Pair Production by γ-Rays in the Field of an Electron

VÁCLAV VOTRUBA
Institute of Theoretical Physics, Charles University,
Prague, Czechoslovakia
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THE present calculation of the cross section of this process is based on Dirac's theory of positrons as formulated in the quantum theory of wave fields and on the general expression for the matrix element $H_{FA}{}^{\text{III}}$, which determines the probability of quantum processes of third order.

Summing the contributions of all possible virtual ways in which the process can take place, we get for our specific $H_{fa}{}^{\text{III}}$ the same expression as in the ordinary quantum electrodynamics of electrons (without theory of holes), although the contributions of individual virtual ways are different in both cases.[1] This shows clearly, that the individual sequences of intermediate states of the usual perturbation theory have no unambiguous meaning.

A rather tedious task is the calculation of the spurs (of products of 12th degree in the Dirac matrices), which occur in the expression $W=\frac{1}{4}\Sigma(H_{fa}{}^{\text{III}})^2$, where $\frac{1}{4}\Sigma$ denotes the averaging over the polarizations of the incident photon and spin states of the initial electron and summing over the spin states of the three particles of the final triplet. This can be done by using some "four-dimensional" generalizations of the usual formulas, e.g., expressing the spurs of the form

$$S_p \prod_{m=1}^{2n} \alpha^{i_m}, \quad (i_m=0,\,1,\,2,\,3), \quad \alpha^i \equiv (1,\boldsymbol{\alpha}),$$

in terms of g_{ij}. The resulting complete formula for W is very complicated.

Choosing the polar coordinate system, in which the initial electron is at rest and the photon moves in the direction of the axis, the formula for the differential cross section can be written in the form

$$d\sigma=\frac{r_0^2}{137}\cdot\frac{S}{\pi}\cdot\frac{dE'dE''d\vartheta'd\vartheta''}{\sin\omega_+},$$

where $S=4\pi^3\mu^7c^8E^+E'E''W/h^6\epsilon^6$ and ω_+ is the angle between the planes $(\mathbf{k},\mathbf{p}')$ and $(\mathbf{k},\mathbf{p}'')$. The energies E and the energy k of the photon are measured in units μc^2. μ is the mass, and ϵ the charge of the electron, and $r_0=\epsilon^2/\mu c^2$.

In computing the total cross section $\sigma(k)$, the two final states with $\mathbf{p}'=\mathbf{A}$, $\mathbf{p}''=\mathbf{B}$ and $\mathbf{p}'=\mathbf{B}$, $\mathbf{p}''=\mathbf{A}$ are not to be considered as different. Because the general and exact integration of $d\sigma$ can hardly be performed, the following special cases were discussed in approximation: (1) The case $0\leqq k-4\ll 1$; (2) the case $k\gg 4$, when, further, either (2a) p' is small (*viz.* $2k^{-1}\leqq p'\leqq 1$) and p'', p^+ large (of the order $k\gg 1$), or (2b) p^+ small and p', p'' large, or, at last, (2c) p', p'', p^+ all large. The case of two particles of the triplet receiving momenta $\ll 1$ does not arise.

The results are as follows:

(1) Because in the limit $k=4$ the quantity $S=S_0=1/12$, we obtain the asymptotic formula[2]

$$\sigma(k)=(r_0^2/137)(\pi\sqrt{3}/2^2 3^5)(k-4)^2. \qquad (1)$$

(2a) In this case (of small recoil) the values of S are very large, although only few terms are significant, and the approximate formula for $\sigma(k)$ becomes

$$\sigma(k)=r_0^2/137[(28/9)\log 2k-(102/9)]. \qquad (2a)$$

The approximation involved results in a not entirely accurate value of the additive constant.

(2b) The probability of this case (of a slow positron) is small, because

$$\sigma(k)=(r_0^2/137)\cdot 2k^{-1}\log k. \qquad (2b)$$

Here again the value of the numerical factor[2] is not exact.

(2c) In this last case I was only able to determine the order of magnitude of σ, which is

$$\sigma\sim r_0^2/137\cdot\bar{S}/64, \qquad (2c)$$

where $\bar{S}$ is some constant (independent of k) of the order unity.

The total cross section of the case (2) will now be approximately equal to the sum of the contributions (2a, b, c), i.e. will be in substance given by (2a).[3] This result confirms the formula of Wheeler and Lamb,[4] and their indirect conclusion, that the contribution of the processes connected with a large momentum transfer to the initial electron should be relatively small and enters into the formula for the total cross section only as a part of the constant term.

A more detailed report will be published shortly in the Bulletin international de l'Académie tchèque des sciences, Prague.

I am indebted to Professor G. Wentzel, who suggested this problem, for valuable stimulating discussions about this subject during my stay in Zurich.

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[1] In the latter case $H_{fa}{}^{\text{III}}$ can also be computed by using the corresponding formula of Moller, Proc. Roy. Soc. **A152**, 481 (1938) for the (inverse) process of radiative collision between two fast electrons.

[2] See also A. Borsellino, Nuovo Cimento **4**, No. 3–4 (1947). The exponent 3 of $(k-4)$ in Borsellino's formula seems to be a misprint.

[3] Contrary to the opinion of K. M. Watson, Phys. Rev. **72**, 1060 (1947) the contribution of the case p'' small and p', p^+ large should not be counted separately, because the interchange of both electrons of the triplet does not lead to a new final state.

[4] J. A. Wheeler and W. E. Lamb, Jr., Phys. Rev. **55**, 858 (1939).

Magnetic Resonance Absorption in Diluted Chrome Alums*

C. A. WHITMER, R. T. WEIDNER, AND P. R. WEISS
Department of Physics, Rutgers University, New Brunswick, New Jersey
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THE microwave measurements on the magnetic resonance absorption in single crystals of ammonium chrome alum reported recently[1] have been repeated with refined techniques, and extended to include potassium chrome alum and diluted crystals of both salts. Troublesome drifts in frequency have been greatly reduced by the use of the Pound stabilized oscillator.[2] The absorption was measured again through the off-balance of a magic-tee bridge at a frequency of 9375 mc/sec. The cavity was tuned to a balance at high fields, of about 7200 oersteds. Compared to the previous method of balancing at zero

TABLE I. Observed and calculated magnetic field values of the absorption maxima for $\nu = 9375$ mc/sec.

Undilute Ammonium Chrome Alum $g = 1.99$, $\delta = 0.143$ cm^{-1}							
111)	1,830	2,780	3,370	3,770	4,910	Calc.	
	1,830	2,720	3,370	Unres.	4,920	exp.	
100)		3,120		3,540		Calc.	
			3,250			exp.	
(110)	2,390	2,630	3,240	3,580	3,910	4,130	Calc.
	2,380	Unres.	3,220	3,600	Unres.	4,190	exp.
Dilute Ammonium Chrome Alum $g = 1.97$, $\delta = 0.100$ cm^{-1}							
(111)	2,310	2,970	3,400	3,720	4,490	Calc.	
	2,270	2,980	3,400	3,710	4,480	exp.	
(100)		3,280		3,480		Calc.	
		3,300		3,480		exp.	
(110)	2,740	2,860	3,330	3,500	3,830	3,910	Calc.
	2,740	2,880	3,330	3,500	3,820	3,920	exp.

d.c. magnetic field, this procedure shifts the maxima by as much as 100 oersteds in some cases. In the new method, the balance is obtained sufficiently far removed from any resonances so that a minimum of magnetic absorption is balanced out. A recalibration of the d.c. field by comparison with the resonance absorption of the proton moment established the field measurements to within 10 gauss for all field strengths used. The zero field splitting previously reported for ammonium chrome alum (0.15 cm^{-1}) is essentially the same, the changes caused by the new method of balancing being largely compensated by the errors in our previous magnetic field calibration.

The diluted crystals were grown from a solution made up by dissolving 1 part by weight of ammonium chrome alum and 5 parts by weight of aluminum ammonium alum. Chemical analysis of the crystals shows the ratio of the number of chrome ions to aluminum ions to be about 1 to 8.5. The crystals of dilute potassium chrome alum were grown in the same way.

Figures 1, 2, and 3 show the absorption for both dilute and undilute ammonium chrome alum single crystals for the orientations in which the d.c. field is perpendicular to: a 111 face, a 100 face, and a 110 face, respectively.

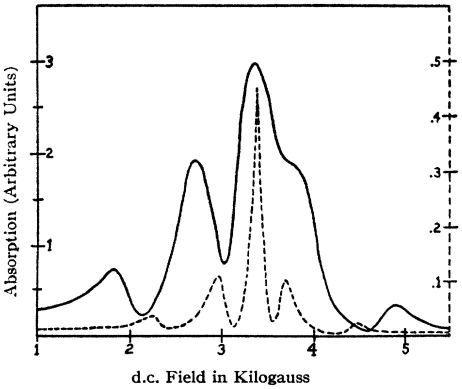

FIG. 1. Relative absorption of ammonium chrome alum with the d.c. field perpendicular to a (111) face. The full line curve refers to the undiluted, the dashed curve to the diluted slat. The scales for the undilute (left) and the dilute (right) salts refer to the same input power.

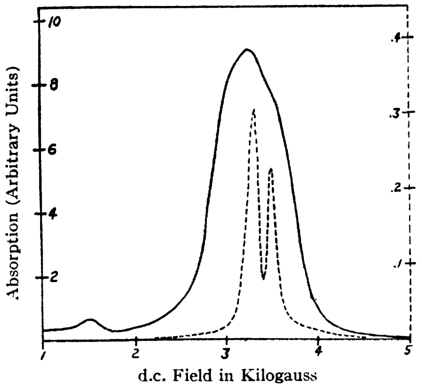

FIG. 2. Relative absorption of ammonium chrome alum with the d.c. field perpendicular to a (100) face. The remarks in the title to Fig. 1 apply here.

The dilution increases the resolution of the peaks by decreasing the widths of the lines. All principal lines predicted by theory[3,4] for the three orientations are resolved. In addition, the dilution decreases the crystalline field splitting. Table I shows the magnetic field strengths of the resonances as observed and as calculated from the theory using the g-values and the zero field splittings shown there. The discrepancies are probably caused by the interference between overlapping resonances.

If the width of the lines were attributed to spin-spin interaction alone the peak absorptions of the lines would be unaffected by the dilution. The measurements show a marked diminution of the peaks with dilution. The ratio of the magnitudes of the peaks vary with the orientation of the d.c. field. This effect is most marked in the (100) orientation, where the ratio can be seen to be about 30. This factor cannot be accounted for by overlapping of the lines, by volume differences of the crystals used, or by changes in the Q of the cavity on account of dielectric absorption. No saturation effects were observed at the power level used. This anomaly is being investigated further.

A comparison of the ammonium and the potassium salts shows that the splitting is smaller in the potassium than

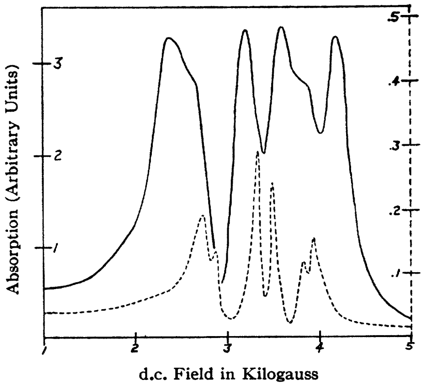

FIG. 3. Relative absorption of ammonium chrome alum with the d.c. field perpendicular to a (110) face. The remarks in the title to Fig. 1 apply here.

in the ammonium salt This applies to both the undiluted and the diluted salts. The splitting $\delta = 0.13$ cm^{-1} for the undilute potassium salt ($\delta = 0.091$ for the diluted salt) is somewhat less than the value quoted above for the corresponding ammonium salt. Bagguley and Griffiths[5] report a value of $\delta = 0.12$ cm^{-1} for chrome alum. The general appearance of their curves indicates that they refer to the potassium salt. Halliday and Wheatley[6] obtained values of 0.12 cm^{-1} for potassium chrome alum and 0.13 cm^{-1} for ammonium chrome alum. The difference between their values and ours for the undiluted salts appears to be greater than the experimental error in measuring the peak positions. It appears that the interference effects mentioned above shift the positions of the maxima in a different way than do the interference effects in the method used by Halliday[6] and by Griffiths.[5]

A detailed report of this investigation will be given in a forthcoming paper.

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[1] P. R. Weiss, C. A. Whitmer, H. C. Torrey, and Jen-Sen Hsiang, Phys. Rev. **72**, 975 (1947).

[2] R. V. Pound, Rev. Sci. Inst. **17**, 490 (1946).

[3] P. R. Weiss, Phys. Rev. **73**, 470 (1948).

[4] C. Kittel and J. M. Luttinger, Phys. Rev. **73**, 162 (1948).

[5] D. M. S. Bagguley and J. H. E. Griffiths, Nature **160**, 532 (1947).

[6] D. Halliday and J. Wheatley, Bull. Am. Phys. Soc. **23**, No. 3, 13 (1948).

Violet Asymmetry of Potassium Resonance Lines under High Rubidium Vapor Pressure

Ch'en Shang-yi
Institute of Physics, National Academy of Peiping, Peiping, China

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It is not surprising at all to conceive that alkali vapors of higher vapor pressure can be used as perturbing atoms, as foreign gases do, on the absorption lines of other alkalis of relatively lower vapor pressure. Because of experimental difficulties, however, experimental research has very seldom been accomplished.

The chief difficulty lies first in the fact that before the discovery of the corrosion-resistant optical windows, it was impossible to get an absorption tube of an alkali vapor in high pressure. Secondly, when the alkali vapor pressure is high, not only its own absorption lines will be extensively broadened but also its molecular absorption bands will be added to the spectrum. Thus the spectral lines of the alkali to be studied will be badly obscured. Füchtbauer and Heimann[1] studied the effects of caesium vapor on the high series lines of sodium by mixing Cs in the discharge lamp of Na. Their pressure was very, very low.

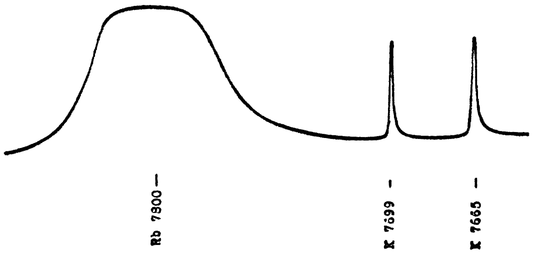

FIG. 1. K resonance lines and the λ7800 of Rb at 426 C. (Rb vapor pressure 19.86 mm; tube length 2 mm.)

If the corrosion resistant MgO windows[2] are used to construct an absorption tube of only 2-mm length,[3] the width of the absorption lines can be considerably narrowed. When the absorption tube of Rb with a trace of K is heated to temperatures as high as 556 C the resonance lines of Rb, $\lambda\lambda$7800.29 and 7947.64, with a separation of 147A, are so much broadened that, even when the 2-mm tube is used, they begin to overlap. The pressure of Rb vapor under that temperature would be not less than 150 mm if one extrapolated from the empirical formula of temperature *vs.* Rb vapor density.[4] Fortunately, the region for the $\lambda\lambda$7664.94, 7699.01 lines of potassium presented as impurity in the Rb metal are not yet masked, rendering observations feasible.

The microphotometer curve shown in Fig. 1 shows clearly the violet asymmetry of the K lines and the broadened Rb 7800.29. The asymmetry becomes appreciable when the vapor pressure of Rb amounts to about 15 mm or up for the 2-mm tube. When the tube length is increased to 7.5 cm, the asymmetry appears at a Rb vapor pressure of about 2 mm. It is also to be noted that the K impurity lines are narrower than they ought to be if the absorption lines of pure potassium[5] are taken. There is no measurable shift.

This sort of research not only can give more extended data on the investigation of the pressure effects on spectral lines, but also suggests a way to investigate quantitatively the variation of the vapor pressure and the line intensities of one element in the presence of other element or elements. By the measurement of the area under the line contour in terms of the absolute absorption coefficients, it should be possible to estimate the concentration of the absorbing atoms in the absorption column.

[1] Füchtbauer and Heimann, Zeits. f. Physik **110**, 8 (1938).

[2] J. Strong and R. T. Brice, J. Opt. Soc. Am. **25**, 207 (1935).

[3] Ch'en Shang-yi, Phys. Rev. **58**, 884 (1940).

[4] T. J. Killian, Phys. Rev. **27**, 578 (1926).

[5] D. S. Hughes and P. E. Lloyd, Phys. Rev. **52**, 1215 (1937).

Production of New Tc Activities from Separated Mo Isotopes*

E. E. Motta and G. E. Boyd
Oak Ridge National Laboratory, Oak Ridge, Tennessee, and Department of Terrestrial Magnetism, Carnegie Institution, Washington, D. C.

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During the past 10 months, a series of 35 bombardments with 15–20 Mev deuterons on the different enriched Mo isotopes has yielded several new Tc activities (element 43). In addition, data on previously uncertain activities have been obtained. In each case the Tc was separated chemically from the bombarded Mo isotopes by means of a highly efficient volatility separation method developed in our chemical studies on the element, and where necessary, the Tc was prepared as a thin sample for counting by co-precipitation with platinum sulfide. All counting was done with mica end-window, bell-type