The decay of the excited nucleus is treated by the usual evaporation model.

On the above basis, the reaction can go in four ways:

(a) $p+C^{12}\rightarrow N^{*13}$; $N^{*13}\rightarrow C^{11}+p+n$,
(b) $p+C^{12}\rightarrow C^{*12}+p$; $C^{*12}\rightarrow C^{11}+n$,
(c) $p+C^{12}\rightarrow N^{*12}+n$; $N^{*12}\rightarrow C^{11}+p$,
(d) $p+C^{12}\rightarrow C^{11}+p+n$ (knock out).

The part played by each of these separate reactions to give the total reaction is shown in Fig. 1 for 50 percent exchange. Reaction (a) contributes chiefly in the 40-Mev region. This is the result (1) of the high probability in this region of the incident particle giving up all of its energy in a few collisions and thus of being captured; and (2) of the energy dependence of the $N^{13}\rightarrow C^{11}+p+n$ reaction. Above the 40-Mev region there is an increased probability of boiling off three or more particles, and below the 40-Mev region, only one particle. Reaction (b) takes place when the incident proton passes through the nucleus and makes few collisions before emerging with most of its original energy. Because of the much greater probability of excited C^{12} breaking down into three α-particles, this reaction contributes very little to the total reaction. It does contribute somewhat more for 50 percent exchange since a one collision non-exchange process can then contribute. Reaction (c) is made possible by a net exchange taking place when the incident proton passes through the nucleus, so that it emerges as a neutron. Excited N^{12} is formed as the intermediate product. For small excitation energies ($\sim$10 Mev)N^{12} will definitely boil off one proton, but this probability rapidly drops off for higher excitation energies because of competing processes coming into play. As a result of this only a single $p-n$ exchange collision is effective in giving reaction (c), a two (or more) collision process leaving too much excitation energy. This results in making the reaction practically directly proportional to the amount of exchange, which, accordingly, exerts a direct influence on the yield of the total reaction at high energies. Reaction (d) is the knock-out reaction. It is assumed that a knock-out reaction can occur only if the nucleon struck by the incident particle travels from the point of collision to the outside of the nucleus without colliding with other nucleons. Otherwise the struck nucleon excites the nucleus by its collisions with the other nucleons and so stays in. The knock-out reaction (d) is probable only if the incident proton makes just the one collision near the edge of the nucleus.

The total reaction for both 50 percent and 100 percent exchange is plotted on a range scale in Fig. 2, assuming 140-Mev protons incident on a carbon block. The attenuation of the incident beam is taken into account. The experimental curve (1) for the reaction is given by the dotted line. The ordinate has been adjusted to bring it into approximate agreement with curve I.

The calculated cross section for the reaction at 62 Mev is: 0.046 barn for 50 percent exchange and 0.062 barn for 100 percent exchange. The experimental value[4] is 0.073 $\pm$0.010 barn for 62-Mev incident protons.

The authors wish to express their great appreciation to

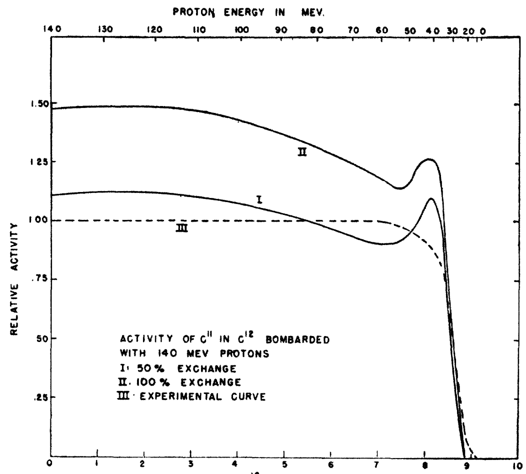

FIG. 2.

Professor Robert Serber for his continued assistance throughout the course of these calculations.

This paper is based on work performed under Contract No. W-7405-eng-48 with the Atomic Energy Commission in connection with the Radiation Laboratory, University of California, Berkeley, California.

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Excitation Function of the Reaction $C^{12}(n, 2n)C^{11}$ at High Energies

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December 22, 1947

THE excitation curve for the reaction $C^{12}(n, 2n)C^{11}$ has been calculated for energies up to 100 Mev. The calculations were done as described in the preceding letter for the similar reaction of C^{12} under proton bombardment.

The reaction can go in three ways:

(a) $n+C^{12}\rightarrow C^{*13}$; $C^{*13}\rightarrow C^{11}+2n$,
(b) $n+C^{12}\rightarrow C^{*12}+n$; $C^{*12}\rightarrow C^{11}+n$,
(c) $n+C^{12}\rightarrow C^{11}+2n$ (knock out).

The results of the calculations for 50 percent exchange are shown in Fig. 1. The calculated cross section for the reaction at 90 Mev is: 0.010 barn for 100 percent exchange and 0.012 barn for 50 percent exchange. The experimental value is 0.022$\pm$0.004 barn.[1]

The ratio of the cross section of the reaction $C^{12}(p, pn)C^{11}$ to the cross section of the above reaction at 90 Mev is: 5.8 for 100 percent exchange and 3.8 for 50 percent exchange. The experimental ratio is 3.3 at 90 Mev.[1]

This difference in cross sections between the two reactions is established by two factors. First, there is the part played by exchange in the $C^{12}(p, pn)C^{11}$ reaction which

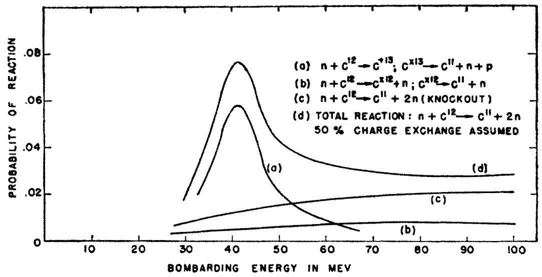

FIG. 1.

leads to excited N^{13} with the subsequent boiling off of a proton, while a similar exchange process cannot take place for the $C^{12}(n, 2n)C^{11}$ reaction. Secondly, there is the difference between the contributions of the knock-out process as a result of the difference in the $n-p$ and the $n-n$ cross sections, which favors the $p+C^{12}$ knock-out reaction. It will be noted that the parts of the reactions which go through excited C^{12}, while practically equal, are so small that they do not greatly affect either reaction.

Although the results of these calculations do not agree too closely with the experimental results, they are probably as good as are to be expected because of the crudity of the assumed model. The results do, however, seem to give a good qualitative picture of the contributing factors affecting the total reactions. Finally, it would seem that the assumption of 50 percent exchange gives better agreement.

The authors wish to express their great appreciation to Professor Robert Serber for his continued assistance throughout the course of these calculations.

This paper is based on work performed under Contract No. W-7405-eng-48 with the Atomic Energy Commission in connection with the Radiation Laboratory, University of California, Berkeley, California.

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Low Energy Pair Production

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THE appreciable effort required to get numerical results from the exact Bethe-Heitler differential cross section for pair production[1] has suggested its imitation by a simpler expression. We first consider the well-known high energy limit,[2] without screening, of the exact formula.

Errors in the high energy formula are positive and <1.5 percent (differential cross section) or <1 percent (total cross section), for $k \geq 15$. (Energies throughout are in units of the electron rest energy.) If one fits the differential formula to exact values at the "midpoint," i.e., for equal electron and positron energies, these limits become +1 percent and $+\frac{1}{2}$ percent, respectively. Comparisons below, however, are to the unfitted formula. Errors for $k>20$ are roughly $(20/k)^2$ times the above limits, for either type of approximation.

TABLE I. Differential cross section at $x=\frac{1}{2}(\phi_0)$.

k		3	4	6	10	20	50
ϕ_0	exact	0.1116	0.4157	1.1419	2.3816	4.3163	6.953
	high energy				2.367	4.342	6.961

The high energy integral formula usually quoted[3] is *not* as good as just stated; one lower power of k must be kept, making the total cross section

$$\Phi=(28/9)\ln 2k-(218/27)+(6.45/k). \tag{1}$$

Here, and throughout, cross sections are in units of $(Z^2/137)(e^2/mc^2)^2$.

As a formula suitable for $2\leq k\leq 15$ and having reasonable overlap with the high energy formula at the upper limit, we propose the following:

$$\phi_x=\phi_0 z[1+0.135(\phi_0-0.52)z(1-z^2)]. \tag{2}$$

$x=(E_+-1)/(k-2)$ is the fraction of *kinetic* energy, $k-2$, given to the positron; $z=2[x(1-x)]^{\frac{1}{2}}$; ϕ_x is the cross section per nucleus per unit x (cf. Heitler, p. 199); ϕ_0 is ϕ_x at $x=\frac{1}{2}$. *The second term in the square bracket is to be dropped when it becomes negative (below $k=4.2$).*

ϕ_0 appears in (2) because no simpler, good k-dependence has been found. At $x=\frac{1}{2}$ the exact formula simplifies appreciably (if not spectacularly), giving

$$\phi_0=(1-\gamma)[\tfrac{1}{3}(4-\gamma^2)(L-1)-\gamma^2\alpha(\alpha-1)-\gamma^4\alpha(L-\alpha)], \tag{3}$$

with $\gamma=(2/k)$, $L=[2/(1-\gamma^2)]\ln(k/2)$, $\alpha=[1/(1-\gamma^2)^{\frac{1}{2}}]\times\ln[(k/2)+((k/2)^2-1)^{\frac{1}{2}}]$. Table I gives ϕ_0 for several values of k. (For comparison, several high energy values are also shown.)

Our formula (2) is, of course, exact at $x=\frac{1}{2}$; its performance at small x (and hence near $x=1$) is shown in Table II. We may suppose on this basis that for $2\leq k\leq 15$ (2) deviates from the exact formula by <2 percent, that deviations of more than 0.2 percent occur only for $x<0.2$ or >0.8, and accordingly, that total cross sections from (2) are wrong by $<\frac{1}{2}$ percent (and <0.1 percent for $k\leq 10$).

We get from (2) for the total cross section

$$\begin{aligned}\Phi&=0.776_0\phi_0+0.0180\phi_0^2 \quad (k>4.2)\\ &=0.785_4\phi_0 \quad (k<4.2).\end{aligned} \tag{4}$$

Values calculated from (4) are presented in Table III,

TABLE II. Exact, Eq. (2), and high energy differential cross sections compared.

		x 0.2	0.1	0.05
$k=4$	exact		0.2492	
	Eq. (2)		0.2494	
$k=6$	exact	0.937		
	Eq. (2)	0.936		
$k=10$	exact	2.046	1.565	1.122
	Eq. (2)	2.043	1.567	1.130
	high energy	2.006	1.497	1.018
$k=20$	exact	3.968	3.194	2.324
	Eq. (2)	3.963	3.100	2.222
	high energy	4.009	3.238	2.343