On the Polarization of Fast Neutrons

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
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ALTHOUGH the production of polarized thermal neutrons has long been an accomplished fact, no such success has been forthcoming with fast neutrons. Only one method for the polarization of fast neutrons has thus far been suggested,[1] of which the essential mechanism is the large, effective nuclear spin-orbit interaction present when neutrons are resonance scattered by helium and similar nuclei. It is the purpose of this note to suggest a second mechanism for polarizing fast neutrons—the spin-orbit interaction arising from the motion of the neutron magnetic moment in the nuclear Coulomb field. Despite the apparent small magnitude of this interaction, the long-range nature of the Coulomb field is such that the use of small scattering angles will produce almost complete polarization under ideal conditions. A closely related phenomenon produced by this electromagnetic interaction is an additional scattering of unpolarized neutrons which increases rapidly with decreasing scattering angle and is comparable with purely nuclear scattering at the small angles effective in producing polarized neutrons.

The energy of a neutron moving in an electric field, $\mathbf{E}=-\nabla\phi$, is described by the following contribution to the neutron Hamiltonian:

$$H'=\mu_n(e\hbar/2M^2c^2)\boldsymbol{\sigma}\cdot\mathbf{E}\times\mathbf{p}, \qquad (1)$$

where $\mu_n=1.91$ is the numerical value of the neutron moment in units of $e\hbar/2Mc$, $\boldsymbol{\sigma}$ is the Pauli spin vector, and $\mathbf{p}$ is the momentum of the neutron. In order that the electric field be fully effective in producing spin-dependent scattering the major portion of this scattering should take place outside of the nucleus ($r\gg R$), but well within the screening radius of the atomic electrons ($r\ll a$). This restricts the range of useful scattering angles, since the waves scattered through an angle ϑ are primarily generated at a distance r from the nucleus, given by

$$2kr\sin\vartheta/2\sim 1, \qquad (2)$$

where $k=p/\hbar$ is the neutron wave number. Hence, the unscreened Coulomb field of a point nucleus will be effective for scattering in the angular range:

$$1/ka\ll 2\sin\vartheta/2\ll 1/kR. \qquad (3)$$

If the nuclear radius and atomic screening radius are taken to be

$$R=1.5\cdot 10^{-13}A^{\frac{1}{3}}\text{ cm} \quad\text{and}\quad a=0.53\cdot 10^{-8}Z^{-\frac{1}{3}}\text{ cm},$$

the angle restrictions for a 1-Mev neutron scattered in Pb, for example, are

$$4\cdot 10^{-4}\ll 2\sin\vartheta/2\ll\tfrac{1}{2}. \qquad (4)$$

The electromagnetic scattering of a neutron under these conditions can be calculated with the plane wave Born approximation, for the nuclear scattered wave is negligible compared with the incident wave at the significant scattering distances. We denote the incident plane wave by

$$\psi_{\text{inc}}=e^{i\mathbf{k}_0\cdot\mathbf{r}}\chi, \qquad (5)$$

where $\mathbf{k}_0$ is the initial propagation vector and χ is a spin function. The asymptotic form of the wave scattered in the direction of the propagation vector $\mathbf{k}$ is then

$$\psi_{sc}\sim(e^{ikr}/r)f(\vartheta)\chi, \qquad (6)$$

with

$$f(\vartheta)=f_0(\vartheta)+\tfrac{1}{2}i\boldsymbol{\sigma}\cdot\mathbf{n}\cot\vartheta/2(\hbar/Mc)(Ze^2/\hbar c). \qquad (7)$$

In this formula, $\mathbf{n}$ is the unit vector defined by

$$\mathbf{k}\times\mathbf{k}_0=\mathbf{n}k^2\sin\vartheta, \qquad (8)$$

and $f_0(\vartheta)$ is the amplitude of the wave scattered by specifically nuclear forces. We assume the latter to be spin-independent and, further, ignore any inelastic nuclear scattering. Both of these assumptions are not unreasonable for neutron energies in the vicinity of 1 Mev.

The intensity of the scattered wave is determined by the following spin scalar product:

$$r^2(\psi_{sc},\psi_{sc})=(\chi, f^{\dagger}(\vartheta)f(\vartheta)\chi), \qquad (9)$$

[1] J. Schwinger, Phys. Rev. 69, 681 (1946).

which yields

$$r^2(\psi_{sc},\psi_{sc})=|f_0(\vartheta)|^2+\gamma^2\cot^2\vartheta/2 + 2\gamma Im\, f_0(\vartheta)\cot\vartheta/2\,\mathbf{n}\cdot\mathbf{P}_{\rm inc}, \quad (10)$$

in which

$$\gamma=\tfrac{1}{2}\mu_n(\hbar/Mc)(Ze^2/\hbar c) \quad (11)$$

and

$$\mathbf{P}_{\rm inc}=(\chi,\boldsymbol{\sigma}\chi) \quad (12)$$

is a vector describing the polarization state of the incident beam. The corresponding vector for the scattered wave is

$$\mathbf{P}_{sc}=(\psi_{sc},\boldsymbol{\sigma}\psi_{sc})/(\psi_{sc},\psi_{sc}), \quad (13)$$

where

$$r^2(\psi_{sc},\boldsymbol{\sigma}\psi_{sc})=\mathbf{n}2\gamma Im\, f_0(\vartheta)\cot\vartheta/2 + |f_0(\vartheta)|^2\mathbf{P}_{\rm inc}+\gamma^2\cot^2\vartheta/2(2\mathbf{nn}\cdot\mathbf{P}_{\rm inc}-\mathbf{P}_{\rm inc}) -2\gamma Re\, f_0(\vartheta)\cot\vartheta/2\,\mathbf{n}\times\mathbf{P}_{\rm inc}. \quad (14)$$

For an initially unpolarized beam:

$$\mathbf{P}_{sc}=\mathbf{n}\frac{2Im\, f_0(\vartheta)\gamma\cot\vartheta/2}{|f_0(\vartheta)|^2+\gamma^2\cot^2\vartheta/2}=\mathbf{n}P(\vartheta). \quad (15)$$

The discussion of the latter formula is greatly simplified by noting that, within the restricted angular range (3), the specifically nuclear scattering must be insensitive to angle and can be replaced by the value appropriate to forward scattering. Now, according to a well-known theorem, $Im\, f_0(0)$ is related to the total scattering cross section by

$$Im\, f_0(0)=(k/4\pi)\sigma, \quad (16)$$

while

$$|f_0(0)|^2=(\sigma/4\pi)G \quad (17)$$

expresses the differential cross section for forward scattering in terms of the "gain," the ratio of the actual forward scattered intensity to that of an isotropic scatterer. It follows from the form of (15) that there is an optimum scattering angle, ϑ_0, for the production of polarized neutrons, namely,

$$\tan\vartheta_0/2=\frac{\gamma}{|f_0(0)|}=\tfrac{1}{2}\mu_n\frac{Ze^2}{\hbar c}\left[\frac{4\pi(\hbar/Mc)^2}{\sigma G}\right]^{\frac{1}{2}}. \quad (18)$$

The maximum polarization is

$$P(\vartheta_0)=\frac{Im\, f_0(0)}{|f_0(0)|}=\left[\frac{k^2\sigma}{4\pi G}\right]^{\frac{1}{2}}. \quad (19)$$

In order to estimate the magnitudes of these quantities, it is necessary to have some knowledge of the energy dependence of σ and G. The model of an impenetrable sphere provides the following information in the limits $kR\ll 1$ and $kR\gg 1$:

$$\begin{array}{lll} kR\ll 1: & \sigma=4\pi R^2, & G=1 \\ kR\gg 1: & \sigma=2\pi R^2, & G=\tfrac{1}{2}(kR)^2. \end{array} \quad (20)$$

The predicted limiting forms of the quantities (18) and (19) are then

$$\tan\vartheta_0/2=\begin{cases}\tfrac{1}{2}\mu_n\dfrac{Ze^2}{\hbar c}\dfrac{\hbar/Mc}{R}, & kR\ll 1\\[2ex] \mu_n\dfrac{Ze^2}{\hbar c}\dfrac{\hbar/Mc}{R}\dfrac{1}{kR}, & kR\gg 1\end{cases} \quad (21)$$

and

$$P(\vartheta_0)=\begin{array}{ll}kR, & kR\ll 1\\ 1, & kR\gg 1.\end{array} \quad (22)$$

It appears to be a reasonable interpolation to place

$$\begin{array}{l}\tan\vartheta_0/2\cong\tfrac{1}{2}\mu_n\dfrac{Ze^2}{\hbar c}\dfrac{\hbar/Mc}{R}\\ P(\vartheta_0)\cong 1\end{array} \quad (23)$$

for $kR\sim 2$, which is the value appropriate to 1-Mev neutrons scattered in Pb. Under these conditions, we expect practically complete polarization for a scattering angle of $\vartheta_0=1.5°$. In view of the stationary character of the polarization in the vicinity of this angle, somewhat larger angles can be employed without undue impairment of the degree of polarization. Thus, for $\vartheta=3°$, $P=0.80$; $\vartheta=6°$, $P=0.47$; $\vartheta=9°$, $P=0.32$.

To detect the polarization produced by scattering, it is necessary to subject the polarized neutrons to a second scattering process. If the two scattering angles are ϑ_1 and ϑ_2, with the normals to the two scattering planes being $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, the intensity after the second deflection is, according to (10) and (15), proportional to

$$1+\mathbf{n}_1\cdot\mathbf{n}_2P(\vartheta_1)P(\vartheta_2). \quad (24)$$

When both scattering events occur in the same plane, the intensity for the situation in which the two deflections occur in the same sense exceeds that for deflections in the opposite sense in the ratio:

$$R=\frac{1+P(\vartheta_1)P(\vartheta_2)}{1-P(\vartheta_1)P(\vartheta_2)}. \tag{25}$$

If both scattering angles equal the optimum angle ϑ_0, this ratio can be very large. Thus the experimental difficulties accompanying the small angles involved are ameliorated to some extent by the large effects under investigation. However, somewhat larger angles can be employed without destroying the experimental effect. For example, under the numerical conditions previously employed, $R=2.0$ for $\vartheta_1=1.5°$, $\vartheta_2=9°$, while $R=1.2$ for $\vartheta_1=\vartheta_2=9°$.

Finally, we note that the differential cross section for the scattering of an unpolarized neutron beam is modified at small angles. According to Eq. (10),

$$\sigma(\vartheta)=\sigma_0(\vartheta)+\gamma^2\cot^2\vartheta/2,$$

where $\sigma_0(\vartheta)$ is the specifically nuclear differential cross section, which assumes the value $(\sigma/4\pi)G$ at the small angles significant for the electromagnetic scattering. Thus the additional contribution to the scattering increases rapidly with diminishing angle and equals the purely nuclear scattering at precisely the angle ϑ_0 that is optimum for polarization. In view of the small angular range over which it is effective, the electromagnetic scattering provides a negligible contribution to the total cross section, namely,

$$\delta\sigma\sim 2\pi\gamma^2\log a/R=2\pi\mu_n{}^2(\hbar/Mc)^2 \times(Ze^2/\hbar c)^2\log 3.5\cdot 10^4(AZ)^{-\frac{1}{3}} \tag{26}$$

for $kR>1$, which has the value $\delta\sigma=2.6\cdot 10^{-26}$ cm^2 for Pb.