hold together as a group and would be captured as such. This group would then be the di-neutron ($_0n^2$), but a di-neutron only under very peculiar physical conditions. It is being found in course of these (H^3, p) reactions only for a very short time, *viz.* the interval between the instant the H^3 begins to be polarized and the instant the capture takes place.

The di-neutron may be an extremely unstable particle and hence, the circumstances of the present investigations may be just the extreme physical conditions under which it may be observed.

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On Infinities in Generalized Meson-Field Theory

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The results of an extension of Podolsky's generalized electrodynamics, corresponding to Proca's extension of Maxwellian electrodynamics, are presented. In this extension the Lagrangian is permitted to depend upon the field coordinates themselves, which is a major step in going from electrodynamics to meson-field theory.

The static interaction and static self-energy, derived by exact classical and quantum methods, as well as the dynamic interaction and dynamic self-energy, obtained by a quantum-mechanical perturbation method, are given. The complete interaction and self-energy are free from singularities and infinities. This is in contrast with the results of ordinary relativistic meson-field theory. It thus appears that these defects may be removed from meson theory, just as in electrodynamics, by going to a generalized field theory in which the Lagrangian contains the second derivatives of the field coordinates.

1. INTRODUCTION

It is well known that the outstanding difficulty in the quantum theory of fields is the problem of infinite self-energies or inertia effects which arise in those theories which meet the requirement of relativistic invariance. In fact, most of the current research in field theory is directed at solving this problem.[1,2]

In a recent series of papers Podolsky,[3] Podolsky and Kikuchi,[4,5] Montgomery,[6] and Green[7] have developed a completely relativistic electrodynamics which appears to be free from the defect of infinite self-energies and which reduces to the Maxwell-Lorentz formulation for low energy phenomenon. In the present paper we extend this generalized field theory by allowing the Lagrangian to contain the field coordinates themselves. The corresponding extension of Maxwellian electrodynamics has been investigated by Proca[8] and others[9,10,11] and is considered to be among the more promising of meson-field theories.

In order to show the essential consequences of this generalization we shall consider only its simplest aspects. It is probable that modification of this theory by the inclusion of such concepts

[1] See G. Wentzel, Rev. Mod. Phys. **19**, 1 (1947).
[2] Also W. Pauli *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946).
[3] B. Podolsky, Phys. Rev. **62**, 68 (1942), to be referred to as GE I.
[4] B. Podolsky and C. Kikuchi, Phys. Rev. **65**, 228 (1944), to be referred to as GE II.
[5] B. Podolsky and C. Kikuchi, Phys. Rev. **67**, 184 (1945), to be referred to as GE III.
[6] D. J. Montgomery, Phys. Rev. **69**, 117 (1946), to be referred to as GE IV.
[7] A. E. S. Green, Phys. Rev. **72**, (1947), to be referred to as GE V.
This series of five papers will be called the GE S.
[8] A. Proca, J. Phys. Rad. (vii) **7**, 347 (1936).
[9] N. Kemmer, Proc. Roy. Soc. **A166**, 127 (1938).
[10] H. Frohlich, W. Heitler, and N. Kemmer, Proc. Roy. Soc. **A166**, 154 (1938).
[11] H. J. Bhabha, Proc. Roy. Soc. **A166**, 501 (1938).

as isotopic spin and symmetry requirements will be necessary to explain nuclear phenomenon. Nevertheless it seems worth while to present the consequences of a development which differs from Podolsky's generalized electrodynamics merely in two respects.

(1) The Lagrangian of the field depends quadratically upon the field coordinates which are components of a four vector $\varphi_\alpha = (\mathbf{A}, i\phi)$.

(2) The parameter coupling the field with a nucleon is a constant g instead of the charge e.

As the mathematical formalism used in this investigation parallels almost exactly that used in the GE S we shall present only the differences which arise as a result of the modification in assumptions. The reader should refer to the GE S for the steps of the calculations and for the notation, nomenclature, and references.

2. LAGRANGIAN, HAMILTONIAN, AND WAVE EQUATION

We choose as our basic Lagrangian of the field

$$L = -(1/2)[\mu^2\varphi_\alpha{}^2 + \varphi_{\alpha,\beta}{}^2 + a^2(\Box\varphi_\alpha)^2], \quad (2.1)$$

where μ is a constant having the dimension of reciprocal length and a is a constant with the dimension of length. The scalar counterpart of this equation has been suggested by Rayski.[12] The development leading to Eqs. Ge II (2.16) and Ge II (2.22) is altered but slightly by the additional term in the Lagrangian. The corresponding field equation is

$$a^2\Box\Box\varphi_\alpha - \Box\varphi_\alpha + \mu^2\varphi_\alpha = 0, \quad (2.2)$$

which is a fourth-order generalization of the Klein-Jordan equation. The Hamiltonian is

$$H = (1/2)[\mu^2\varphi_\alpha{}^2 + \varphi_{\alpha,\beta}{}^2 + a^2(\Box\varphi_\alpha)^2] - \varphi_{\alpha,4}(1 - a^2\Box)\varphi_{\alpha,4} - a^2\varphi_{\alpha,44}\Box\varphi_{\alpha,4}. \quad (2.3)$$

3. CLASSICAL TREATMENT OF THE STATIC CASE

For the static case the field equation becomes

$$a^2\nabla^2\nabla^2\phi - \nabla^2\phi + \mu^2\phi = 0. \quad (3.1)$$

If we consider the interaction of the field with a point nucleon at the origin then we must, as in GE I, place on the right the nucleon density

$$\rho = g\delta(\mathbf{R}). \quad (3.2)$$

A well behaved solution of (3.1) which gives rise to the delta-function is

$$\phi = (g/4\pi\gamma R)(e^{-\mu_0 R} - e^{-\mu_1 R}), \quad (3.3)$$

where

$$\mu_0 = [(1+2a\mu)^{\frac{1}{2}} - (1-2a\mu)^{\frac{1}{2}}]/2a, \quad (3.4)$$

$$\mu_1 = [(1+2a\mu)^{\frac{1}{2}} + (1-2a\mu)^{\frac{1}{2}}]/2a, \quad (3.5)$$

and

$$\gamma = a^2(\mu_1{}^2 - \mu_0{}^2) = (1 - 4a^2\mu^2)^{\frac{1}{2}}. \quad (3.6)$$

The volume integral of the Hamiltonian (henceforth called simply the Hamiltonian) is

$$\bar{H} = \int H dV = (1/2)\int [\mu^2\phi^2 + (\nabla\phi)^2 + a^2(\nabla^2\phi)^2] dV. \quad (3.7)$$

Using (3.3) we obtain for the energy of the field

$$\bar{H} = g^2/8\pi a(1+2a\mu)^{\frac{1}{2}}. \quad (3.8)$$

4. CLASSICAL SOLUTION OF THE GENERAL CASE

A general solution of the field equation (2.2), expressed in terms of Fourier integrals, is[13]

$$\varphi_\alpha(\mathbf{r}, t) = (1/2\pi)^{\frac{3}{2}} \int [\varphi_\alpha(\mathbf{k})\exp(-i\phi_0) + \varphi_\alpha{}^*(\mathbf{k})\exp i\phi_0 + \bar{\varphi}_\alpha(\mathbf{k})\exp(-i\phi_1) + \bar{\varphi}_\alpha{}^*(\mathbf{k})\exp i\phi_1] d\mathbf{k}, \quad (4.1)$$

where

$$\phi_0 = ck_0 t - \mathbf{k}\cdot\mathbf{r}, \quad \phi_1 = ck_1 t - \mathbf{k}\cdot\mathbf{r}, \quad (4.2)$$

$$k_0{}^2 = k^2 + \mu_0{}^2, \quad \text{and} \quad k_1{}^2 = k^2 + \mu_1{}^2. \quad (4.3)$$

From the standpoint of quantum mechanics, Eqs. (4.3) indicate that our field is associated with particles having the masses

$$m_0 = \hbar\mu_0/c \quad \text{and} \quad m_1 = \hbar\mu_1/c. \quad (4.4)$$

To avoid particles of complex mass, Eqs. (3.4) and (3.5) require that

$$2a\mu \leq 1. \quad (4.5)$$

By expressing all the quantities in (2.3) in terms of Fourier amplitudes and carrying out the

[12] G. Rayski, Phys. Rev. **70**, 573 (1946).

[13] In this paper bars are used in place of tildas.

integrations over all space we obtain, after a tedious calculation, the Hamiltonian

$$\bar{H}=\gamma\int[k_0^2(\varphi_\alpha\varphi_\alpha^*+\varphi_\alpha^*\varphi_\alpha)-k_1^2(\bar{\varphi}_\alpha\bar{\varphi}_\alpha^*+\bar{\varphi}_\alpha^*\bar{\varphi}_\alpha)]d\mathbf{k}. \quad (4.6)$$

5. QUANTIZATION AND AUXILIARY CONDITIONS

Accepting (4.6) as the quantum-mechanical Hamiltonian, we may obtain the commutation rules for amplitudes by requiring that

$$\dot{\varphi}_\alpha(\mathbf{r},t)=(i/\hbar)[\bar{H},\varphi_\alpha(\mathbf{r},t)]. \quad (5.1)$$

Expressing both sides in terms of Fourier integrals and equating coefficients of corresponding exponentials, gives

$$[\varphi_\alpha^*(\mathbf{k}),\varphi_\beta(\mathbf{k}')]=-\delta_{\alpha\beta}\delta(\mathbf{k}-\mathbf{k}')c\hbar/2k_0\gamma, \quad (5.2)$$

and

$$[\bar{\varphi}_\alpha^*(\mathbf{k}),\bar{\varphi}_\beta(\mathbf{k}')]=\delta_{\alpha\beta}\delta(\mathbf{k}-\mathbf{k}')c\hbar/2k_1\gamma. \quad (5.3)$$

To reduce the number of degrees of freedom of the field, we take as auxiliary conditions on our wave functional

$$C(\mathbf{k})\psi=[Q(\mathbf{k})-\phi(\mathbf{k})]\psi=0, \quad (5.4)$$

$$\bar{C}(\mathbf{k})\psi=[\bar{Q}(\mathbf{k})-\bar{\phi}(\mathbf{k})]\psi=0, \quad (5.5)$$

and their complex conjugates where

$$[Q^*(\mathbf{k}),Q(\mathbf{k}')]=-\delta(\mathbf{k}-\mathbf{k}')c\hbar/2k_0\gamma, \quad (5.6)$$

and

$$[\bar{Q}^*(\mathbf{k}),\bar{Q}(\mathbf{k}')]=\delta(\mathbf{k}-\mathbf{k}')c\hbar/2k_1\gamma. \quad (5.7)$$

This insures that $C(\mathbf{r},t)$ commutes with $C(\mathbf{r}',t')$ taken at another space-time point.

When nucleons are present in the field, the modified auxiliary conditions are

$$C(\mathbf{k})\psi=[Q(\mathbf{k})-\phi(\mathbf{k})+f(\mathbf{r}_st_s)/2k_0^2\gamma]\psi=0, \quad (5.8)$$

and

$$\bar{C}(\mathbf{k})\psi=[\bar{Q}(\mathbf{k})-\bar{\phi}(\mathbf{k})-\bar{f}(\mathbf{r}_st_s)/2k_1^2\gamma]\psi=0. \quad (5.9)$$

This modification is made so that $C(\mathbf{r},t)$ commutes with the operator $R_s-i\hbar\partial/\partial t_s$, where R_s, the sum of the relativistic Hamiltonian of the particle and the field-particle interaction, is given by

$$R_s=c\boldsymbol{\alpha}_s\cdot\mathbf{p}_s+m_sc^2\beta_s+g_s\phi(\mathbf{r}_st_s)-g_s\boldsymbol{\alpha}_s\cdot\mathbf{A}(\mathbf{r}_st_s). \quad (5.10)$$

6. THE INTERACTION ENERGY

The extension of the mathematical formalism of Fock, given in GE III, may now be used to eliminate the scalar potential and part of the longitudinal component of the vector potential from the wave functional. We thus obtain for the static interaction of a system of particles

$$V=\sum\nolimits'_{s,u}(g_sg_u/8\pi\gamma R)(e^{-\mu_0R}-e^{-\mu_1R}), \quad (6.1)$$

and the static self-energy of a single particle

$$V_s=g^2/8\pi a(1+2a\mu)^{\frac{1}{2}}. \quad (6.2)$$

These are in agreement with the results obtained classically. Applying the procedure given at the end of GE III and beginning of GE IV gives, as the wave equation for a system of particles,

$$\{\sum\nolimits_s[c\boldsymbol{\alpha}_s\cdot\mathbf{p}_s-g_s\boldsymbol{\alpha}_s\cdot\mathbf{D}(\mathbf{r}_st)+m_sc^2\beta_s+g_s^2/8\pi a(1+2a\mu)^{\frac{1}{2}}]+\sum\nolimits'_{s,u}(g_sg_u/8\pi\gamma R)\times(e^{-\mu_0R}-e^{-\mu_1R})\}\Omega=[i\hbar\partial/\partial t]\Omega, \quad (6.3)$$

where

$$\mathbf{D}(\mathbf{k})=\mathbf{A}(\mathbf{k})-\mathbf{k}Q(\mathbf{k})/k_0$$

and

$$\bar{\mathbf{D}}(\mathbf{k})=\bar{\mathbf{A}}(\mathbf{k})-\mathbf{k}\bar{Q}(\mathbf{k})/k_1 \quad (6.4)$$

To calculate the dynamic interaction we write the wave equation (6.3) as

$$[H-i\hbar\partial/\partial t]\Omega=[\sum\nolimits_s g_s\boldsymbol{\alpha}_s\cdot\mathbf{D}(\mathbf{r}_st)]\Omega, \quad (6.5)$$

and treat the right-hand side as a perturbation.

An analysis corresponding to that given in GE V gives as a first-order approximation the perturbation energy

$$U_d=-\sum\nolimits_{s,u}(g_sg_u/16\pi^3\gamma)\times\left[\int\boldsymbol{\alpha}_s\cdot\boldsymbol{\alpha}_u\exp i\mathbf{k}\cdot\mathbf{R}(1/k_0^2-1/k_1^2)d\mathbf{k}-\int(\boldsymbol{\alpha}_s\cdot\mathbf{k}/k)(\boldsymbol{\alpha}_u\cdot\mathbf{k}/k)\times\exp i\mathbf{k}\cdot\mathbf{R}(k^2/k_0^4-k^2/k_1^4)d\mathbf{k}\right]. \quad (6.6)$$

After integration this becomes

$$U_d=-\sum_{s,u}(g_sg_u/16\pi\gamma)\times\{\boldsymbol{\alpha}_s\cdot\boldsymbol{\alpha}_u(e^{-\mu_0R}-e^{-\mu_1R})/R+(\boldsymbol{\alpha}_s\cdot\mathbf{R}/R)(\boldsymbol{\alpha}_u\cdot\mathbf{R}/R)\times[e^{-\mu_0R}(\mu_0+1/R)-e^{-\mu_1R}(\mu_1+1/R)]\}. \quad (6.7)$$

The dynamic self-energy of a single particle is thus

$$U_{ds}=-\boldsymbol{\alpha}\cdot\boldsymbol{\alpha}g^2/16\pi a(1+2a\mu)^{\frac{1}{2}}=-3g^2/16\pi a(1+2a\mu)^{\frac{1}{2}}. \quad (6.8)$$

Considering the static terms, we have for the complete interaction

$$U=\sum{}'_{s,u}(g_sg_u/8\pi\gamma)\times\{(1-\boldsymbol{\alpha}_s\cdot\boldsymbol{\alpha}_u/2)(e^{-\mu_0R}-e^{-\mu_1R})/R-(1/2)(\boldsymbol{\alpha}_s\cdot\mathbf{R}/R)(\boldsymbol{\alpha}_u\cdot\mathbf{R}/R)\times[e^{-\mu_0R}(\mu_0+1/R)-e^{-\mu_1R}(\mu_1+1/R)]\}. \quad (6.9)$$

This result resembles in its spatial and spin dependence the interaction function used in several of the "mixed" theories of meson forces.[14] An important difference is the fact that the present interaction function contains no singularities. We also do not have as a factor the isotopic spin operator, as the isotopic spin formalism,[15] which seems to be necessary in order to predict the correct sign for the interaction energy, is not considered in this paper.

An equivalent expression for the interaction energy is

$$U=\sum{}'_{s,u}(g_sg_u/16\pi a^2\eta)\times[(1-\boldsymbol{\alpha}_s\cdot\boldsymbol{\alpha}_u/2)e^{-\eta R}\sinh\xi R/\xi R-(1/2)(\boldsymbol{\alpha}_s\cdot\mathbf{R}/R)(\boldsymbol{\alpha}_u\cdot\mathbf{R}/R)e^{-\eta R}(\sinh\xi R/\xi R+\eta R\sinh\xi R/\xi R-\cosh\xi R)], \quad (6.10)$$

where

$$\eta=(1+2a\mu)^{\frac{1}{2}}/2a \quad\text{and}\quad \xi=(1-2a\mu)^{\frac{1}{2}}/2a. \quad (6.11)$$

The complete self-energy is finite in contrast with the results of most relativistic theories. It is given by

$$U_s=-g^2/16\pi a(1+2a\mu)^{\frac{1}{2}} \quad (6.12)$$

Generalized electrodynamics is now a special case in which $\mu=0$ and $g=e$. With these constants Eq. (6.9) reduces to GE V (4.1). Letting $a=0$, this further reduces to Breit's formula, a result of ordinary quantum electrodynamics.

Placing $a=0$ in (6.9) gives

$$U=\sum{}'_{s,u}(g_sg_u/8\pi)\times[(1-\boldsymbol{\alpha}_s\cdot\boldsymbol{\alpha}_u/2)e^{-\mu R}/R-(1/2)(\boldsymbol{\alpha}_s\cdot\mathbf{R}/R)\times(\boldsymbol{\alpha}_u\cdot\mathbf{R}/R)e^{-\mu R}(\mu+1/R)]. \quad (6.13)$$

As might be expected, this result, apart from a matter of constants, is the same as that obtained from the ordinary Proca equations.

7. CONCLUSION

It appears from the present investigation that infinities and singularities may be removed from meson theory, just as in the case of electrodynamics, by allowing the Lagrangian to contain the second derivatives of the field coordinates.

The writer wishes to acknowledge his indebtedness to Dr. Boris Podolsky for his valuable suggestions and criticism.

[14] See references 1 and 2.

[15] B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936).