comments will show why this is true and to what extent it may be experimentally observed.

If recombination is disregarded, the term in Eq. (2) containing R is absent. Similarity of φ then requires only E/N, ω/N, and Nx to remain unchanged. The electron density n is quite unaffected by the changes in E, N, ω, and x. Hence one might conclude that n remains the same. But this would be fallacious, for Eq. (2) in this case does not determine n at all and is satisfied for any n. Variation of n in discharges is therefore a very critical matter; diffusion alone leaves it indeterminate, the presence of recombination causes it to vary with N, other mechanisms impose a more peculiar variation. All this is true, of course, for d.c. as well as for h.f. discharges. It shows that similarity applies strictly only to the energy dependence of φ, not necessarily to the observable properties of discharges.

If diffusion is disregarded in Eq. (2), D is zero and the constancy of Nx is no longer required for similarity of φ. The role of diffusion is thus rather distinctive, and its importance should be settled by experimentation. The obvious procedure would be to study discharges with the same E, ω, and N but with different dispositions of electrodes and sizes or shapes of discharge vessels. A difficulty encountered in this attempt is that, in practice, one rarely gets the same E in a cavity if its dimensions or its shape are altered. Here the similarity principle would help, for it says that a change in E can be compensated by changes in ω and in N. In performing such experiments it would be desirable primarily to measure quantities depending on φ, such as breakdown and maintenance potentials. Though these are ill defined theoretically, it would seem that equality of these potentials indicates equality of energy distributions.[5]

The importance of the principle arises from the fact that failure of a given type of discharge to conform to it exposes the presence of processes whose cross sections have a peculiar dependence on n and N.

[5] Cf. L. M. Hartman, Phys. Rev. 73, 316 (1948).

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Electronic Component of Cosmic Rays in the Low Atmosphere. I. Theoretical

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The authors discuss the problem of the equilibrium conditions between meson and secondary electronic component. Following William's concept of *electron track*, some formulas are deduced for the number of electrons with energy $\geqslant \eta$ generated in coulombian collisions and in equilibrium with the meson component, in a substance having a critical energy ϵ_c. Similarly a general formula is given for the number of disintegration electrons in equilibrium with the meson component in atmosphere. Such formulas present a remarkable generality because they include the fundamental parameters explicitly, and the meson spectrum appears in them in integral form; this makes them particularly suitable for comparison with experiments.

With the aid of these formulas the following calculations have been performed: (1) Number of secondary electrons generated in Pb and in equilibrium with the meson component for $\eta=2$, 3, 4, 5 Mev and for different values of the lower limit of meson energy. (2) Number of secondary collision electrons arising from the atmosphere for different values of η at sea level and at 3500 m. above sea level. (3) Number of secondary disintegration electrons for different values of the cut-off energy, etc.

I. INTRODUCTION

MEASUREMENTS concerned with the electronic component of cosmic rays near the earth's surface are not well understood in terms of the usual theories of their origin. According to the theoretical work of Heitler,[1] Euler and

[1] W. Heitler, Proc. Roy. Soc. 161, 261 (1937).

Heisenberg,[2] Ferretti,[3] Bhabha,[4] Lyons,[5] etc., only electrons in equilibrium with the hard component should be expected between sea level and an altitude of 4–5 km.

On the other hand, experiments performed by Cacciapuoti[6] and Bernardini, Cacciapuoti, Ferretti, Piccioni and Wick[7] seem to indicate that the soft component is not in equilibrium with the hard one; in fact, the increase in intensity of the soft component with height is somewhat faster than is theoretically predicted. These results were confirmed by further experiments performed by Cocconi and Tongiorgi[8] and by Rossi and Greisen.[9]

Many theoretical and experimental investigations have since been performed to establish the equilibrium conditions between electronic and meson components but the conclusions are not as yet in satisfactory agreement. In most of these works,[10–19] the excess in the intensity of the soft component is ascribed mainly to the presence of slow mesons (energy between 0 and 200 Mev). According to Greisen,[14] the number of these slow mesons would be, even at sea level, so high as to rule out the normal process of disintegration of a meson into an electron and a neutrino.[20]

Since several experiments performed by the Rome Center of Nuclear Physics during the war seem to suggest a rather different conclusion, we thought it would be useful to discuss the whole problem over again, taking also into account the recent results published by Hazen,[21] Nassar and Hazen,[22] and by Powell.[23]

II. GENERAL REMARKS ON THE EQUILIBRIUM CONDITIONS BETWEEN MESON AND SECONDARY ELECTRONIC COMPONENTS

Following the well-known Williams' argument, Ferretti[24] and Rossi and Klapman[25] have pointed out that in treating the equilibrium conditions between meson and electronic components, it is useful to introduce the concept of *electron track*. If $N(\epsilon, \eta, x)$ is the number of electrons with energy $\geqq\eta$ in a shower originated by an "ancestor electron" having the initial energy ϵ and present at a distance x from the origin of the shower, the electron track is defined by the integral:

$$z(\epsilon, \eta)=\int_0^\infty N(\epsilon, \eta, x)dx.$$

This expression denotes the total length of the paths described by the electrons of the shower while their energy is larger than η. According to Ferretti,[24] if β denotes the ionization loss per unit length and ϵ_c the critical energy of the substance in which the shower has originated, a sufficiently approximated value of $z(\epsilon, \eta)$ is given by:

$$z(\epsilon, \eta, \epsilon_c)=\frac{\epsilon}{\beta}\left\{1-\frac{\eta}{\epsilon}\exp\left[-\frac{2(\epsilon-\eta)}{\epsilon_c}\right]\right.$$
$$\left.-\frac{2\eta}{\epsilon_c}\exp\left[\frac{2\eta}{\epsilon_c}\right]\left[E_i\left(-\frac{2\epsilon}{\epsilon_c}\right)-E_i\left(-\frac{2\eta}{\epsilon_c}\right)\right]\right\}. \quad (1)$$

Ferretti's relation is particularly convenient because of its simplicity and its explicit dependence upon the fundamental parameters ϵ, η, ϵ_c.

Now, according to the already mentioned intuitive considerations of Williams,[26] when the genetic equilibrium between electronic and meson

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[2] H. Euler and W. Heisenberg, Ergeb. d. exakt Naturwiss. **17**, 1 (1938).
[3] B. Ferretti, Ricerca Scient. **7–8**, 736 (1939).
[4] H. J. Bhabha, Proc. Roy. Soc. **164**, 25 (1938).
[5] Lyons, Phys. Zeits. **42**, 166 (1941).
[6] B. N. Cacciapuoti, Ricerca Scient. **12**, 1082 (1939).
[7] G. Bernardini, B. N. Cacciapuoti, B. Ferretti, O. Piccioni, and G. C. Wick, Phys. Rev. **58**, 1017 (1940).
[8] G. Cocconi and V. Tongiorgi, Zeits. f. Physik **118**, 88 (1941).
[9] B. Rossi and K. Greisen, Phys. Rev. **61**, 121 (1942).
[10] Tamm and Belenky, Z. Phys. Ac. Sc. USRR **1**, 177 (1939).
[11] J. A. Richards and L. W. Nordheim, Phys. Rev. **61**, 735 (1942).
[12] P. V. Auger, Phys. Rev. **61**, 684 (1942).
[13] W. E. Hazen, Phys. Rev. **65**, 67 (1944).
[14] K. Greisen, Phys. Rev. **63**, 323 (1943).
[15] H. E. Stanton, Phys. Rev. **66**, 48 (1944).
[16] D. B. Hall, Phys. Rev. **66**, 321 (1944).
[17] S. V. Chandrashekhar Aiya and R. C. Saxena, Phys. Rev. **66**, 183 (1944).
[18] A. Alichanow and A. Alichanian, J. Phys. U.S.S.R. **9**, 73 (1945).
[19] S. Nassar and W. E. Hazen, Phys. Rev. **69**, 298 (1946).
[20] The different behavior of positive and negative mesons predicted by Tomonaga and Araky (Phys. Rev. **58**, 90 (1940)) and confirmed experimentally by Conversi, Pancini and Piccioni (Phys. Rev. **68**, 232 (1945)) does not affect the mesons in the atmosphere because it concerns only mesons at the end of their range and practically at rest.

[21] W. E. Hazen, Phys. Rev. **64**, 7 (1943) and **65**, 67 (1944).
[22] S. Nassar and W. E. Hazen, Phys. Rev. **69**, 298 (1946).
[23] W. M. Powell Phys. Rev. **69**, 385 (1946).
[24] B. Ferretti, Ricerca Scient. **13**, 532 (1942).
[25] B. Rossi and S. J. Klapman, Phys. Rev. **61**, 414 (1942).
[26] E. J. Williams, Proc. Camb. Phys. Soc. **36**, 183 (1940).

components is reached, the number of electrons with energy $\geqq\eta$ generated by coulombian collision processes which accompany the mesons of energy E to $E+dE$, is given by

$$J(E)dE\int_{\eta}^{\epsilon_m(E)}\sigma(\epsilon,E)z(\epsilon,\eta)d\epsilon,$$

where $\sigma(\epsilon, E)$ denotes the total effective cross section for coulombian collisions, $\epsilon_m(E)$ the maximum energy that can be transferred from a meson to an electron in the collision and $J(E)$ the meson differential spectrum: The total number of electrons with energy $\geqq\eta$ generated in coulombian collisions and in equilibrium with the meson component, in a substance having a critical energy ϵ_c, is therefore

$$n_c(\eta)=\int_{E^*}^{\infty}J(E)dE\int_{\eta}^{\epsilon_m}\sigma(\epsilon,E)z(\epsilon,\eta)d\epsilon, \quad (2)$$

where E^* denotes the minimum energy a meson must have in order to generate, through a collision, an electron having at least an energy equal to η.

For the study of the equilibrium conditions near sea level, it may be assumed that the spin of the meson is zero.[27] In this case, if k indicates a well-known coefficient which depends on the substance (see below), we can write with good approximation

$$\sigma(\epsilon,E)=\sigma(\epsilon,\epsilon_m)=[k/\epsilon^2][1-(\epsilon/\epsilon_m(E))], \quad (3)$$

where $\epsilon_m(E)$ is the maximum energy that a meson with energy E can transfer to an electron in a coulombian collision, and is given by

$$\epsilon_m=[E^2-(\mu c^2)^2]/[\mu c^2(\mu/2m)-(E/\mu c^2)], \quad (4)$$

where μ is the mass of the meson, m the mass of the electron. The value of E^* to be taken as a lower limit for the first integral in (2) is therefore

$$E^*(\eta)=(\eta/2)+\{(\eta^2/4)+(\mu c^2)^2[(\eta/2mc^2)+1]\}^{\frac{1}{2}}. \quad (5)$$

Moreover, for an electron generating a shower and having an energy much greater than the cut-off energy, Ferretti's relation assumes the following simple form*

$$z_\infty(\epsilon,\eta)=S_\infty\epsilon/\beta,$$

with

$$S_\infty=1+\frac{2\eta}{\epsilon_c}\exp\left[\frac{2\eta}{\epsilon_c}\right]E_i\left(-\frac{2\eta}{\epsilon_c}\right). \quad (6)$$

In order to calculate the integral (2) we have split it in the following way:

$$n_c(\eta)=\int_{E^*}^{E_c}J(E)dE\int_{\eta}^{\epsilon_m}\sigma z d\epsilon+\int_{E_c}^{\infty}J(E)dE\int_{\eta}^{\epsilon_c}\sigma z d\epsilon+\int_{E_c}^{\infty}J(E)dE\int_{\epsilon_c}^{\epsilon_m}\sigma z d\epsilon$$

where E_c is the energy of a meson which can transfer to an electron only an energy $\leqq\epsilon_c$.

Since the differential spectrum of mesons deduced from Wilson chamber measurements is not known with great precision, neither at high nor at low energies, we have proceeded with an integration by parts so as to express the results in terms of the integral spectrum.

Indicating with

$$I(E)=\int_E^{\infty}J(E)dE, \quad (7)$$

the integral spectrum, it can easily be seen that the number of collision electrons is given by the following general formula:

$$\frac{n_c}{k}=I(E^*)\phi(E^*)+I(E_c)\{\psi(E_c)+\theta(E_c)-\phi(E_c)\}+\int_{E^*}^{E_c}I(E)\frac{\partial\phi}{\partial E}dE+\int_{E_c}^{\infty}I(E)\frac{\partial\psi}{\partial E}dE+\int_{E_c}^{\infty}I(E)\frac{\partial\theta}{\partial E}dE, \quad (8)$$

[27] R. E. Lapp, Phys. Rev. 69, 321 (1946). The value of the spin of the mesons is not important for this purpose on account of the low mean energy of the mesons.

* The critical energy ϵ_c is very often much larger than the cut-off energy and, in these cases, the asymptotic formula can be used when $\epsilon>\epsilon_c$.

where

$$\phi(E)=\frac{1}{k}\int_{\eta}^{\epsilon_m}\sigma z d\epsilon=\frac{1}{\beta}\left\{S_\infty\left(\log\frac{\epsilon_m}{\eta}+\frac{\eta}{\epsilon_m}-1\right)-\eta\left(\frac{2}{\epsilon_c}+\frac{1}{\epsilon_m}\right)\left[E_i\left(-\frac{2\eta}{\epsilon_c}\right)-E_i\left(-\frac{2\epsilon_m}{\epsilon_c}\right)\right]\exp\left[\frac{2\eta}{\epsilon_c}\right]\right.$$
$$\left.+\frac{\eta}{\epsilon_m}\exp\left[\frac{2(\eta-\epsilon_m)}{\epsilon_c}\right]-1+\frac{\eta}{\epsilon_m}\exp\left[\frac{2\eta}{\epsilon_c}\right]\int_{2\mu/\epsilon_c}^{2\epsilon_m/\epsilon_c}E_i(-x)dx-\frac{2\eta}{\epsilon_c}\exp\left[\frac{2\eta}{\epsilon_c}\right]\int_{2\mu/\epsilon_c}^{2\epsilon_m/\epsilon_c}E_i(-x)\frac{dx}{x}\right\}, \quad (9)$$

$$\theta(E)=\frac{1}{k}\int_{\epsilon_c}^{\epsilon_m}\sigma S_\infty\epsilon\frac{d\epsilon}{\beta}=S_\infty\left[\log\frac{E_m}{\epsilon_c}+\frac{\epsilon_c}{\epsilon_m}-1\right]\frac{1}{\beta}, \quad (9')$$

and the functions ψ_1 and ψ_2 are respectively:

$$\psi_1=S_\infty\log\frac{\epsilon_c}{\eta}+\frac{\eta}{\epsilon_c}\exp\left[\frac{2(\eta-\epsilon_c)}{\epsilon_c}\right]-1-\frac{2\eta}{\epsilon_c}\exp\left[\frac{2\eta}{\epsilon_c}\right]\left\{E_i\left(-\frac{2\eta}{\epsilon_c}\right)-E_i(-2)+\int_{2\eta/\epsilon_c}^{2}E_i(-x)\frac{dx}{x}, \quad (10)\right.$$

$$\psi_2=(\epsilon_c-\eta)S_\infty+\eta\exp\left[\frac{2\eta}{\epsilon_c}\right]\left\{E_i\left(-\frac{2\eta}{\epsilon_c}\right)-E_i(-2)-\int_{2\eta/\epsilon_c}^{2}E_i(-x)dx\right\}, \quad (10')$$

$$\psi(E)=\frac{1}{k}\int_{\eta}^{\epsilon_c}\sigma z d\epsilon=\frac{1}{\beta}\left\{\psi_1(\eta,\epsilon_c)-\frac{1}{\epsilon_m}\psi_2(\eta,\epsilon_c)\right\}. \quad (9'')$$

Remembering (4) we can write (8) in the following form

$$\frac{n_c}{k}=I(E^*)\phi(E^*)+I(E_c)\{\psi(E_c)+\theta(E_c)-\phi(E_c)\}-\int_{E^*}^{E_c}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE\int_{\eta}^{\epsilon_m}\frac{z}{\epsilon}d\epsilon$$
$$-\frac{\psi_2-S_\infty\epsilon_c}{\beta}\int_{E_c}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE-\frac{S_\infty}{\beta}\int_{E_c}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)\epsilon_m dE. \quad (11)$$

We can now consider three cases:

(1) $E^*>E_c$. In this case, if $\Theta=\vartheta/\beta$ and $\Psi=\psi/\beta$, we can write (11) as follows:

$$\frac{\beta}{k}n_c=I(E^*)[\Psi(E^*)-\Theta(E^*)]-S_\infty\int_{E^*}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)\epsilon_m dE-\{\psi_2-S_\infty\epsilon_c\}\int_{E^*}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE, \quad (11')$$

where the last two integrals are independent of the substance, that is of β and ϵ_c.

(2) $-E^*=E_c$. In this case (11′) becomes

$$\frac{\beta}{k}n_c=I(E_c)\Psi(E_c)-S_\infty\int_{E_c}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)(\epsilon_m-\epsilon_c)dE \quad (11'')$$

(3) $E^*<E_c$. This case is the most complicated one because it is not possible to isolate completely the dependence of $I(E)$ in the first integral of (11).

It is easily seen that in this case we obtain

$$\frac{\beta}{k}n_c=I(E^*)\phi(E^*)-S_\infty\int_{E^*}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)\epsilon_m dE-\eta\exp\left[\frac{2\eta}{\epsilon_c}\right]E_i\left(-\frac{2\eta}{\epsilon_c}\right)\int_{E^*}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE$$
$$-0.014\eta\exp\left[\frac{2\eta}{\epsilon_c}\right]\int_{E_c}^{\infty}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE+\eta\exp\left[\frac{2\eta}{\epsilon_c}\right]\int_{E^*}^{E_c}I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)$$
$$\times\left\{\left(1+\frac{2\epsilon_m}{\epsilon_c}\right)E_i\left(-\frac{2\epsilon_m}{\epsilon_c}\right)+\exp\left[\frac{-2\epsilon_m}{\epsilon_c}\right]\right\}dE, \quad (11''')$$

that obviously reduces to (11″) when $E^*=E_c$. The relations (11) present the following advantages over those generally used by other authors:

(1) a remarkable generality, because they include the fundamental parameters, explicitly, for comparison with experiments;

(2) the meson spectrum appears in integral form and can therefore be obtained directly with great accuracy from the ordinary absorption curves.

The validity of (11), however, fails for $\eta\rightarrow 0$, which is an interesting case, for instance, when the interpretation of Wilson chamber experiments is required. Yet it is obvious that we can find some relations similar to (11) in the way we followed for the preceding calculations. In this case, the calculations are even easier because Ferretti's "track" reduces to the electron track of Williams, that is $z=\epsilon/\beta$. For the electrons arising from coulombian collision (spin = 0) in any kind of material, we have, therefore, this limit value of

$$\frac{\beta}{k}n_c=\int_{E^*}^{\infty} I(E)\log\frac{\epsilon_m}{\eta}dE+\eta\int_{E^*}^{\infty} I(E)\frac{dE}{\epsilon_m}-\int_{E^*}^{\infty} I(E)dE, \tag{12}$$

from which we easily obtain

$$\frac{\beta}{k}n_c=\int_{E^*}^{\infty} I(E)\frac{\partial}{\partial E}(\epsilon_m)\frac{dE}{\epsilon_m}+\eta\int_{E^*}^{\infty} I(E)\frac{\partial}{\partial E}\left(\frac{1}{\epsilon_m}\right)dE, \tag{13}$$

where the lower limits of the integrals do not now correspond to the cut-off of the apparatus, but to the maximum value we can give to the energy of the electrons so as to maintain the validity of the formula used for the collision cross section. If we take $\epsilon=3$ mc², where m is the electron mass, we surely reach a sufficient approximation. We shall emphasize that William's concept of "electron track" must be understood likewise.

At sea level, we deduced from (13) a limiting value for collision electrons which amounts to ~ 12 percent of the meson components.

With a procedure similar to those just described, we have found a formula giving the number n_d of disintegration electrons in equilibrium with the meson component in atmosphere, which contains the parameters, and the integral meson spectrum. Starting from the following relation, the meaning of which is obvious,

$$n_d=\frac{\mu}{\tau c\rho}\left[\int_0^{p(\epsilon_c)}\frac{I(p)}{p^2}dp\int_\eta^{\epsilon(p)} z(\epsilon,\eta)d\epsilon+\int_{p(\epsilon_c)}^{\infty}\frac{I(p)}{p^2}dp\int_\eta^{\epsilon_c} z(\epsilon,\eta)d\epsilon+\int_{p(\epsilon_c)}^{\infty}\frac{I(p)}{p^2}dp\int_{\epsilon_c}^{\epsilon_p} z(\epsilon,\eta)d\epsilon\right], \tag{14}$$

where p indicates the momentum of the mesons, $\epsilon(p)=\frac{1}{2}[c\{p^2+(\mu c)^2\}^{\frac{1}{2}}+p]$, the maximum energy of a disintegration electron, and ρ and $I(p)$ indicate the density of the atmosphere and the differential spectrum at a height of about 130 g/cm² above the observation station (9), it is readily seen that the first integral gives a vanishing contribution and, therefore,

$$n_d=\frac{1}{\tau c\rho\mu c^2}\left\{\int_\eta^{\epsilon_c} z(\epsilon,\eta)d\epsilon+\frac{S_\infty}{2\beta}\left[\frac{(\mu c^2)^2}{4}-\epsilon_c^2\right]\right\}\int_{\lambda_c}^{\infty}\frac{I(\lambda)}{\lambda^2}d\lambda+\frac{\mu c^2 S_\infty}{4\tau c\beta\rho}\left\{\int_{\lambda_c}^{\infty}\frac{I(\lambda)}{\lambda}(\lambda^2+1)^{\frac{1}{2}}d\lambda+I(\lambda_c)\right\}, \tag{15}$$

where we have introduced the numerical variable $\lambda=p/\mu c$. We can substitute in (15) the differential spectrum with the integral spectrum $I(\lambda)$ by means of an integration by parts. We see then explicitly that n_d does not depend much upon the particular form of the spectrum, especially when high energies are concerned. For low energies instead there is an appreciable dependence and, for this reason, it may be convenient to substitute the differential spectrum with the integral one also for the disintegration electrons. In the calculations of the following paragraph, we proceeded in this way.

We also calculated the number of disintegration electrons in the case $\eta\rightarrow 0$, that is, substituting

Ferretti's track with the simplified one of Williams. However, we proceeded with a greater accuracy than in the case of collision electrons, finding the limit value of the integral,

$$n_d=\frac{\mu}{cp\tau}\int_0^\infty \frac{I(p)}{p^2}dp\int_\eta^{\epsilon_p}\frac{\epsilon}{\beta}d\epsilon,$$

for $\eta\to 0$.

We found, for this extrapolated value, $n_d=30.7$ for 100 mesons.

III. NUMERICAL VALUES

Using the preceding formulae, we performed the following calculations:

(1) Number of secondary electrons generated in lead and in equilibrium with the meson component, for $\eta=2$, 3, 4, 5 Mev and for different values of the lower limit of meson energy. In Fig. 1 the results relative to the cut-off energy $\eta=2$ Mev are plotted in a graph.

Table I gives the number of knock-on electrons generated in lead from the total meson spectrum. These values agree rather well with those of Tamm and Belenky.

(2) Number of secondary collision electrons arising from the atmosphere for different values of η at sea level and at 3.500 meters above sea level.

(3) Number of secondary disintegration electrons for different values of the cut-off energy, etc. In these calculations the paths are measured in g/cm^2, the energies in Mev and we have used the following notations

$k=2\pi NZr_0^2mc^2/A$, where Z is the atomic number, A the atomic weight, N Avogadro's number.

$\mu c^2=90$ Mev,

$\tau=2.3\times 10^{-6}$ sec.,

$\beta_{Pb}=1.18$ Mev/(g/cm²); $\epsilon_c=7$ Mev,

$\beta_{air}=2.27$ Mev/(g/cm²); $\epsilon_c=98$ Mev.

For the integral spectra we took the absorption curves at sea level and at 3500 meters obtained from many experiments performed by different authors and by us. These curves were reduced in an energy scale (expressed in μc^2) by means of Wick's nomogram.[28] For energies higher than 25 μc^2 we used the spectra obtained by the law $E^{-2.87}$, taking into account the disintegration processes, after having verified their agreement with the spectra of Blackett and Jones.[29]

TABLE I. Number of knock-on electrons generated in lead from the total meson spectrum.

η (in Mev)	2	3	4	5
η_ϵ (percent)	9	7	5.5	4.5

In Table II we report the calculated values, in percent, of the number of secondary electrons present in air at sea level and at 3500 meters.

4. Comparison of theoretical results with Wilson-Camber observations.

(*a*) Knock-on electrons in lead. In this case a direct comparison of our calculation with the experimental results does not give too significative results because, on account of the low energies of knock-on electrons, the scattering becomes quite important. Nevertheless, considering Wilson chamber experiments, the comparison is roughly possible, because, in this case, we observe also electrons with very large scattering angles.

Stuhlinger,[30] with $\eta\sim 0$ found about 10 percent of secondary electrons. Hazen, for an η estimated about 2 Mev, finds, at 3000 meters altitude,

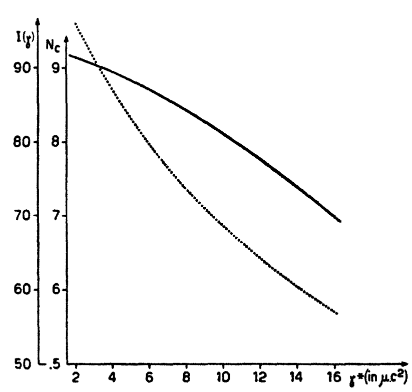

FIG. 1. Number of knock-on electrons generated in lead by mesons having an energy $\geqslant\gamma^*$ ($\eta=2$ Mev). The dotted line gives the corresponding integral meson's spectrum $I(\gamma^*)$; ($I(0)=100$).

[28] G. C. Wick, Nuovo Cimento X, 1, 302 (1943).

[29] It is well known that the greatest uncertainty is at low energies.

[30] E. Stuhlinger, Zeits. f. Physik 116, 281 (1940).

TABLE II. Calculated value in percent of the number of secondary electrons in air at sea level.

Sea level			3500 meters		
η	n_e	n_d	η	n_e	n_d
5	7.0	23.5	10	3.8	33.5
10	5.2	20.0	20	2.7	25.9
20	3.8	15.1	50	1.0	12.7
50	1.7	6.5			
~ 0	12.4	30.7			

about 7.4 knock-on electrons for 100 mesons. Considering the error due to the scattering, both the results agree with our calculations.

(*b*) Secondary electrons in the atmosphere. Hazen also measured very carefully, still at 3000 meters above sea level, the ratio between the number of electrons and mesons in the atmosphere. He discriminated between the two types of particles through their behavior in lead screens. For electrons, the energy was measured by counting the number of shower electrons at the maximum of the shower's development. As the energy of the descendant's at the maximum is ϵ_c (i.e., 7.0 Mev in lead), it is clear that the scattering effect will reduce the number of secondary electrons actually observed. Therefore, the energy values estimated in this way are surely too low.

Hazen discovered over 8678 particles, which were presumably mesons, 1090 showers having at their maximum four or more electrons and 605 showers with five or more electrons. According to the shower theory, this means that at 3000 meters, for every 100 mesons there are about 10 electrons with an energy of at least about 150 Mev.

But as it was pointed out by Belenky,[31] the simplifying assumptions usually used in the shower theory are inapplicable to heavy elements, like lead, in which the shower production is extremely intensive. Using a more accurate expression for pair production, but still disregarding the scattering,** one finds, for electrons which generate in lead, a shower with four or more electrons of the maximum, an energy of about 350 Mev. Now, according to Stanton,[15] whose calculations complete ours for high energies, the number of the secondary electrons of 350 Mev, generated by the mesons of 2.3^{-6} sec. lifetime, does not exceed the 5 percent.

The comparison shows that at 3000 m. above sea level *the electronic component is largely constituted by electrons which cannot be secondary to ordinary mesons*. This is especially true for high energies. This conclusion will be amphasized in the second part of this paper.

[31] S. Belenky, J. Phys. U.S.S.R. **8**, 305 (1944).
** Unfortunately we are not acquainted with a successive work published by Belenky.