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The Limits of Beta-Stability

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The beta-stable nuclides are plotted in such a manner as to indicate the charge of maximum stability for odd mass numbers and the upper and lower limits of beta-stability for even mass and charge numbers. Curves representing these three quantities are drawn in following considerations based on the liquid-drop nuclear model, by which their positions can be determined fairly closely. The spacing between the curves is related to the parameters of the empirical mass equation of Bohr and Wheeler. The diagram is used to indicate the possibility of undetected natural radioactivities, and to suggest possible undiscovered or "missing" beta-stable nuclides.

I. INTRODUCTION: THE LIQUID-DROP NUCLEAR MODEL

MANY of the general properties of the atomic nucleus can be explained by the liquid-drop model proposed by Gamow[1] and developed by Heisenberg,[2] Wick,[3] Gamow,[4] Weizsäcker,[5] Bethe and Bacher,[6] Bohr and Kalckar,[7] Bohr and Wheeler,[8] and others. According to this model, which is applicable when the number of nucleons is sufficiently large ($\gtrsim 30$), nuclear binding energies and atomic masses are smoothly varying functions of the numbers of constituent protons and neutrons. By treating the various nuclear forces in a statistical manner and estimating their magnitudes empirically, it is possible to reproduce the general trends of binding energies and masses. Superposed on these general trends are more rapid variations caused by quantum-mechanical effects not well understood, and alternations with even and odd character of the proton and neutron numbers caused by the pairing of like elementary particles with opposite spin in otherwise identical quantum states.

As a result of the latter effect, all nuclides[9] may be divided into three general classes, those of odd A, those of even A and even Z, and those of even A and odd Z, where A is the mass number and Z the charge number. For each class, the nuclidic mass[10] as a function of A and Z lies on a surface having the general shape of a valley. The three surfaces have nearly the same shapes but are displaced vertically with respect to one another, the even-A, even-Z surface lying lowest, the even-A, odd-Z highest, and the odd-A midway between. The cross sections of the surfaces are approximately symmetrical in the region of the valley.

Beta-transitions occur when a nuclide occupies a higher position than that of a neighboring isobar.[11] In beta-transitions all nuclides of odd A

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[1] G. Gamow, Proc. Roy. Soc. (London) **A123**, 386 (1929), **A126**, 632 (1930); Physik Zeits. **30**, 717 (1929); *Constitution of Atomic Nuclei and Radioactivity* (Clarendon Press, Oxford, 1931), Chapter I.

[2] W. Heisenberg, Zeits. f. Physik **77**, 1 (1932), **78**, 156, **80**, 587 (1933); "Rapport du VIIième Congrès Solvay, Brussels, 1933," 289 (1934).

[3] G. C. Wick, Nuovo Cimento **11**, 227 (1934).

[4] G. Gamow, Zeits. f. Physik **89**, 592 (1934); *Structure of Atomic Nuclei and Nuclear Transformations* (Clarendon Press, Oxford, 1937), Chapter II.

[5] C. F. v. Weizsäcker, Zeits. f. Physik **96**, 431 (1935); *Die Atomkerne* (Akademische Verlagsgesellschaft, Leipzig, 1937), Chapter II.

[6] H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936); H. A. Bethe, Rev. Mod. Phys. **9**, 69 (1937).

[7] N. Bohr and F. Kalckar, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **14**, No. 10 (1937).

[8] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[9] The term *nuclide*, derived from *nucle-* (nuclear) and εἶδος (species), is used for "a species of atom characterized by the constitution of its nucleus, in particular by the numbers of protons and neutrons in its nucleus." T. P. Kohman, Am. J. Phys. **15**, 356 (1947).

[10] In this paper the masses considered are *atomic* rather then *nuclear* masses. Thus the energy release in a beta-transition (negatron emission, electron capture, or positron emission), being determined by the difference in mass between the initial and final *atom*, is given by the sum of the kinetic energies of the particles emitted, the rest-mass (if any) of the neutrino, and any quantum radiation accompanying the decay. The rest-masses of emitted or captured negative electrons are not included, but in the case of positron emission two election rest-masses must be included, since these are ultimately transformed to annihilation radiation.

[11] This is strictly true in all cases only if the rest-mass of the neutrino is zero, as is assumed for the present discussion. This is known to be at least very nearly true and is believed to be exactly true: E. Fermi, Zeits. f. Physik **88**, 161 (1934); H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936); E. J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943; W. E. Stephens, Rev. Mod. Phys. **19**, 19 (1947).

remain on the middle surface, while isobars of even A alternate between the upper and lower. The beta-stable nuclides are those having smaller masses than either of their neighboring isobars. For each odd A there is one and only one beta-stable isobar, for which Z is within one-half unit of Z_A, the coordinate of the bottom of the valley. For even A, in general, one, two or three isobars of even A are beta-stable and all others, including those of odd Z, are unstable. According to the statistical considerations on which the liquid-drop model is based, Z_A should be a smoothly varying function of A, and the region of stability for even-A, even-Z nuclides should be bounded by smoothly varying limits symmetrically disposed with respect to Z_A. This paper describes a method for evaluating these quantities by an examination of the distribution of the beta-stable nuclides.

II. DIAGRAMATIC REPRESENTATION OF THE BETA STABILITY LIMITS

Diagrams and plots of the stable nuclides too numerous to cite have been published in early demonstrations of regularities and irregularities in their pattern of distribution. Gamow[4] in 1934 used plots of the neutron-proton ratio *versus* A to locate Z_A as a function of A within narrow confines. Separate plots were made for even A and odd A, each plot revealing definite windings in the path of beta-stability. The accuracy was limited by the then incomplete knowledge of the beta-stable nuclides. Bohr and Wheeler[8] determined Z_A more accurately by plotting the beta-stable odd-A nuclides, each with a vertical line extending one-half charge unit above and below the stable charge number to indicate the confines of the curve. A similar plot was made by I. Joliot-Curie,[12] who used additional data from the beta-stable even-A nuclides and from beta-disintegration energies to assist in localizing the curve.

At the present time the isotopic constituents of the naturally occurring forms of the elements are known fairly completely.[13] By making use of the previously mentioned considerations based on the liquid-drop nuclear model, it is possible to determine the limits of beta-stability from a single plot in which both even-A and odd-A nuclides are represented.

In Fig. 1 the abscissa is A and the ordinate $U=Z-0.4\,A$. The latter function of the charge has been chosen instead of the charge itself to eliminate the main part of the variation of Z with A, represented approximately by the term $0.4\,A$. This allows all of the points of interest to be contained on one horizontal plot with large vertical separations. Thus, deviations from the general trend can be clearly seen, and a given change of A or of Z is represented similarly in any part of the chart. Values of Z are indicated at the top and by the sloping lines.

Beta-stable odd-A nuclides are plotted as solid circles and even-A ones as open circles. From each solid circle there has been drawn, following the method of Bohr and Wheeler,[8] a vertical line extending upward and downward by one-half unit. The curve representing Z_A as a function of A must cross each of these lines. From the open circle representing the lowest beta-stable charge for each even A there has been drawn a line extending downward by two units. The curve representing the lower limit for beta-stability must cross each of these lines. From the open circle representing the highest beta-stable charge for even A there has been drawn a line extending upward by two units. The curve representing the upper limit for beta-stability must cross each of these lines. Where an even A has only one beta-stable isobar, both lines originate from the same point.

The three curves are drawn in such a manner that each stays within its prescribed limits at all points, fluctuations are minimized, and the center line is vertically midway between the other two insofar as is possible. By this procedure the localization of each curve is influenced by the limits of the other two, and together their positions can be determined more closely than when each is considered separately. Only in the region $150<A<190$ is it impossible to keep the vertical spacings equal.

This procedure can be followed only up to the region of the heavy natural radioelements. Beyond lead, many of the beta-stable nuclides

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[12] I. Joliot-Curie, J. de phys. et rad. [8] 6, 209 (1945).

[13] F. W. Aston, *Mass Spectra and Isotopes* (Edward Arnold and Company, London, 1942), second edition; J. Mattauch and S. Flügge, *Nuclear Physics Tables* (Verlag Julius Springer, Berlin, 1942—trans. Interscience Publishers, Inc., New York, 1946); G. T. Seaborg, Rev. Mod. Phys. 16, 1 (1944).

are missing because of alpha-instability, and the beta-stability or -lability of the short-lived alpha-emitters cannot be determined in all cases. Such of the heavy natural radionuclides as are fairly certainly beta-stable, and those of the heavy synthetic radionuclides for which information has been published,[14] are included in the plot, but the positions of the lines for $A > 200$ are only rough approximations. The treatment for small values of A is described below.

Viewing Fig. 1 as a whole, it is seen that the beta-stable even-A even-Z nuclides occupy a definite path bounded by the upper and lower curves. In the center of this path is a narrower path occupied by the beta-stable odd-A nuclides. The boundaries of the latter path are two lines, respectively, one-half charge unit above and below the center line; these have been omitted from the diagram to avoid crowding. The curve of maximum stability shows definite wave-like variations superposed on the long-range variation, as was shown by Gamow.[4] The Z_A values of Bohr and Wheeler[8] follow those of Fig. 1 closely, especially in the region $97 \leqslant A \leqslant 145$ where their curve is shown in detail. The plot of Joliot-Curie,[12] when replotted on the coordinate system of Fig. 1, shows numerous irregular fluctuations about the position of the present curve.[15] It will be recalled that the latter was intentionally made as smooth as was consistent with the data used. Undoubtedly short-range fluctuations do occur, and the curves of Fig. 1 are, therefore, to be considered as mean positions only.

III. THE BETA-STABILITY LIMITS IN TERMS OF THE BOHR-WHEELER EMPIRICAL MASS EQUATION

Bohr and Wheeler[8] have shown that the semi-theoretical semi-empirical atomic-mass equation given by Bethe and Bacher[16] can be transposed into the following more convenient form:

$$M_{A,Z} = A(1+f_A) + \tfrac{1}{2}B_A(Z-Z_A)^2 \pm \tfrac{1}{2}\delta_A \text{ or } 0.$$

Here $M_{A,Z}$ is the mass of a nuclide of mass number A and charge number Z, f_A is the packing fraction of the hypothetical odd-A isobar of greatest stability having charge Z_A (not necessarily integral), B_A represents the steepness of the walls of the valley of the mass surface, and δ_A is the apparent energy of pairing of nucleonic spins, the last term[17] being taken negative for even A and even Z, positive for even A and odd Z, and zero for odd A.

The parameters f_A, B_A, and Z_A can be expressed in terms of the more fundamental parameters representing the properties of the nuclear fluid. When the latter parameters are evaluated empirically, the Bohr-Wheeler equation correctly expresses the observed general trends of atomic masses. However, the short-period variations are not reproduced. Consequently, Bohr and Wheeler proposed to use their equation with its parameters determined empirically as functions of A, independently of the theory. They evaluated f_A from mass-spectrographic data and estimated Z_A from the values of Z for stable odd-A nuclides. In the absence of sufficient data on radioactive disintegration energies, B_A was calculated from Z_A according to a theoretical relationship. δ_A was then estimated from the limits of stable even-A nuclides and from some radioactive disintegration energies. Subsequently, Joliot-Curie[12] undertook a completely empirical evaluation of Z_A, B_A, and δ_A as functions of A. With the parameters so determined, the Bohr-Wheeler empirical equation, though divested of much of its theoretical significance, is quite useful in practical correlations of atomic masses and predictions of disintegration energies.

The beta-stability limits are readily expressed in terms of the parameters of the Bohr-Wheeler

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[14] G. T. Seaborg, E. M. McMillan, J. W. Kennedy, and A. C. Wahl, Phys. Rev. **69**, 366 (1946); G. T. Seaborg, A. C. Wahl, and J. W. Kennedy, Phys. Rev. **69**, 367 (1946); G. T. Seaborg, Chem. Eng. News **23**, 2190 (1945), **25**, 358 (1947), Science **104**, 379 (1946); G. T. Seaborg, J. W. Gofman, and R. W. Stoughton, Phys. Rev. **71**, 378 (1947); G. T. Seaborg, and others, reported by J. M. Cork, *Radioactivity and Nuclear Physics* (Edwards Brothers, Inc., Ann Arbor, 1946 and D. Van Nostrand Company, Inc., New York, 1947), Chapter XII; F. Hagemann, L. I. Katzin, M. H. Studier, A. Ghiorso, and G. T. Seaborg, Phys. Rev. **72**, 252 (1947); A. C. English, T. E. Cranshaw, P. Demers, J. A. Harvey, E. P. Hincks, J. V. Jelley, and A. N. May, Phys. Rev. **72**, 253 (1947).

[15] Joliot-Curie defined Z_A in terms of nuclear rather than atomic masses. Thus her curve of maximum stability is not centrally disposed with respect to the distribution of the stable nuclides, but lies above the present curve on the average by the electron rest-mass (0.51 Mev) divided by B_A (see next section), or a few tenths of a charge unit.

[16] Equation (182) of reference 6, based on the liquid-drop nuclear model, and modified from Weizsäcker (reference 5).

[17] Added by Bohr and Wheeler following Heisenberg (reference 2).

equation. The equation yields for the energy, E_-, released by the emission of a negative beta-particle by an even-A even-Z nuclide of charge Z

$$E_- = B_A(Z_A - Z - \tfrac{1}{2}) - \delta_A.$$

For a hypothetical nuclide just at the lower limit of stability, whose charge is Z_A',

$$E_- = B_A(Z_A - Z_A' - \tfrac{1}{2}) - \delta_A = 0.$$

Consequently,

$$Z_A' = Z_A - \frac{\delta_A}{B_A} - \tfrac{1}{2} = Z_A - S_A,$$

where

$$S_A = \frac{\delta_A}{B_A} + \tfrac{1}{2}.$$

Similarly, the upper limit, Z_A'', is given by

$$Z_A'' = Z_A + S_A.$$

S_A is thus the vertical spacing between the curves of Fig. 1. The lower and upper stability limits for odd-A nuclides are $Z_A - \frac{1}{2}$ and $Z_A + \frac{1}{2}$, respectively.

Although the statistical treatment of the nucleus is not valid for small numbers of nucleons ($A \lesssim 30$), it is of interest that a slight modification allows the scheme of the diagram to be extended all the way down to $A = 1$. Throughout the region of validity of the statistical model, $B_A < 2\delta_A$, $S_A > 1$, and all even-A odd-Z nuclides are unstable. But if $B_A > 2\delta_A$, as is the case for $A \lesssim 30$, then $S_A < 1$; and if for a given A no even Z lies between the limits $Z_A \pm S_A$, the isobar of odd Z which does will be beta-stable. This may be considered to be the case for mass numbers 2, 6, 10, and 14; therefore, from the points in Fig. 1 representing ${}_1\mathrm{H}^2$, ${}_3\mathrm{Li}^6$, ${}_5\mathrm{B}^{10}$, and ${}_7\mathrm{N}^{14}$, a line is drawn extending one unit upward and one unit downward. The limiting even-A curves must cross these lines, which fix their positions fairly closely in this region. The curves as drawn correspond to such known facts as the small disintegration energies of H^3 and C^{14} and the approximate equality in mass of the ground states of He^5 and Li^5.

Figure 1 was constructed as a part of a plan

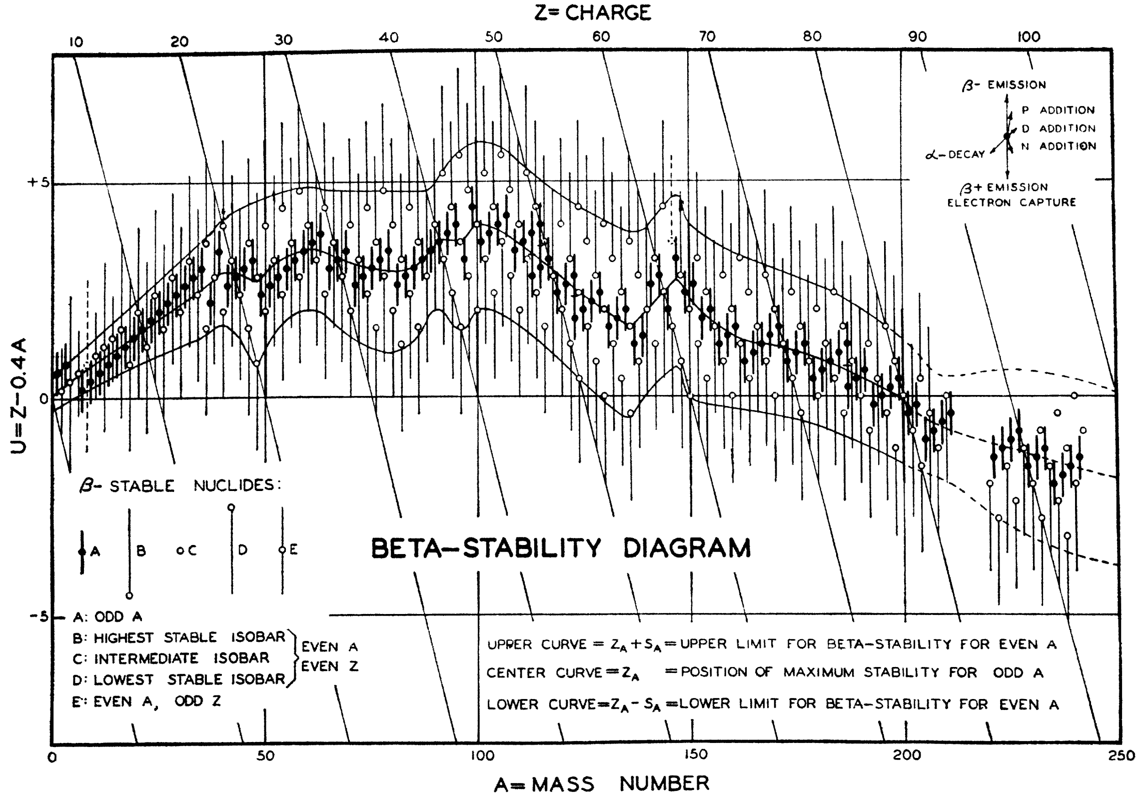

FIG. 1. The limits of beta-stability as determined from the beta-stable nuclides.

to determine the validity of the parabolic mass expression of Bohr and Wheeler and to evaluate the best empirical values of the parameters. However, this relation was not involved in the construction of the diagram. The curves can, of course, be placed more accurately by making use of disintegration energies, but for the present purpose only a few beta-disintegration energies were used as qualitative guides. It is evident that a knowledge of the beta-stable nuclides is by itself sufficient for a fairly close determination of the charge of maximum stability and the limits of beta-stability.

IV. APPLICATIONS OF THE BETA-STABILITY DIAGRAM

Besides clearly indicating the trends of nuclear stability, the beta-stability diagram has additional value, as may be demonstrated by several examples of its usefulness.

Relative Stability of Neighboring Odd-*A* Isobars

Four odd mass numbers, 113, 115, 123, and 187, have two neighboring apparently beta-stable isobars. In each case one should have a greater mass than the other and transform to it by a beta-transition.[11] From the position of the Z_A curve it can be predicted that Sn^{115} is probably unstable with respect to electron capture[18] to form In^{115}, and Sb^{123} is probably unstable with respect to negatron emission to form Te^{123}. For the other two pairs, Cd^{113}-In^{113} and Re^{187}-Os^{187}, no choice can be made. The reported[19] activity of Os^{187} has recently been questioned, with indications that Re^{187} instead is radioactive.[20] If this is the case, the point representing Re^{187} must be removed and the stability curves raised slightly in this region.

TABLE I.

A		B		C	
−	+	−*	+*	−*	+*
P^{33}	Mn^{53}	Si^{32}	Ti^{44}	Ca^{48}	Ni^{58}
A^{39}	Sb^{119}	Ti^{52}	Ge^{68}	Se^{82}	Kr^{78}
Ni^{63}	La^{137}	Cr^{56}	Zr^{88}	Kr^{86}	Mo^{92}
Se^{79}	61^{145}	Fe^{60}	Pd^{100}	Zr^{96}	Ru^{96}
Pd^{107}	Tb^{157}	Ni^{66}	Te^{118}	Mo^{100}	Cd^{106}
I^{129}	Ho^{163}	Cd^{118}	Nd^{140}	Sn^{124}	Sn^{112}
Cs^{135}	Ta^{179}	Dy^{166}	Gd^{150}	Te^{130}	Xe^{124}
Tm^{171}	Pt^{193}	Hf^{182}	Dy^{156}	Xe^{136}	Ba^{130}
	Pb^{205}		Hf^{172}	Nd^{150}	Ce^{136}
			W^{178}	Yb^{176}	Sm^{144}
			Pt^{190}	Hg^{204}	Er^{162}
			Pb^{202}		Yb^{168}
					Os^{184}
					Hg^{196}

A: unknown odd-*A* nuclides of low disintegration energy, some of which may be sufficiently long-lived to exist as natural radioactivities.
B: unknown even-*A*, even-*Z* nuclides on or near the limits of beta-stability, some of which may be beta-stable but undiscovered or missing in nature, and some of which may be sufficiently long-lived to exist as natural radioactivities.
C: apparently stable even-*A*, even-*Z* nuclides on or near the limits of beta-stability, some of which may be beta-labile with long lifetimes.
−: nuclides which if beta-labile would emit negative beta-particles.
+: nuclides which if beta-labile would capture electrons.
*: nuclides which if beta-active would be accompanied by daughters of large disintegration energies and relatively short lifetimes.

Odd-*A* Nuclides of Low Beta-Disintegration Energy

By inspection of the diagram it is possible to locate positions corresponding to odd-*A* nuclides of low beta-disintegration energy. Those whose radioactive properties are not known are listed in Table I-A. These should all be long-lived, and in some cases the long lifetimes have been the reason for their non-discovery.

If it is assumed that the universe is 3×10^9 years old and that at the time of its origin all stable and nearly stable isotopes of an element were present in comparable amounts, the half-life of a nuclide would, in general, have to be $\gtrsim 1.5\times10^8$ years in order for detectable amounts to remain, $\lesssim 4\times10^8$ years in order not to have been discovered mass spectrometrically, and $\lesssim 2\times10^8$ years not to have been discovered radiometrically unless the radiations are extremely soft. There is thus a small but finite chance that traces of one or more of the nuclides of Table I-A may remain as natural radioactivities.[21]

Even-*A*, Even-*Z* Nuclides Possibly Beta-Stable and "Missing," or of Low Disintegration Energy

The position of Sm^{146} is well within the region of beta-stability, and its absence (at least in

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[18] A similar prediction has been made by E. D. Eastman on the basis of atomic-mass calculations, Phys. Rev. **46**, 1 (1934), and of the common occurrence of indium in tin ores, Phys. Rev. **52**, 1226 (1937).

[19] E. T. Lougher and S. Rowlands, Nature **153**, 374 (1944).

[20] W. F. Libby and S. N. Naldrett, private communication.

[21] S. Katcoff, Phys. Rev. **71**, 826 (1947) has presented evidence for the possible presence of I^{129} to the extent of about one part per million in natural iodine, suggesting a minimum half-life of 10^8 years.

amounts sufficient to be detected mass spectrometrically[22]) is undoubtedly to be attributed to alpha-instability.[23] Positions of even A and Z which lie on or just outside of the stability limits as drawn are listed in Table I-B.[24] Some of these may actually be beta-stable and either completely "missing" in nature or present in such low abundance as to be still undiscovered. Considerations based on the liquid-drop nuclear model indicate that the lightest beta-stable isotopes of some of the heavy even elements may be alpha-unstable with cosmologically short lifetimes,[25] and some of the nuclides in the latter part of list I-B (+) may be missing in nature for that reason. Pt^{190} in particular seems to lie within the stability limits, and if the low energy electrons[26] emitted by Ir^{190} are actually negative beta-particles, Pt^{190} must owe its absence to alpha-decay.

On the other hand, any nuclide in Table I-B which is beta-labile will have a low disintegration energy and will be long-lived. If any should be present in nature it would be accompanied by a daughter of large disintegration energy and short lifetime. By chemical extraction of the active daughter the sensitivity of detection could be enormously enhanced, so that the half-life might be as low as $\sim 1.0 \times 10^8$ years, on the above assumption regarding the formation of the elements, and still permit detection in the presence of stable isotopes.

Naturally Occurring Even-*A*, Even-*Z* Nuclides Which May Possibly Be Beta-Labile

Apparently stable nuclides which lie on or just within the stability limits as drawn are listed in Table I-C. Some of these may actually be unstable but with lifetimes sufficiently long to have prevented appreciable decay since their formation. The absence of detectable radiations would indicate minimum lifetimes of about 10^{13} years for most of the nuclides listed. Any which are beta-active, however, would be accompanied by short-lived daughters with energetic radiations, so that by use of the active-daughter extraction technique it should be possible to detect activities with half-lives as great as $\sim 10^{19}$ years.

Ca^{48} is of particular interest, since it lies well below the main region of stability. Its natural occurrence, at least as a stable nuclide, has been doubted,[12, 27] but the mass-spectrometric[28] and nuclear-reaction[29, 30] evidence for its existence in natural calcium appears conclusive. It has been reported[30] that Sc^{48} apparently undergoes electron capture, in which case Ca^{48} would be stable. If, however, Ca^{48} is radioactive, the accumulation of Ti^{48} would make it a valuable geological and paleontological time index.

A natural radioactivity of neodymium has been reported[31] though never confirmed. If the activity actually belongs to this element, the isotope of mass 150 is probably responsible, since it seems to be outside the stability region, with Nd^{140} as an alternate possibility. If Nd^{150} is unstable, its daughter would be an active isotope of element 61, which element has not yet been found naturally in either stable or radioactive forms.[32] Similarly, if Sm^{144} undergoes electron capture, an element 61 activity would result. If Mo^{100} or Ru^{96} is unstable, an active isotope of technecium, the missing element 43, would be present naturally.

A search is being made for natural radioactivities such as those indicated as possibilities in this paper.

ACKNOWLEDGMENT

I wish to thank Mr. Anthony Van Zervic for his careful reproduction of the drawing for Fig. 1.

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[22] F. W. Aston, Proc. Roy. Soc. (London) A146, 46 (1934).

[23] This has been suggested previously, for example, by Flügge (reference 12), p. 103.

[24] Several nuclides which an inspection of the chart would place in this group are known to be radioactive, being produced in fission (for example, Sr^{90}) or by alpha-decay (for example, Pb^{210}), and hence are not listed.

[25] T. P. Kohman, paper in preparation.

[26] L. J. Goodman and M. L. Pool, Phys. Rev. **71**, 288 (1947).

[27] M. N. Saha and A. K. Saha, Trans. Nat. Inst. Sci. India 2, 193 (1946); Nature **158**, 6 (1946).

[28] A. O. Nier, Phys. Rev. **53**, 282 (1938); C. W. Sherwin and A. J. Dempster, Phys. Rev. **59**, 114 (1941).

[29] H. Walke, F. C. Thompson, and J. Holt, Phys. Rev. **57**, 177 (1940); H. Walke, Phys. Rev. **57**, 163 (1940).

[30] C. T. Hibdon and M. L. Pool, Phys. Rev. **67**, 313 (1945).

[31] W. F. Libby, Phys. Rev. **45**, 845 (1934); **46**, 196 (1934).

[32] D. M. Yost, H. Russell, Jr., and C. S. Garner, *The Rare-Earth Elements and their Compounds* (John Wiley and Sons, Inc., New York, 1947), Chapter 4. See, however, B. S. Hopkins, *Chapters in the Chemistry of the Less Familiar Elements* (Stipes Publishing Co., Champaign, 1938), Chapter 6.