earlier paper[2] by the writer. The relevant part is quoted below:

"On this assumption substances which have no conductivity electrons and substances in which the interaction between the conductivity electrons and the lattice vibrations is not negligible will not show superconductivity. According to Debye (Ann. d. Physik 32, 85 (1938)) the effect of the lattice oscillations is very small for all temperatures below 1°K. From this we can conclude that substances which do not become superconducting by about 1°K will not become superconducting at all. This seems to agree with the observed fact that the lowest transition temperature recorded (Smith and Wilhelm, Rev. Mod. Phys. 7, 237 (1935)) is 0.6°K for cadmium."

[1] J. de Launay and R. L. Dolecek, Phys. Rev. 72, 141 (1947).
[2] Dixit, Curr. Sci. 9, 274 (1940).

Index of Refraction for Electrons in Crystalline Media

LORENZO STURKEY
The Dow Chemical Company, Midland, Michigan
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THE Schrödinger equation for electrons in a crystalline medium may be written[1]

$$\nabla^2\psi + (8\pi^2 me/h^2)(E + V_0 + \sum_{g \neq 0} V_g e^{2\pi i g \cdot r})\psi = 0,$$

where E is the accelerating potential of the electrons, V_0 the "inner potential," and the V_g are the coefficients in the Fourier expansion of the periodic potential in the crystal.

It is usually assumed that the index of refraction n for electrons is given by

$$n^2 = (E + V_0)/E.$$

However, if electrons are incident on a crystal at the Bragg angle for a set of planes (hkl), a correction must be made to the inner potential. This correction may be calculated by finding the value of ΔE that diagonalizes the energy matrix

$$\begin{pmatrix} \Delta E & -V_{hkl} \\ -V_{-h-k-l} & \Delta E \end{pmatrix},$$

i.e., $\Delta E = \pm V_{hkl}$. In this case

$$n^2 = (E + V_0 \pm V_{hkl})/E$$

and double refraction occurs, with $P = V_0 \pm V_{hkl}$ as effective inner potential.

In a previous communication,[2] refraction effects were discussed for electrons in crystals of cubic habit like MgO and CdO. The vector angular deviations from the Bragg reflection position due to refraction for such crystals may be expressed as:

$$\delta = (P/2E)(u \tan <r, n_1 + V \tan <r, n_2) = (P/2E)K, \quad (1)$$

where

$$u = (r \times n_1)/|r \times n_1|,$$
$$V = (r \times n_2)/|r \times n_2|,$$

r is unit vector in direction of electron ray,

n_1 and n_2 are face normals of crystal.

Since $P = V_0 \pm V_{hkl}$ when the Bragg conditions are satisfied, one should expect two values of δ for each incident direc-

TABLE I.

	{hkl}	meas. V_{hkl}/V_0	calc. V_{hkl}	calc. V_0
MgO	200	0.54	7.42 volts	13.7 volts
		0.59		12.6
		0.58		12.8
	220	0.415	5.37	12.9
		0.425		12.6
CdO	200	0.69	13.9	20.0
	220	0.55	9.8	18.0
		0.57		17.5
		0.52		19.0
		0.55		18.0

tion. On account of the cubic shape of the crystal, equal positive and negative values of δ will occur for the same setting of the crystal, making a total of four spots in a line:

$$\delta_1 = +K(V_0 + V_{hkl})/E,$$
$$\delta_2 = -K(V_0 + V_{hkl})/E,$$
$$\delta_3 = +K(V_0 - V_{hkl})/E,$$
$$\delta_4 = -K(V_0 - V_{hkl})/E.$$

Since V_{hkl} may be calculated with a much higher degree of accuracy than V_0, this phenomenon allows V_0 to be calculated from V_{hkl} without evaluating K or even knowing E.

$$V_0 = V_{hkl}(\delta_1 - \delta_2 + \delta_3 - \delta_4)/(\delta_1 - \delta_2 - \delta_3 + \delta_4). \quad (2)$$

In electron diffraction patterns from MgO and CdO smokes, groups of spots as described are frequently observed on diffraction rings having a V_{hkl} of the same order of magnitude as V_0—i.e., [200] and [220] rings. Since K in (1) often cannot be determined unambiguously, (2) provides a more reliable method for obtaining V_0. Some representative measurements are found in Table I.

In cases where the angle of incidence is such that the Bragg reflection conditions are satisfied for more than one set of planes, multiple values of the inner potential occur. Such cases often occur in electron diffraction since the wave-length is usually much smaller than the interplanar spacings. For an angle of incidence such that the (200), (022), and (222) reflections are obtained simultaneously, four values of P should occur, with eight values of δ:

$$\Delta E = \pm (V_1 + V_2) - V_3, \quad V_1 = V_{200},$$
$$\Delta E = \pm (V_1 - V_2) + V_3, \quad V_2 = V_{022},$$
$$V_3 = V_{222}.$$

When measurements are to be made on spots in diffraction patterns from crystals of various geometrical shapes, it should be remembered that each member of a group may be affected differently by extinction. Consequently, measurements using (1) on a single spot may give an error of $\pm V_{hkl}$, and such measurements should be made only when $V_{hkl} \ll V_0$, a condition not frequently valid for planes of low indices. Observations made on a large number of patterns indicate that spots resulting from the smallest values of the inner potential occur most frequently. Such considerations should explain anomalous inner potential values reported in the literature for planes of low indices.[3, 4]

[1] H. A. Bethe, Ann. d. Physik 87, 55 (1928).
[2] Sturkey and Frevel, Phys. Rev. 68, 56 (1945). An extended discussion was given in a paper by L. Sturkey at the 1946 meeting of ASX-RED at Silver Bay, Lake George, New York.
[3] Yamaguti, Proc. Phys. Math. Soc. Japan 16, 95 (1934).
[4] Thomson and Cochrane, Theory and Practice of Electron Diffraction (The Macmillan Company, New York, 1939), Chapter 10.