



FIG. 1.

leads to excited N^{13} with the subsequent boiling off of a proton, while a similar exchange process cannot take place for the $C^{13}(n, 2n)C^{11}$ reaction. Secondly, there is the difference between the contributions of the knock-out process as a result of the difference in the $n-p$ and the $n-n$ cross sections, which favors the $p+C^{12}$ knock-out reaction. It will be noted that the parts of the reactions which go through excited C^{13} , while practically equal, are so small that they do not greatly affect either reaction.

Although the results of these calculations do not agree too closely with the experimental results, they are probably as good as are to be expected because of the crudity of the assumed model. The results do, however, seem to give a good qualitative picture of the contributing factors affecting the total reactions. Finally, it would seem that the assumption of 50 percent exchange gives better agreement.

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¹ E. M. McMillan and H. F. York, Phys. Rev. 73, 262 (1948).

Low Energy Pair Production

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THE appreciable effort required to get numerical results from the exact Bethe-Heitler differential cross section for pair production¹ has suggested its imitation by a simpler expression. We first consider the well-known high energy limit,² without screening, of the exact formula.

Errors in the high energy formula are positive and <1.5 percent (differential cross section) or <1 percent

TABLE I. Differential cross section at $x = \frac{1}{2}(\phi_0)$.

k	3	4	6	10	20	50
ϕ_0 exact	0.1116	0.4157	1.1419	2.3816	4.3163	6.953
high energy				2.367	4.342	6.961

(total cross section), for $k \geq 15$. (Energies throughout are in units of the electron rest energy.) If one fits the differential formula to exact values at the "midpoint," i.e., for equal electron and positron energies, these limits become +1 percent and + $\frac{1}{2}$ percent, respectively. Comparisons below, however, are to the unfitted formula. Errors for $k > 20$ are roughly $(20/k)^2$ times the above limits, for either type of approximation.

The high energy integral formula usually quoted³ is not as good as just stated; one lower power of k must be kept, making the total cross section

$$\Phi = (28/9) \ln 2k - (218/27) + (6.45/k). \quad (1)$$

Here, and throughout, cross sections are in units of $(Z^2/137)(e^2/mc^2)^2$.

As a formula suitable for $2 \leq k \leq 15$ and having reasonable overlap with the high energy formula at the upper limit, we propose the following:

$$\phi_x = \phi_0 z [1 + 0.135(\phi_0 - 0.52)z(1 - z^2)]. \quad (2)$$

$x = (E_+ - 1)/(k - 2)$ is the fraction of kinetic energy, $k - 2$, given to the positron; $z = 2[x(1 - x)]^{1/2}$; ϕ_x is the cross section per nucleus per unit x (cf. Heitler, p. 199); ϕ_0 is ϕ_x at $x = \frac{1}{2}$. The second term in the square bracket is to be dropped when it becomes negative (below $k = 4.2$).

ϕ_0 appears in (2) because no simpler, good k -dependence has been found. At $x = \frac{1}{2}$ the exact formula simplifies appreciably (if not spectacularly), giving

$$\phi_0 = (1 - \gamma) \left[\frac{1}{2}(4 - \gamma^2)(L - 1) - \gamma^2 \alpha(\alpha - 1) - \gamma^4 \alpha(L - \alpha) \right], \quad (3)$$

with $\gamma = (2/k)$, $L = [2/(1 - \gamma^2)] \ln(k/2)$, $\alpha = [1/(1 - \gamma^2)^{1/2}] \times \ln[(k/2) + ((k/2)^2 - 1)^{1/2}]$. Table I gives ϕ_0 for several values of k . (For comparison, several high energy values are also shown.)

Our formula (2) is, of course, exact at $x = \frac{1}{2}$; its performance at small x (and hence near $x = 1$) is shown in Table II. We may suppose on this basis that for $2 \leq k \leq 15$ (2) deviates from the exact formula by <2 percent, that deviations of more than 0.2 percent occur only for $x < 0.2$ or > 0.8 , and accordingly, that total cross sections from (2) are wrong by < $\frac{1}{2}$ percent (and <0.1 percent for $k \leq 10$).

We get from (2) for the total cross section

$$\begin{aligned} \Phi &= 0.776\phi_0 + 0.0180\phi_0^2 \quad (k > 4.2) \\ &= 0.785\phi_0 \quad (k < 4.2). \end{aligned} \quad (4)$$

Values calculated from (4) are presented in Table III,

TABLE II. Exact, Eq. (2), and high energy differential cross sections compared.

		x		
		0.2	0.1	0.05
$k = 4$	exact		0.2492	
	Eq. (2)		0.2494	
$k = 6$	exact	0.937		
	Eq. (2)	0.936		
$k = 10$	exact	2.046	1.565	1.122
	Eq. (2)	2.043	1.567	1.130
	high energy	2.006	1.497	1.018
$k = 20$	exact	3.968	3.194	2.324
	Eq. (2)	3.963	3.100	2.222
	high energy	4.009	3.238	2.343

TABLE III. Total cross section.

k	3	4	6	10	20
Eq. (4)	0.0876(0)	0.3265(± 1)	0.9096(± 8)	1.950(± 2)	3.69(-2)
Heitler	0.085	0.32	0.89	1.94	3.75
Eq. (1)				1.89	3.73(+2)

where we have also reproduced results given by Heitler,⁴ and at $k=10, 20$ numbers from the high energy formula (1). The figures in parentheses are expected errors in the last digit shown. One should note the considerable improvement in accuracy over Heitler's results, and the overlap of (4) and (1) at $k=20$. At $k=15$, the two are still closer.

The low energy form of (2), $\phi_z = \phi_0 z$, was found by expanding the exact formula in powers of the excess energy, $k-2$; the limiting k -dependence found for ϕ_0 is

$$\phi_0 = \left(\frac{1}{3}\right)(k-2)^3 \quad (k-2) \ll 1. \quad (5)$$

(5) is interesting in that the coefficients of three lower powers of $(k-2)$ vanish, so there results a higher order of contact of the total cross section curve at $k=2$ than might have been expected. The range of validity shown in (5) must be taken seriously: at $k=2.5$ (5) is already almost double the true value. As pointed out to me by Professor H. A. Bethe, multiplying (5) by $(2/k)^3$ gives the best possible power law approximation for large k ; the improved formula is 12 percent low at $k=3$, 50 percent low at $k=6$, and gives a total cross section at high energies ~ 2 instead of actual limits (if screening is included) of ~ 12 .

I wish to thank Professor Bethe for suggesting these calculations and for helpful discussions of many points while carrying them out.

¹ H. Bethe and W. Heitler, Proc. Roy. Soc. 146, 90 (1934), Eq. (21); or W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, 1936), p. 196, Eq. (8).

² W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, 1936), p. 197, Eq. (9).

³ See reference 2, p. 200, Eq. (15).

⁴ See reference 2, p. 200, Table V.

On the Structure of Grain Boundaries in Metals

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THERE is no method known at present of determining the structure of grain boundaries in metals. Considerations based on the atomistic viewpoint suggest that the grain boundary is a transition region, where the atom positions represent a compromise between the crystalline arrangements in the two adjoining grains. Extensive experiments on internal friction and related effects have demonstrated that this transition region behaves in a viscous manner in the sense that it cannot sustain permanently a shear stress and it has a coefficient of viscosity decreasing with an increase of temperature.^{1,2} This does not, however, tell us about the structure of this transition region, since any layer of disturbed crystallinity when considered as an entity may manifest a viscous behavior.

A study of the activation energy associated with the viscous slip along grain boundaries seems to contribute to our understanding of the structure of the grain boundary. It has been found that the activation energy associated with the grain boundary slip in alpha-brass is close to the value for the diffusion of zinc in alpha-brass of the same composition.³ Recently, in the study of the anelastic effects in alpha-iron,⁴ the value obtained for the activation energy of grain boundary slip agrees, within experimental error, with the value for the self-diffusion in alpha-iron reported by Birchenall and Mehl.⁵ Such an agreement has also been found in the case of aluminum. The activation energies for volume diffusion and grain boundary slip of these metals are summarized in the second and third columns of Table I.

If such an agreement in activation energies were found to be a general phenomenon in all metals, this would indicate that the grain boundary slip involves the same mechanism as does volume diffusion. This would also indicate that, at least as far as the local order is concerned, the structure of the transition region at the grain boundary cannot be markedly different from that of the interior of the grains. The grain boundary slip can thus be considered as creep on a microscopic scale, and this creep occurs at a lower temperature than creep in single crystals because of the disturbed crystallinity at the grain boundary. This viewpoint is strengthened by the findings that the activation energies for the creep in these metals (shown in the fourth column of Table I) are comparable with that of grain boundary slip.

The upper limit of the width of grain boundary has been suggested to be five atomic diameters on the basis that the forces between atoms in solids are of short-range type extending with appreciable intensity only over a few atomic distances.⁶ It seems that the boundary region within which creep may take place must have a width of at least a few atomic diameters. Actually the mere fact of short-ranged forces in solids does not necessarily imply that the transitional region can only be very thin. Reference can be made

TABLE I. Different types of activation energy of a few metals (in calories per mole).

Type of metal	Volume diffusion	Grain boundary slip	Creep
Alpha-brass	41,700 ^{a,*}	41,000 ^{b,*}	42,000 ^{c,**}
Alpha-iron	78,000 ^{d,***}	85,000 ^{e,***}	90,000 ^f
Aluminum	37,500 ^g	34,500 ^{h,†}	37,000 ^{i,†}

^{*} 29 percent Zn.

^{**} 40 percent Zn.

^{***} Westinghouse Puron.

[†] 99.99 percent aluminum.

^a A. E. Van Arkel, *Metallwirtschaft* 7, 656 (1928); see also R. F. Mehl, *Trans. A.I.M.E.* 122, 11 (1936).

^b See reference 3.

^c H. Tapsell, A. Johnson, and W. Clenshaw, *Eng. Research Report No. 13*, Dept. Sci. and Ind. Research, London (1932).

^d See reference 5.

^e See reference 4.

^f J. J. Kanter, *Trans. A.I.M.E.* 131, 385 (1938).

^g Estimated on the basis of binding energy and of melting temperature according to an empirical rule for face-centered cubic metals observed by W. A. Johnson, *Trans. A.I.M.E.* 143, 107 (1941); also private communication to the writer.

^h See reference 1.

ⁱ S. Dushman, L. W. Dunbar, and H. Huthstener, *J. App. Phys.* 15, 108 (1944); extrapolated to zero stress by C. Zener and J. H. Hollomon, *J. App. Phys.* 17, 69 (1946).