{}^\alpha$ are Gaussian integers not divisible by $1+i$. III. $\lambda_\beta{}^\alpha = 2^{\frac{1}{2}}\mu_\beta{}^\alpha/(1+i)$, where $\mu_\beta{}^\alpha$ are Gaussian integers such that $\Sigma_{\alpha\beta}\,\mu_\beta{}^\alpha$ is divisible by $1+i$. IV. $\lambda_\beta{}^\alpha = \frac{1}{2}2^{\frac{1}{2}}\mu_\beta{}^\alpha$, where $\mu_\beta{}^\alpha$ are Gaussian integers not divisible by $1+i$. The theory of Gaussian integers shows immediately that each of the above cases includes an infinity of spin transformations. Thus, *the integral Lorentz group is infinite*, though discrete.

From the above it can be deduced that all primitive integral null vectors are equivalent, in the sense that any two of them are integral transforms of one another.

Consider any integral vector and form the set of all its integral transforms. Project each of these vectors onto the *xyz* space. Then the directions defined by these projections in 3-space are everywhere dense. This shows that our discrete space-time model possesses a large measure of *spatial isotropy*. It is obvious that our cubic lattice is invariant under all translations which map one lattice point into another. In this sense our discrete model is *homogeneous.*

Finally we must mention a property of our model which constitutes a drawback as far as hopes for physical application are concerned. The velocities associated with integral Lorentz transformations are given by the formula $v = (n^2-1)^{\frac{1}{2}}/n$, where n is any positive integer. The smallest non-zero velocity is $\frac{1}{2}3^{\frac{1}{2}} = 0.866$ times the velocity of light.

[1] Frank B. Jewett Fellow, on leave from Carnegie Institute of Technology, Pittsburgh, Pennsylvania.
[2] V. Ambarzumian and D. Iwanenko, Zeits. f. Physik **64**, 563 (1930); L. Silberstein, *Discrete Space-Time* (University of Toronto Studies, Physics Series, 1936).
[3] The coordinates are integral multiples of a "fundamental length" ϵ (probably of the order of nuclear dimensions). We choose ϵ as the unit of length.
[4] O. Laporte and G. E. Uhlenbeck, Phys. Rev. **37**, 1381 (1931).
[5] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers* (Oxford University Press, New York, 1908), Chapter XII.

Range and Energy of Beta-Radiation from Calcium 45

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THE radiations given off by Ca^{45} have been investigated by Walke, Thompson, and Holt[1] who report beta-radiations of maximum energy 0.2 and 0.9 Mev, gamma-radiation of 0.7 Mev, and a half-life of 180 days. The energies of the radiation were determined by absorption measurements. In view of the importance of this isotope in biological research, it has seemed advisable to reinvestigate the radiation characteristics.

Experimental details.—Carrier-free Ca^{45}, produced by $n-p$ reaction on monoisotopic Sc^{45}, was obtained from the Atomic Energy Commission. The counting apparatus was the same as that previously described;[2] a thin-window (1.9 mg/cm^2) Geiger counter was used. The source was essentially carrier-free, and was deposited in a thin (0.017-inch) aluminum stamping. The beta-radiation was measured by absorption in aluminum foils.

Results and discussion.—The method of Feather[3] was used in analyzing the results, as previously described.[2] The initial strength of the sources varied from 3000 to

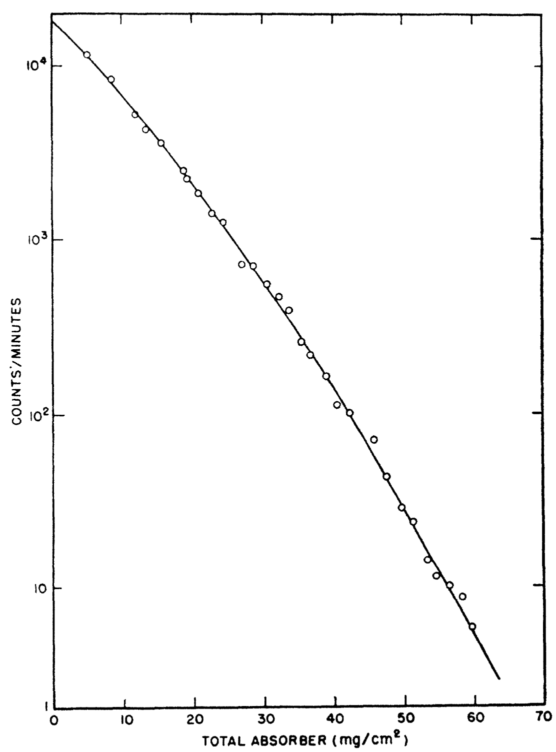

FIG. 1. Aluminum absorption curve of Ca^{4} β-radiation.

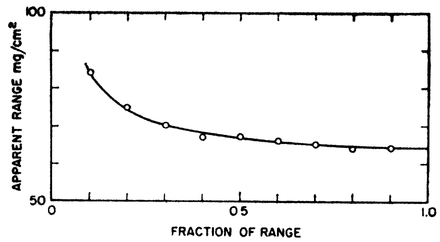

FIG. 2. Feather plot for Ca^{45}.

12,000 counts per minute, and the contribution due to gamma-rays and other unabsorbed contaminants was less than one part in 3000 with the strongest source, thus indicating the absence of any appreciable amount of gamma-radiation. The absorption curve obtained with the strongest source is shown in Fig. 1. The Feather plot, shown in Fig. 2, gives a range of 64 ± 1 mg/cm^2.

Glendenin[4] has shown that a reliable range-energy curve for the low energy region can be derived from the data of Marshall and Ward[5] for monoenergetic electrons and beta-ray spectrograph data on low energy beta-emitters. Glendenin's curve is identical with that of Marshall and Ward below 0.5 Mev. Using this range-energy curve, we have found that the Ca^{45} beta-radiation has a maximum energy of 260 ± 5 kev. We have found no evidence of any harder beta-radiation, or of any gamma-radiation at all in the course of this investigation.[6]

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6 This result is in good agreement with a value of 250 kev, given in *Radioisotopes, Catalog and Price List No. 2*, revised September, 1947, distributed by Isotopes Branch, United States Atomic Energy Commission. Unfortunately, the Atomic Energy Commission's result is not supported by any published experimental evidence.

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

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ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction $(\sim(e^2/\hbar c)m_0)$ to the mechanical mass m_0.

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;* the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu=(\frac{1}{2}\pi)e^2/\hbar c=0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium[1] have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.[2] Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydrogen and deuterium hyperfine structures to be $\delta\mu/\mu=0.00126\pm0.00019$ and $\delta\mu/\mu=0.00131\pm0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the $^2S_{\frac{1}{2}}$, $^2P_{\frac{1}{2}}$, and $^2P_{3/2}$ states of sodium and gallium.[3] To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu=0.00118\pm0.00003$.