Letters to the Editor

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On the Hamiltonian for a Particle in an Electromagnetic Field

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December 19, 1947

IN connection with a study of the "infra-red catastrophe," it was found that a contact transformation may be used to eliminate certain low frequency divergences. The classical Lagrangian for a particle with charge e and mass m in a field specified by the potentials $\mathbf{A}$, ϕ is:

$$L_0=\frac{m}{2}|\dot{\mathbf{r}}|^2+\frac{e}{c}\mathbf{A}\cdot\dot{\mathbf{r}}-e\phi, \tag{1}$$

where dots indicate (total) differentiation with respect to time. This Lagrangian L_0 leads to the Hamiltonian usually given in the literature.

The equations of motion, however, are not changed when an arbitrary total time derivative is added to the Lagrangian. If we subtract $(d/dt)(\mathbf{A}\cdot\mathbf{r})$ from (1), we obtain:

$$\begin{aligned}L&=\frac{m}{2}|\dot{\mathbf{r}}|^2-e\phi-\frac{e}{c}\mathbf{r}\cdot\frac{d\mathbf{A}}{dt}\\&=\frac{m}{2}|\dot{\mathbf{r}}|^2-e\phi-\frac{e}{c}\left(\frac{\partial\mathbf{A}}{\partial t}+\dot{\mathbf{r}}(\nabla\mathbf{A})\right)\cdot\mathbf{r},\end{aligned} \tag{2}$$

where $\nabla\mathbf{A}$ is the dyadic $\partial A_i/\partial x_j$. It may be directly verified that (2) gives the Lorentz force equations, though in highly disguised form.

The Hamiltonian derived from (2) is found to be:

$$H=\frac{1}{2m}|\mathbf{p}+\frac{e}{c}(\nabla\mathbf{A})\cdot\mathbf{r}|^2+\frac{e}{c}\mathbf{r}\cdot\frac{\partial\mathbf{A}}{\partial t}+e\phi. \tag{3}$$

It has been pointed out to the author* that (3) may also be obtained from the usual Hamiltonian by carrying out the quantum-mechanical contact transformation

$$\psi=\exp\left(i\frac{e}{c}\mathbf{A}\cdot\mathbf{r}\right)\psi'.$$

For a plane wave, those terms linear in $\mathbf{A}$ which arise from the first part of (3) are smaller than the second term by essentially the factor v/c. Thus it is legitimate to treat the perturbing energy as $-e\mathbf{r}\cdot\mathbf{E}$. This procedure eliminates those low frequency divergences which formerly arose upon taking matrix elements of

$$\frac{e}{c}\mathbf{A}\cdot\mathbf{p}_{ik}\approx(v_{ik}/v)(e\mathbf{r}_{ik}\cdot\mathbf{E}).$$

Since the two Hamiltonians involved are completely equivalent, the original divergence must have arisen from neglecting terms which were actually not small, though it is very difficult to see just where this is done in the usual treatment.

The form (3) is interesting in that it roughly splits the energy into elementary "moment" interactions. Thus, the second term is the electric dipole energy, while the linear parts of the first term are easily seen to contain the magnetic dipole and electric quadrupole interactions.

* Private discussion with H. S. Snyder.
** Research carried out at the Brookhaven National Laboratory under the auspices of the Atomic Energy Commission.

Nuclear Quadrupole Interaction in the ICl Rotational Spectrum*

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December 22, 1947

RECENTLY reported[1] microwave absorption measurements with ICl vapor in the mm pressure range showed absorption peaks at 6980 and 6684 mc/sec. These were interpreted as arising from $J=0$ to $J=1$ rotational transitions in ICl^{35} and ICl^{37} and indicated rotational constants some 2 percent greater than the accepted values.[2] As a result of private communications with Dr. C. H. Townes this discrepancy has been removed and the observations reinterpreted. In the $J=3$ to $J=4$ spectrum of ICl Townes has found large nuclear quadrupole interactions, $eQ(\partial^2V/\partial Z^2)$, -2930 cm/sec. for the I nucleus and -82.5 cm/sec. for the Cl^{35} nucleus, and rotational constants in agreement with previous determinations.[2] Using these values, the $J=0$ to $J=1$ spectra have been recalculated assuming the Cl interaction as a perturbation of the I interaction. ICl^{35} and ICl^{37} each give three close-spaced triplets; the strongest of the ICl^{35} triplets and a weak ICl^{37} triplet fall near 6980 mc/sec., and a relatively strong ICl^{37} triplet is near 6684 mc/sec. The other three triplets predicted are outside of the frequency range of our measurements.

* Assisted by the Office of Naval Research under Contract N6ori-44, Task Order II.
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[1] R. T. Weidner, Phys. Rev. 72, 1268 (1947).
[2] W. E. Curtis and J. Patkowski, Phil. Trans. Roy. Soc. 232, 395 (1934).

Radioactive Carbon of High Specific Activity*

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December 22, 1947

AS we have described briefly elsewhere,[1] the first 225 millicuries of carbon 14 to be produced and distributed under the isotope program of the Atomic Energy Commission were made in an apparatus which circulated saturated ammonium nitrate solution through an irradiation tube in the Clinton nuclear reactor. The major portion of the carbon 14 was found to be converted to $C^{14}O_2$,