Pair Production by γ-Rays in the Field of an Electron

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THE present calculation of the cross section of this process is based on Dirac's theory of positrons as formulated in the quantum theory of wave fields and on the general expression for the matrix element $H_{FA}{}^{III}$, which determines the probability of quantum processes of third order.

Summing the contributions of all possible virtual ways in which the process can take place, we get for our specific $H_{fa}{}^{III}$ the same expression as in the ordinary quantum electrodynamics of electrons (without theory of holes), although the contributions of individual virtual ways are different in both cases.[1] This shows clearly, that the individual sequences of intermediate states of the usual perturbation theory have no unambiguous meaning.

A rather tedious task is the calculation of the spurs (of products of 12th degree in the Dirac matrices), which occur in the expression $W=\frac{1}{4}\Sigma(H_{fa}{}^{III})^2$, where $\frac{1}{4}\Sigma$ denotes the averaging over the polarizations of the incident photon and spin states of the initial electron and summing over the spin states of the three particles of the final triplet. This can be done by using some "four-dimensional" generalizations of the usual formulas, e.g., expressing the spurs of the form

$$S_p \prod_{m=1}^{2n} \alpha^{i_m}, \quad (i_m=0,\,1,\,2,\,3), \quad \alpha^i\equiv(1,\boldsymbol{\alpha}),$$

in terms of g_{ij}. The resulting complete formula for W is very complicated.

Choosing the polar coordinate system, in which the initial electron is at rest and the photon moves in the direction of the axis, the formula for the differential cross section can be written in the form

$$d\sigma=\frac{r_0^2}{137}\cdot\frac{S}{\pi}\cdot\frac{dE'dE''d\vartheta'd\vartheta''}{\sin\omega_+},$$

where $S=4\pi^3\mu^7c^8E^+E'E''W/h^6\epsilon^6$ and ω_+ is the angle between the planes $(\mathbf{k},\mathbf{p}')$ and $(\mathbf{k},\mathbf{p}'')$. The energies E and the energy k of the photon are measured in units μc^2. μ is the mass, and ϵ the charge of the electron, and $r_0=\epsilon^2/\mu c^2$.

In computing the total cross section $\sigma(k)$, the two final states with $\mathbf{p}'=\mathbf{A}$, $\mathbf{p}''=\mathbf{B}$ and $\mathbf{p}'=\mathbf{B}$, $\mathbf{p}''=\mathbf{A}$ are not to be considered as different. Because the general and exact integration of $d\sigma$ can hardly be performed, the following special cases were discussed in approximation: (1) The case $0\leqq k-4\ll 1$; (2) the case $k\gg 4$, when, further, either (2a) p' is small (*viz.* $2k^{-1}\leqq p'\leqq 1$) and p'', p^+ large (of the order $k\gg 1$), or (2b) p^+ small and p', p'' large, or, at last, (2c) p', p'', p^+ all large. The case of two particles of the triplet receiving momenta $\ll 1$ does not arise.

The results are as follows:

(1) Because in the limit $k=4$ the quantity $S=S_0=1/12$, we obtain the asymptotic formula[2]

$$\sigma(k)=(r_0^2/137)(\pi\sqrt{3}/2^23^5)(k-4)^2. \tag{1}$$

(2a) In this case (of small recoil) the values of S are very large, although only few terms are significant, and the approximate formula for $\sigma(k)$ becomes

$$\sigma(k)=r_0^2/137[(28/9)\log 2k-(102/9)]. \tag{2a}$$

The approximation involved results in a not entirely accurate value of the additive constant.

(2b) The probability of this case (of a slow positron) is small, because

$$\sigma(k)=(r_0^2/137)\cdot 2k^{-1}\log k. \tag{2b}$$

Here again the value of the numerical factor[2] is not exact.

(2c) In this last case I was only able to determine the order of magnitude of σ, which is

$$\sigma\sim r_0^2/137\cdot\bar{S}/64, \tag{2c}$$

where $\bar{S}$ is some constant (independent of k) of the order unity.

The total cross section of the case (2) will now be approximately equal to the sum of the contributions (2a, b, c), i.e. will be in substance given by (2a).[3] This result confirms the formula of Wheeler and Lamb,[4] and their indirect conclusion, that the contribution of the processes connected with a large momentum transfer to the initial electron should be relatively small and enters into the formula for the total cross section only as a part of the constant term.

A more detailed report will be published shortly in the Bulletin international de l'Académie tchèque des sciences, Prague.

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[1] In the latter case $H_{fa}{}^{III}$ can also be computed by using the corresponding formula of Moller, Proc. Roy. Soc. **A152**, 481 (1938) for the (inverse) process of radiative collision between two fast electrons.
[2] See also A. Borsellino, Nuovo Cimento **4**, No. 3–4 (1947). The exponent 3 of $(k-4)$ in Borsellino's formula seems to be a misprint.
[3] Contrary to the opinion of K. M. Watson, Phys. Rev. **72**, 1060 (1947) the contribution of the case p'' small and p', p^+ large should not be counted separately, because the interchange of both electrons of the triplet does not lead to a new final state.
[4] J. A. Wheeler and W. E. Lamb, Jr., Phys. Rev. **55**, 858 (1939).

Magnetic Resonance Absorption in Diluted Chrome Alums*

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THE microwave measurements on the magnetic resonance absorption in single crystals of ammonium chrome alum reported recently[1] have been repeated with refined techniques, and extended to include potassium chrome alum and diluted crystals of both salts. Troublesome drifts in frequency have been greatly reduced by the use of the Pound stabilized oscillator.[2] The absorption was measured again through the off-balance of a magic-tee bridge at a frequency of 9375 mc/sec. The cavity was tuned to a balance at high fields, of about 7200 oersteds. Compared to the previous method of balancing at zero