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On the Theory of Ferromagnetic Resonance Absorption

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The theory of ferromagnetic resonance absorption previously developed is extended to include the effect of the shape of the specimen and, in the case of a single crystal, the effect of crystal orientation. The resonance condition may be written $\omega_0=\gamma H_{eff}$, where H_{eff} is equal to $(BH)^{\frac{1}{2}}$ for a plane surface, $H+2\pi M$ for a long circular cylinder, and H for a sphere; the latter two values apply only to the situation in which the eddy current skin depth is large in comparison with the radius of the specimen. In the case of an uniaxial crystal with the axis parallel to the static magnetic field, the value of H to be used in the resonance conditions is increased by $2K/M$, where K is the anisotropy constant. The case of a cubic crystal is also considered. A detailed discussion of macroscopic eddy current effects is given, and it is shown that the usual eddy current losses do not introduce damping terms into the expression for the permeability, when properly interpreted.

I. INTRODUCTION

FERROMAGNETIC resonance absorption is the analog of paramagnetic and nuclear resonance absorption. The ferromagnetic effect was found originally by Griffiths,[1] and further confirmation has been reported by Yager and Bozorth.[2] A theory of the resonance effect has been proposed by the present author.[3]

In a typical experimental arrangement the ferromagnetic specimen is in the form of a thin sheet or foil, which is employed as one wall of a rectangular cavity terminating a wave guide fed by a microwave generator. The ferromagnetic side of the cavity is chosen so that the magnetic vector of the microwave field is constant in direction in the plane of the wall. A static magnetic field is applied (by means of an electromagnet) also in the plane of the wall but perpendicular to the microwave magnetic field. It is found experimentally that the energy loss in the cavity goes through a maximum as the strength of the static magnetic field is increased. For example, Yager and Bozorth found a sharp resonance peak in a Supermalloy specimen for a field strength near 5000 oersted when using a frequency near 24,000 Mc/sec. The measured Q of their cavity dropped sharply from a value of several thousand on either side of resonance to a value of several hundred at resonance.

The principal result of the theory[3] is that the resonance condition for a plane surface should be given by

$$\omega_0=\gamma(BH)^{\frac{1}{2}}, \tag{1}$$

instead of the Larmor condition $\omega_0=\gamma H$; here ω_0 is the frequency at resonance; γ is the magnetomechanical ratio for an electron spin; H is the strength of the static magnetic field, and B is the magnetic induction in the specimen.

The considerations set forth below elucidate and extend certain aspects of the original theory. The discussion generally follows macroscopic and classical lines, analogous to the Bloch[4] treatment of the nuclear induction experiment. In a paper which is to appear in *The Physical Review*, Van Vleck has shown that a quantum-mechanical treatment of ferromagnetic resonance leads to the identical resonance condition as the classical theory.

II. DEPENDENCE OF RESONANCE CONDITION ON THE SHAPE OF THE SPECIMEN

The equation $\omega_0=\gamma(BH)^{\frac{1}{2}}$ was derived specifically for a plane surface. The derivation of the resonance condition is intimately related to the nature of the demagnetizing field, and it is reasonable to expect that the equation will depend on the shape of the specimen. It turns out, for example, that the resonance condition in a *small sphere* of ferromagnetic material is given by the Larmor equation $\omega_0=\gamma H_z$.

[1] J. H. E. Griffiths, Nature **158**, 670 (1946).
[2] W. A. Yager and R. M. Bozorth, Phys. Rev. **72**, 80 (1947).
[3] C. Kittel, Phys. Rev. **71**, 270 (1947).
[4] F. Bloch, Phys. Rev. **70**, 460 (1946).

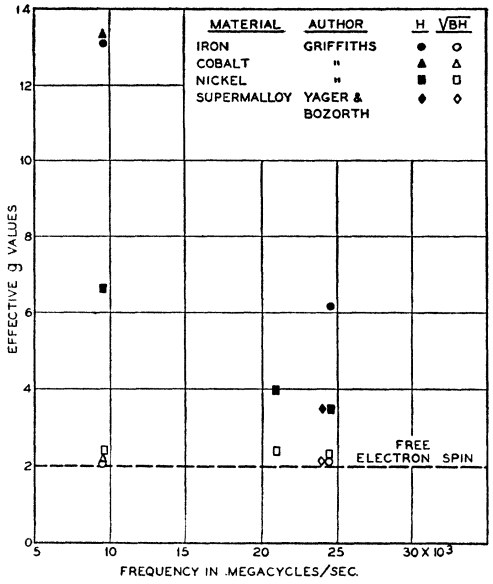

FIG. 1. Summary of data on ferromagnetic resonance in plane specimens: comparison of effective g values using both H and $(BH)^{\frac{1}{2}}$ as the effective magnetic field in the resonance condition. For an electron spin $g=2.00$.

We shall limit the discussion to shapes for which the demagnetizing factors have a rigorous meaning. We must accordingly discuss only objects which are uniformly magnetized at the frequencies under consideration. It is therefore necessary that certain dimensions of the specimens be small in comparison with the eddy current skin depth. A plane surface is in a sense an exception to this requirement.

The skin depth in the ferromagnetic metals at microwave frequencies is of the order of 10^{-5} to 10^{-4} cm, so that only very fine particles will satisfy the size requirement. However, the new ferromagnetic semi-conductor materials offer unusual possibilities for working with specimens of a convenient size. For example, Snoek[5] gives the resistivity of "Ferroxcube 4," which is a Ni-Zn ferrite, as 10^6 ohm-cm, or about 10^{11} higher than the resistivity of iron. The skin depth is ~ 3 cm at $\lambda=1$ cm and $\mu=100$. One should be able to work at microwave frequencies with a single shaped specimen of such material with dimensions of the order of 0.1 to 1 cm.

The magnetization $\mathbf{M}$ and angular momentum density $\mathbf{J}$ are related by

$$\mathbf{M}=\gamma\mathbf{J}, \tag{2}$$

where γ is the magneto-mechanical ratio and is equal to e/mc for electron spins; numerically, $\gamma/2\pi=2.80$ megacycles/oersted. The equation of motion referred to unit volume of material is

$$d\mathbf{J}/dt=\mathbf{M}\times\mathbf{H}, \tag{3}$$

since the expression on the right is the torque acting on a unit volume. This equation may be written

$$d\mathbf{M}/dt=\gamma\mathbf{M}\times\mathbf{H}. \tag{4}$$

We now consider the resonance condition for a general ellipsoid with principal axes parallel to the x, y, z axes of the coordinate system. The demagnetizing factors are N_x, N_y, N_z. The static magnetic field is H_z; the r-f field* is H_x. The effective values of the magnetic field components inside the material are:

$$H_x^i=H_x-N_xM_x; \tag{5a}$$

$$H_y^i=-N_yM_y; \tag{5b}$$

$$H_z^i=H_z-N_zM_z. \tag{5c}$$

The values H_x^i, H_y^i, H_z^i should be used when substituting for H in Eq. (4). The component equations of Eq. (4) become

$$dM_x/dt=\gamma[H_z+(N_y-N_z)M_z]M_y; \tag{6a}$$

$$dM_y/dt=\gamma[M_zH_x-(N_x-N_z)M_xM_z-M_xH_z]; \tag{6b}$$

$$dM_z/dt\cong 0. \tag{6c}$$

On solving these equations with time dependence $\exp[j\omega t]$, the susceptibility $\chi_x(=M_x/H_x)$ is found to be

$$\chi_x=\frac{\chi_0}{1-(\omega/\omega_0)^2}, \tag{7}$$

[5] J. L. Snoek, *New Developments in Ferromagnetic Materials* (Elsevier, Amsterdam, 1947), p. 97; Philips Technical Review 8, 353 (1946).

* This convention for the labeling of static and r-f fields will be followed throughout the present paper. It may be noted that the Weiss exchange field does not enter into the problem, since the exchange field is parallel to $\mathbf{M}$ and hence its vector product with $\mathbf{M}$ is identically zero. Any anisotropic term in the Lorentz local field is lumped in with the crystalline anisotropy energy, which is treated below. In the quantum-mechanical treatment the Weiss field does not enter because the operator $\Sigma\sigma_i^x$ commutes with the exchange interaction term in the Hamiltonian, as Van Vleck has shown.

where

$$\chi_0=\frac{M_z}{H_z+(N_x-N_z)M_z} \tag{8}$$

and the *resonance frequency* is given by

$$\omega_0=\gamma\{[H_z+(N_y-N_z)M_z] \times[H_z+(N_x-N_z)M_z]\}^{\frac{1}{2}}. \tag{9}$$

We shall consider some special cases of Eq. (9):

(a) *Plane* ($N_x=N_z=0$; $N_y=4\pi$)

$$\omega_0=\gamma(B_zH_z)^{\frac{1}{2}}. \tag{10}$$

(b) *Sphere* ($N_x=N_y=N_z=4\pi/3$)

$$\omega_0=\gamma H_z. \tag{11}$$

(c) *Infinite Circular Cylinder* ($N_x=N_y=2\pi$; $N_z=0$)

$$\omega_0=\gamma(H_z+2\pi M_z). \tag{12}$$

It is seen that, except for the special cases of a plane and a sphere, Eq. (7) predicts the *existence of a finite resonance frequency even for zero static field.* It should however be emphasized at this point that the ferromagnetic resonance effect may only be conspicuous when the entire specimen is magnetized to saturation as a *single domain;* otherwise the inhomogeneous magnetization may lead to high damping by eddy currents. It will commonly be necessary to apply at least a small static field in order to insure approximate saturation.

It is clear from the behavior in the general case just considered that the demagnetizing fields affect the resonance condition by changing the magnetic field energy associated with a given direction of the magnetization vector.

Some numerical comparisons are given below in order to show the importance of the shape effect. The material considered is iron, for which $M_z\cong1700$; for electron spins $\gamma/2\pi=2.80$ Mc/oersted.

Example A $H=1000$ oersted.
Plane: $f_0=13{,}300$ Mc/sec.
Sphere: $f_0=2{,}800$ Mc/sec.
Cylinder: $f_0=32{,}800$ Mc/sec.

Example B $H\cong0$.
Plane: $f_0=0$.
Sphere: $f_0=0$.
Cylinder: $f_0=30{,}000$ Mc/sec.

Figure 1 is based on the published data on ferromagnetic resonance in *plane* specimens. The scale of ordinates is the effective g value as calculated from the observed frequency and magnetic field at resonance. The solid points are the g-values as calculated assuming the ordinary Larmor resonance condition with H_z as the effective field. The hollow points are calculated for the same observations but using $(BH)^{\frac{1}{2}}$ as the effective field; it is seen that this assumption leads to g values very much closer to the value $g=2.00$ which obtains for an electron spin. The values remain slightly high, however; no satisfactory explanation of the residual deviation has been put forward.

Figure 2 compares the theoretical resonance conditions for a plane surface and for a small sphere. The value of the saturation magnetization is taken as for a representative ferrite.

III. EFFECT OF CRYSTALLINE ANISOTROPY ENERGY ON THE RESONANCE CONDITION

The energy of ferromagnetic crystals depends in part on the magnetization direction relative to the crystal axes; this part of the energy is called the anisotropy energy. In an uniaxial

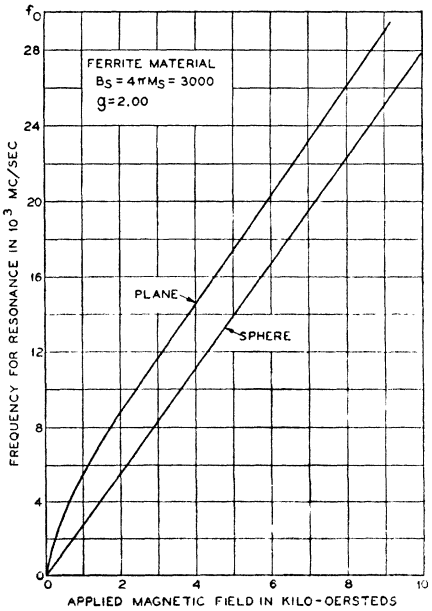

FIG. 2. Comparison of theoretical resonance conditions for a plane surface and for a small sphere (diameter small in comparison with eddy current skin depth).

crystal such as cobalt, the major term in the anisotropy energy may be written

$$f=K_1' \sin^2\theta, \tag{13}$$

where f refers to unit volume of material; θ is the angle between the magnetization and the principal axis of the crystal; K_1' is the first order anisotropy constant. Similarly in a cubic crystal

$$f=K_1(\alpha_1^2\alpha_2^2+\alpha_2^2\alpha_3^2+\alpha_1^2\alpha_3^2), \tag{14}$$

where α_1, α_2, α_3 are the direction cosines of the magnetization relative to the cube edges.

The anisotropy energy will in general effect an alteration in the resonance condition. In a single crystal one expects to find that the value of the magnetic field required for resonance at a fixed frequency will depend on the direction of the crystal axes relative to the shape axes of the specimen. In a polycrystalline specimen the resonance in general will be broader than in a single crystal of the same material, since the distribution in direction of the crystal axes causes a distribution in field strengths for resonance.

It is convenient to consider the effect of the anisotropy energy in terms of an equivalent magnetic field.[5a] The equivalent field $\mathbf{H}^e$ is defined such that the torque exerted on the specimen by such a field is equal to the torque exerted by the anisotropy energy: that is,

$$\partial f/\partial\theta=\mathbf{M}_s\times\mathbf{H}^e \tag{15}$$

where θ is a rotation about an axis parallel to the direction of $\mathbf{M}_s\times\mathbf{H}^e$. *The vector* $\mathbf{H}^e$ *is not completely determined by Eq. (15), since either the magnitude or the direction are still arbitrary.* It is often appropriate to take $\mathbf{H}^e$ parallel to the x or y axes of the specimen** and to express its magnitude in terms of an *effective demagnetizing factor* N^e defined by

$$H_x^e=-N_x^eM_x \tag{16a}$$

and

$$H_y^e=-N_y^eM_y. \tag{16b}$$

[5a] In general the torque vector resulting from the anisotropy energy is to be treated directly as an additional torque in the equations of motion. The artifice of the effective field or effective demagnetizing factors is applicable to planes and directions of high symmetry, and is a convenience in that it often avoids solving the equations of motion over again for each situation.

** When $N_x^e=N_y^e$ it may be convenient to consider $\mathbf{H}^e$ as parallel to the z axis; see, for example, Eq. (20).

The resonance condition is then determined by adding N_x^e, N_y^e to the usual demagnetizing factors in Eq. (9), giving

$$\omega_0=\gamma\{[H_z+(N_y+N_y^e-N_z)M_z] \times[H_z+(N_x+N_x^e-N_z)M_z]\}^{\frac{1}{2}}, \tag{17}$$

as the general condition for resonance in an ellipsoidal single crystal when the static magnetic field H_z is along one of the principal axes of the ellipsoid.

It remains now to determine the values of the effective demagnetizing factors N_x^e and N_y^e for representative experimental situations.

(a) *Uniaxial crystal; H_z along axis.*

$$\partial f/\partial\theta=2K_1' \sin\theta\cos\theta=\mathbf{M}_s\times\mathbf{H}^e \cong N_x^eM_zM_x\cong N_x^eM_z^2\sin\theta, \tag{18}$$

so that, for angles near $\theta=0$,

$$N_x^e=(2K_1'/M_z^2); \tag{19a}$$

similarly,

$$N_y^e=(2K_1'/M_z^2). \tag{19b}$$

The resonance condition for a plane specimen is

$$\omega_0=\gamma\left\{\left(H_z+4\pi M_z+\frac{2K_1'}{M_z}\right)\left(H_z+\frac{2K_1'}{M_z}\right)\right\}^{\frac{1}{2}}. \tag{20}$$

This equation predicts a finite limiting frequency as $H\rightarrow 0$; in practice a small biasing field will usually be required in order to eliminate domain boundary effects. The possibility of resonance in the "anisotropy field" was first considered by Landau and Lifshitz,[6] for the special case of uniaxial crystals. Their treatment neglects the effects of demagnetizing fields and the eddy current damping associated with the movement of domain boundaries.

(b) *Cubic crystal; H_z, H_z along [100] directions.*

In a (100) plane Eq. (14) for the anisotropy energy reduces to

$$f=(K_1/4)\sin^2 2\theta, \tag{21}$$

where θ is the angle between the magnetization and a [100] axis. We have

$$\partial f/\partial\theta=K_1 \sin 2\theta\cos 2\theta=N_x^eM_z^2\sin\theta; \tag{22}$$

[6] L. Landau and E. Lifshitz, Physik. Zeits. Sowjetunion **8**, 153 (1935); see also J. L. Snoek, Nature **160**, 90 (1947).

so that for angles near $\theta=0$,

$$N_x^e=(2K_1/M_z^2); \tag{23a}$$

similarly,

$$N_y^e=(2K_1/M_z^2). \tag{23b}$$

The results for this orientation are similar to those worked out above for cobalt. The numerical value of the effective field in the z-direction is $2K_1/M_s\sim 7000$ oersted in Co and ~ 500 oersted in Fe, at room temperature.

(c) *Cubic crystal; H_x, H_z along [110] directions.*

Let $\epsilon=\theta-\pi/4$; then

$$\partial f/\partial\epsilon=-K_1\sin 2\epsilon\cos 2\epsilon=N_x^e M_z^2\sin\epsilon, \tag{24}$$

so that for angles near $\epsilon=0$,

$$N_x^e=-(2K_1/M_z^2); \tag{25a}$$

the negative sign should be noted. To determine N_y^e we write the anisotropy energy as

$$f=K_1(\tfrac{1}{4}\sin^2 2\theta+\alpha_3^2), \tag{26}$$

so that

$$-\partial f/\partial\theta_3=2K_1\cos\theta_3\sin\theta_3=N_y^e M_z^2\cos\theta_3, \tag{27}$$

giving, for angles near $\theta_3=\pi/2$,

$$N_y^e=(2K_1/M_z^2). \tag{25b}$$

The difference in sign between N_x^e and N_y^e may be confirmed by considering the general shape of the anisotropy energy surface. An instructive photograph of a model of the surface is given by Bitter.[7]

A comparison of cases (b) and (c) shows that the magnetic field required for resonance with a (100) crystal face is expected to be greater when H_z is in the [110] direction than when H_z is in the [100] direction. The difference ΔH_z is of the order of $4K_1/M_s$ for $H_z\ll 4\pi M_s$. The theoretical resonance conditions for [100] and [110] directions are compared in Fig. 3.

The general expression for the resonance condition in the (001) plane is given by Eq. (17) with

$$N_x^e=2K_1\cos 4\theta/M_s^2, \tag{27a}$$

$$N_y^e=(2K_1+\tfrac{1}{2}K_2\sin^2 2\theta)/M_s^2, \tag{27b}$$

where θ is the angle between the z-axis and the [100] direction; here we have taken account of the second-order term $K_2\alpha_1^2\alpha_2^2\alpha_3^2$ in the anisotropy energy.

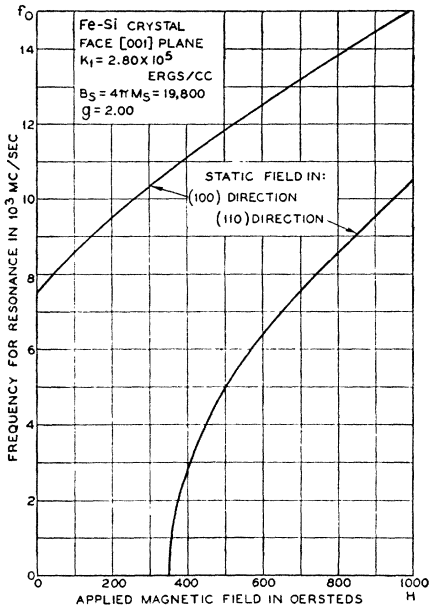

FIG. 3. Comparison of theoretical resonance condition for [100] and [110] directions in an Fe-Si single crystal with a (001) plane surface.

The discussion has supposed that the static magnetic field H_z is sufficiently large so that the static magnetization is in the direction of H_z. If this condition is not satisfied, we may under certain conditions still obtain resonance, but in the calculations it is then necessary to use as H_z the projection of the static field on the direction of the static magnetization; also, θ should be taken as the angle between the magnetization and the [100] direction.

Note added in proof: The predicted dependence of the resonance condition on angle in the (001) plane in ion has been verified approximately by Kip and Arnold (to be published) and by Yager (to be published). Kip and Arnold have also observed at 3 cm wave-length two resonance peaks for angles near [110]. Qualitatively it seems that this observation may be explained by the fact that the resonance condition may sometimes have two roots if the static field H_z is of the same order of magnitude as K_1/M_s, so that the angle between the static field and magnetization may be appreciable. The quantitative treatment of the condition for the appearance of the second peak is difficult to carry through in

[7] F. Bitter, *Introduction to Ferromagnetism* (McGraw-Hill Book Company, Inc., New York, 1937), p. 195.

detail because of uncertainty[7a] as to the actual domain structure of the crystal in the relevant range of field strength.

One may treat magnetostrictive energy due to strains on the same basis as the anisotropy energy, so that one would expect the resonance condition to depend on the state of strain of the material.

IV. FERROMAGNETIC RESONANCE IN THE PRESENCE OF MACROSCOPIC EDDY CURRENTS—PLANE SURFACE

The usual measurements and computations involved in deducing values of the effective permeability μ_R as a function of the static field H_z are listed below. In the experiments it is found that μ_R as determined in this way shows a well-defined maximum for values of H somewhere near the value predicted by Eq. (1).

(a) The cavity Q as a function of H_z is determined by means of standing wave ratio measurements[8] in a slotted section of wave guide connecting the r-f power supply to the resonant cavity.

(b) From the measured Q's and the known geometry of the cavity one can calculate[9] (at least approximately) the value of the "loss factor" $(\mu_R\rho)^{\frac{1}{2}}$ for each of the walls of the cavity separately.

(c) Taking ρ from d.c. measurements, we are able to calculate μ_R *vs.* H_z as desired.

Mention should be made of the connection between the complex permeability $\mu=\mu_1-j\mu_2$ with which the theory commonly deals and the effective permeability μ_R which emerges from the interpretation, as above, of the results of a resistive type of measurement, such as a cavity Q measurement.[10] The *resistive loss* from the skin effect is determined by the *imaginary* part of the complex wave number $k=k_1-jk_2$, which describes the spatial variation of the magnetic field within a conducting surface according to an equation of the form $H_x\sim e^{-jky}=e^{-k_2y}e^{-ik_1y}$. From the differential equation for eddy currents one finds

$$k\sim(-j\mu)^{\frac{1}{2}}=j(\mu_2+j\mu_1)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}(\mu_L^{\frac{1}{2}}-j\mu_R^{\frac{1}{2}}), \quad (28)$$

where the term on the extreme right may be viewed as defining the real quantities μ_L and μ_R. This definition is consistent with the characteristic properties of μ_R and μ_L.

From Eq. (28) we find the well-known result

$$\mu_R=(\mu_1^2+\mu_2^2)^{\frac{1}{2}}+\mu_2; \quad (29a)$$

$$\mu_L=(\mu_1^2+\mu_2^2)^{\frac{1}{2}}-\mu_2. \quad (29b)$$

If μ is real, $\mu_R=\mu_L=\mu$.

By a Q measurement only μ_R is determined; μ_L is the effective permeability for an inductive measurement and may be determined from the shift in resonance frequency of the *cavity*—this is a far more difficult determination than an ordinary Q measurement.

The above discussion enables one to interpret the results of theoretical derivations for uniformly magnetized specimens—uniform with respect to both static and r-f components of magnetization—in terms of the experimental situation, which in normal ferromagnetic conductors involves the non-uniform r-f magnetization associated with eddy currents.

Because the argument may seem somewhat devious, it is intended below to treat the matter in a more direct way. In particular it will emerge clearly that, when the results of an experiment are properly interpreted, the *usual eddy current losses do not as such introduce damping terms into the expression for the permeability.*

In addition to Eq. (4) we must consider the relation between M and H given by the eddy current equation:

$$\nabla^2\mathbf{H}=(4\pi/\rho c^2)\left(\frac{\mathbf{dH}}{dt}+4\pi\frac{\mathbf{dM}}{dt}\right), \quad (30)$$

which is a consequence of Maxwell's equations for curl$\mathbf{E}$ and curl$\mathbf{H}$; here ρ is the resistivity.

[7a] A. von Engel and M. S. Wills, Proc. Roy. Soc. **A188**, 464 (1947).

[8] See, for example, J. C. Slater, Rev. Mod. Phys. **18**, 441 (1946).

[9] Actually the observed Q's do not usually agree very closely with the values calculated for cases where the permeability μ is unity and the resistivity ρ is taken from d.c. measurements. The resulting uncertainty in μ_R (in cases where this is not unity) does not affect appreciably the deduced values of the resonance frequency and the half-width of the resonance. It seems reasonable to require that μ_R approach unity as H approaches infinity, and this requirement can be invoked as a partial check on the normalization of μ_R.

[10] For a more detailed discussion of this point, see C. Kittel, Phys. Rev. **70**, 281 (1946).

For a plane surface the components of **H** are $(H_x, -4\pi M_y, H_z)$. Here H_x is the r-f field; H_z is the static field;[11] $H_y = -4\pi M_y$ as a consequence of the demagnetization factor $N_y = 4\pi$.

Neglecting products of small quantities, Eqs. (4) and (30) reduce to the following, where we have assumed solutions of the form $H_x, M_x, M_y \sim \exp[j(\omega t - ky)]$:

$$j\omega M_x - \gamma B_z M_y = 0, \tag{31a}$$

$$j\omega M_y - \gamma(M_z H_x - M_x H_z) = 0, \tag{31b}$$

$$(jp^2 + k^2)H_x + j4\pi p^2 M_x = 0. \tag{31c}$$

Here $p^2 = 4\pi\omega/\rho c^2$. The set of three homogeneous linear equations in the three unknown H_x, M_x, M_y has a non-trivial solution if the determinant

$$\begin{vmatrix} 0 & j\omega & -\gamma B_z \\ -\gamma M_z & \gamma H_z & j\omega \\ (jp^2+k^2) & j4\pi p^2 & 0 \end{vmatrix} = 0. \tag{32}$$

The determinantal equation reduces to

$$k^2 = jp^2 \frac{\mu_0\omega_0^2 - \omega^2}{\omega^2 - \omega_0^2}, \tag{33}$$

where

$$\omega_0 = \gamma(B_z/H_z)^{\frac{1}{2}} \tag{34}$$

and

$$\mu_0 = B_z/H_z. \tag{35}$$

Now the ordinary eddy current equation for permeability μ_R leads to

$$k^2 = -j\mu_R p^2, \tag{36}$$

which is identical with Eq. (33) if we set

$$\mu = \mu_R = (\omega_0^2\mu_0 - \omega^2)/(\omega_0^2 - \omega^2). \tag{37}$$

The susceptibility χ_x is therefore

$$\chi_x = \chi_0/[1 - (\omega/\omega_0)^2], \tag{38}$$

with $\chi_0 = M_{sat}/H_z$ and $\omega_0 = \gamma(B_z H_z)^{\frac{1}{2}}$. This result is identical with that derived in the earlier paper,[3] where eddy currents were not considered explicitly; we may therefore conclude that the macroscopic eddy current losses do not enter in the expression for the susceptibility.

[11] It is assumed here that the specimen is a thin plane sheet, so that we may neglect demagnetizing corrections to the field H_z.

It is of interest to consider the form of the orbit described by the magnetization vector. Equation (31a) shows that M_x and M_y differ in phase by $\pi/2$, and $|M_x|/|M_y| = \gamma B_z/\omega$. The orbit is therefore an ellipse; at resonance the ratio of the principal axes is $(B_z/H_z)^{\frac{1}{2}}$, with the long axis parallel to the plane surface of the specimen and the short axis normal to the surface.

V. CONCLUSION

The results of this paper predict several new effects which have not as yet been reported experimentally. The resonance condition is expected to depend on the shape of the specimen, and, in the case of a single crystal, also on the orientation of the crystal. It may be possible to test the predicted shape dependence using ferromagnetic semi-conducting materials, such as the ferrites.

The considerations given here throw no light on the anomalous g value of 2.17 reported by Yager and Bozorth[2] using a Supermalloy specimen, since the anisotropy and magnetostriction of this material are very low. The quantum mechanical treatment by Van Vleck also does not account for this anomaly, since his treatment leads to the usual resonance condition.

In connection with relaxation or damping effects, it may be remarked in passing that the failure[2] of the relaxation term $-\lambda(\mathbf{M} - \chi_0\mathbf{H})$ proposed previously[3] is not particularly alarming in the absence of a detailed physical description of the relaxation process. The basis for the above term is purely formal, and the term could have been written $-\lambda'[(\mathbf{M}/\chi_0) - \mathbf{H}]$, which will fit the data of Yager and Bozorth in a satisfactory manner.

I am indebted to Professor J. H. Van Vleck for a discussion of the quantum-mechanical treatment of the problem, and to W. A. Yager for discussions of the effect of various relaxation terms. Part of the work reported here was done at the Massachusetts Institute of Technology during the tenure of a fellowship of the John Simon Guggenheim Memorial Foundation. I wish to thank Professor J. C. Slater and Professor J. A. Stratton for their hospitality, and the trustees of the Foundation for financial support.